Spectral Separation of Two Unresolved White Dwarf Binary Populations with LISA

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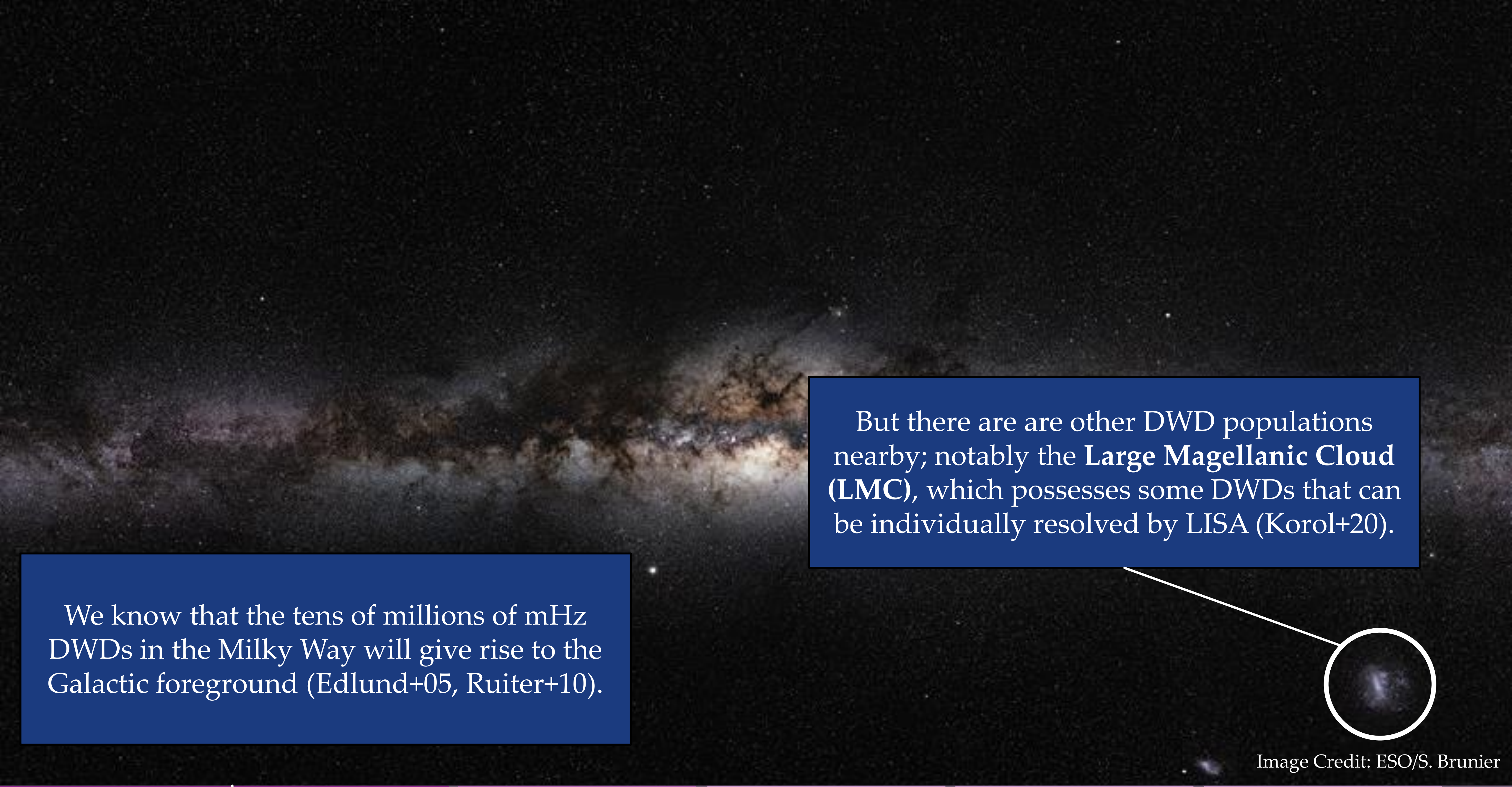
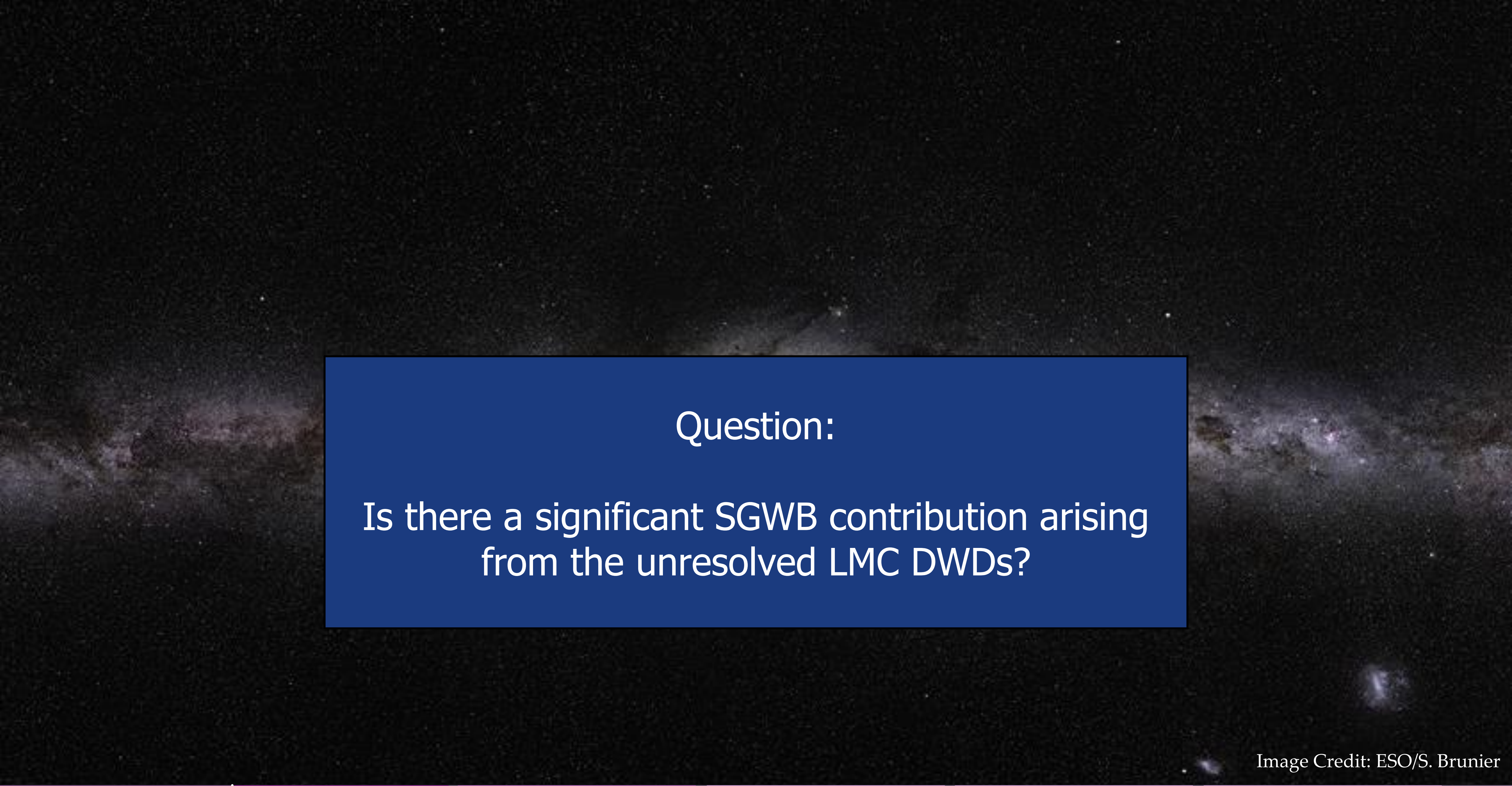
Nov. 7th, 2024 | LISA AstroWG Meeting (MPA/Online)



We know that the tens of millions of mHz DWDs in the Milky Way will give rise to the Galactic foreground (Edlund+05, Ruiter+10).

But there are are other DWD populations nearby; notably the **Large Magellanic Cloud (LMC)**, which possesses some DWDs that can be individually resolved by LISA (Korol+20).

Image Credit: ESO/S. Brunier



Question:

Is there a significant SGWB contribution arising from the unresolved LMC DWDs?

Image Credit: ESO/S. Brunier

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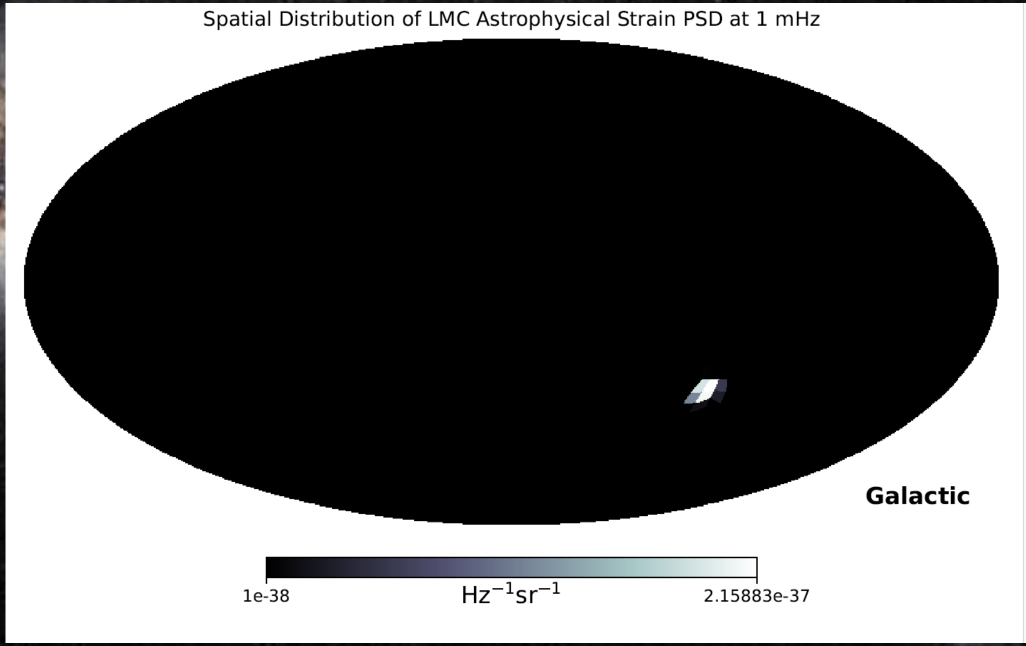
The LMC SGWB

Steven Rieck

Worked with UMN undergraduate (now PhD student @ University of Cincinatti) Steven Rieck, using the DWD population synthesis catalogue of Keim+22, to **establish for the first time** that:

Yes! There will be a significant SGWB in LISA from the LMC DWDs!

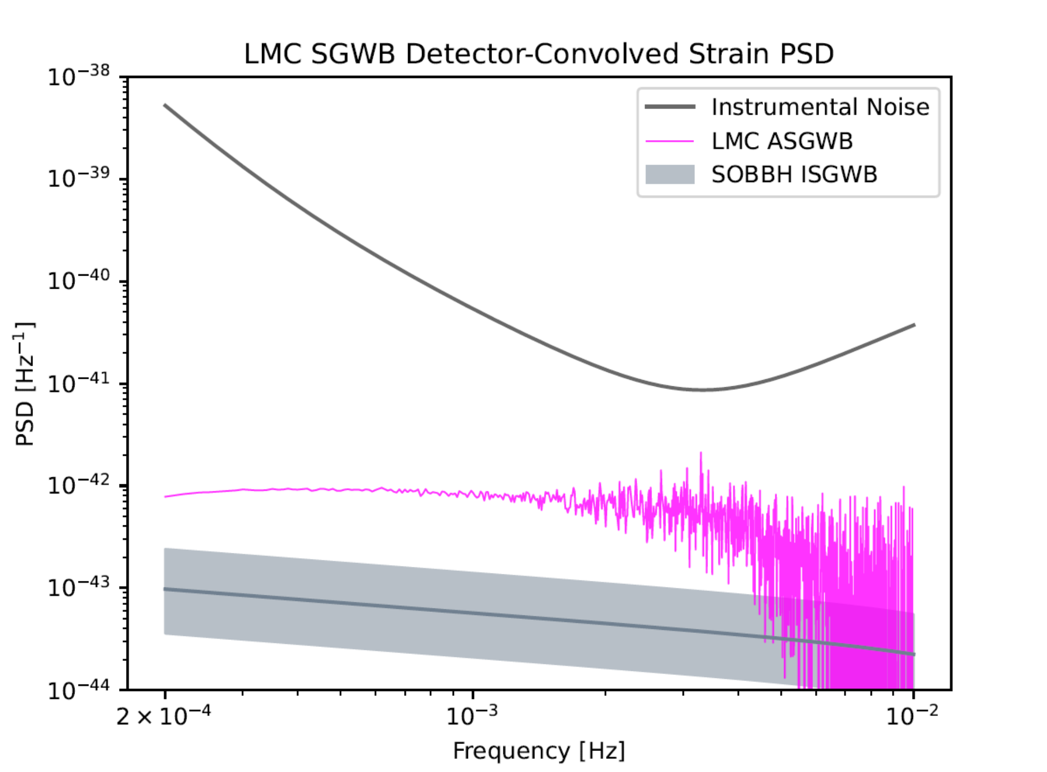


Rieck/AWC+24 (joint first authorship)

Image Credit: ESO/S. Brunier

The LMC SGWB

Not only that, but the amplitude of the LMC SGWB will be **greater than what is expected for the stellar-origin black hole binary SGWB**



LISA SOBBH SGWB as inferred from GWTC-3 per Babak+23

Image Credit: ESO/S. Brunier

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The LMC SGWB

Properly characterizing the LMC SGWB will likely be crucial for SGWB science with LISA – and gives us another angle with which to learn about this satellite of our Galaxy!

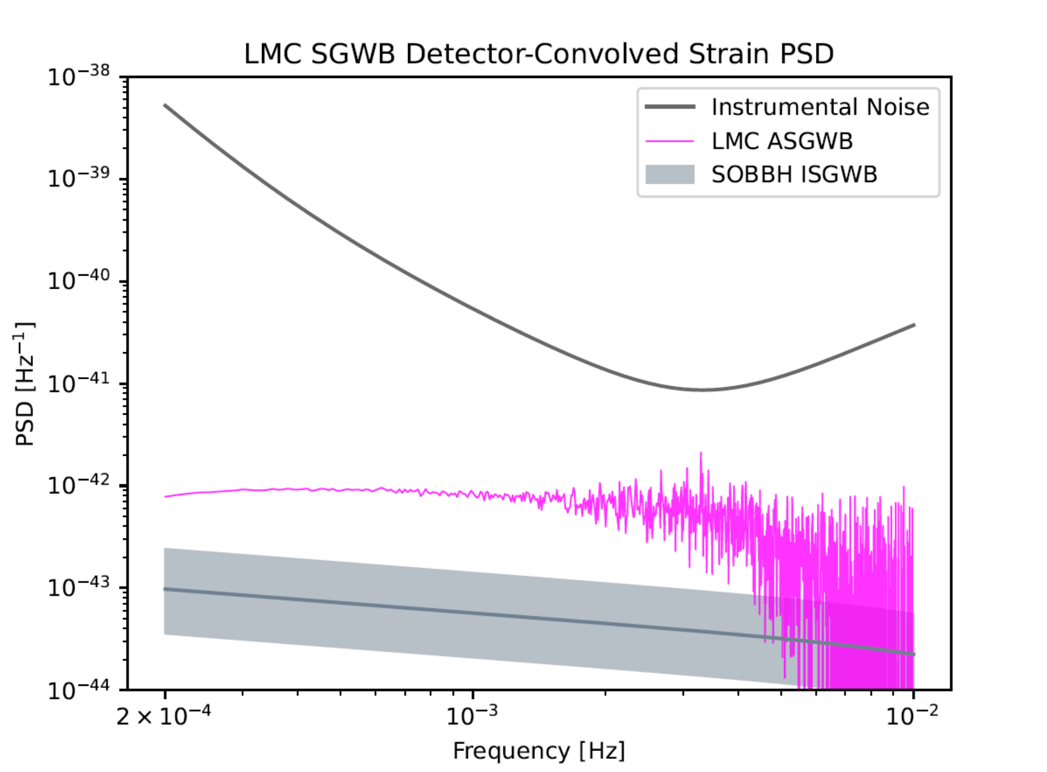


Image Credit: ESO/S. Brunier

The LMC SGWB

Of course, this depends on being able to separate it from the similar Galactic foreground from the Milky Way's DWD population…

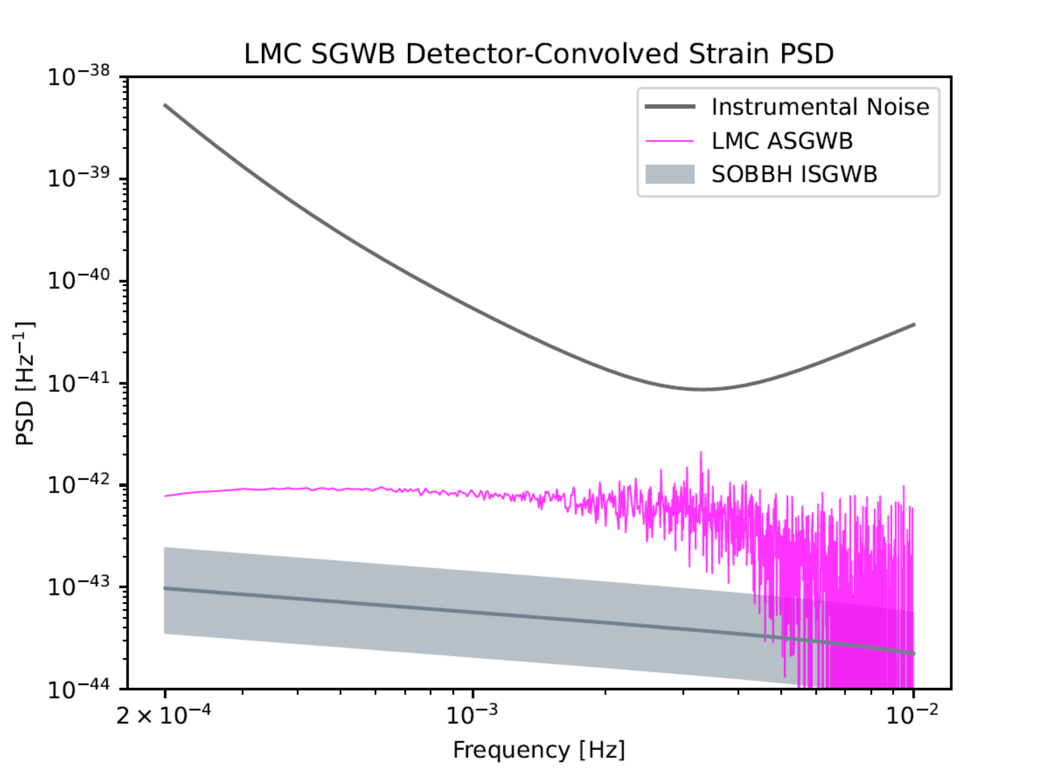


Image Credit: ESO/S. Brunier

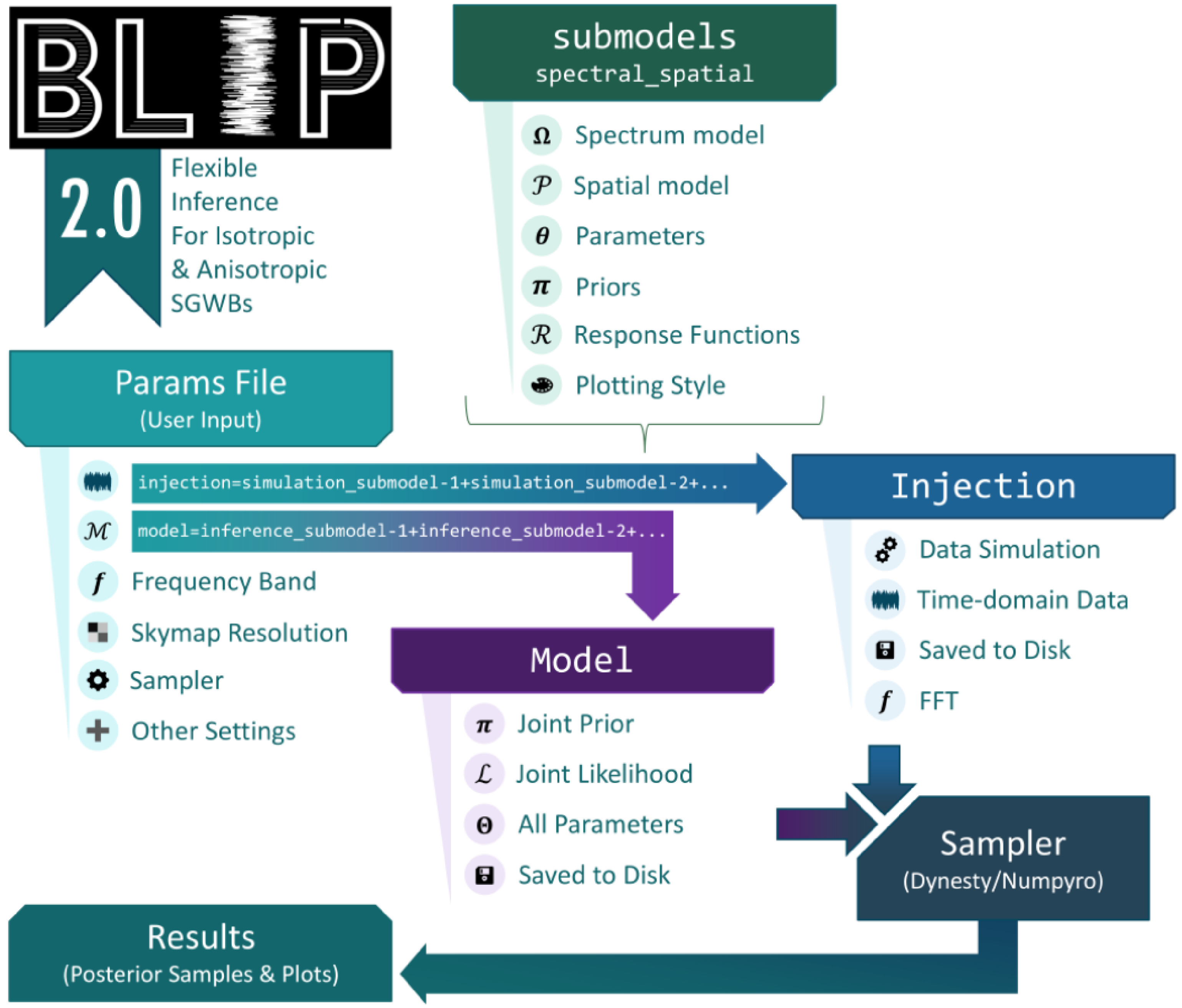
The Bayesian LISA Inference Package (BLIP)

A Python package for simulation and Bayesian inference of isotropic and anisotropic SGWBs in LISA

v1.0: Banagiri, AWC+21; v2.0: AWC+ (in prep)

BLIP 2.0

- Parallelization
- GPU-accelerated sampling
- Dozens of quality-of-life features
- New models
- Pixel basis analyses
- Modular Structure
  - submodel=spectral+spatial
- Simultaneous Inference with arbitrary number and combination of SGWBs



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Simultaneous Inference in BLIP 2.0

LISA Instrumental Noise + Anisotropic Galactic Foreground + Anisotropic LMC SGWB

Simultaneous Inference in BLIP 2.0

```
model=noise+mwspec_mwtemplate+lmcspec_lmctemplate
```

Simultaneous Inference in BLIP 2.0

```
model=noise+mwspec_mwtemplate+lmcspec_lmctemplate
```

```
spectral_spatial
```

spectral_

`model=noise+mwspec_mwtemplate+lmcspec_lmctemplate`

mwspec – a tanh-truncated power law with astrophysical priors

→ Has a fixed low-frequency slope and priors such that:

$$\Omega_{\rm GW}(f) = \frac{1}{2}\,\Omega_{\rm ref}\left(\frac{f}{f_{\rm ref}}\right)^{\alpha}\left(1 + \tanh\left(\frac{f_{\rm cut} - f}{f_{\rm scale}}\right)\right)$$

$$\pi(\log\Omega_{\rm ref}) = \mathcal{U}(-6,-4)$$
$$\pi(\log_{10} f_{\rm cut}) = \mathcal{U}(-3.1,-2.4)$$
$$\pi(\log_{10} f_{\rm scale}) = \mathcal{U}(-4,-2)$$

$$\alpha = 2/3$$

spectral_

`model=noise+mwspec_mwtemplate+lmcspec_lmctemplate`

lmcspec – a broken power law with astrophysical priors

→ Has a fixed low-frequency slope and priors such that:

$$\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{ref}} \left(\frac{f}{f_{\mathrm{ref}}}\right)^{\alpha_1} \left(1 + \left(\frac{f}{f_{\mathrm{break}}}\right)^{\frac{1}{\delta}}\right)^{\delta(\alpha_1 - \alpha_2)}$$

$$\pi(\log_{10}\Omega_{\mathrm{ref}}) = \mathcal{U}(-12, -8)$$
$$\pi(\alpha_2) = \mathcal{U}(2/3, 8/3)$$
$$\pi(\log_{10} f_{\mathrm{break}}) = \mathcal{U}(-3, -2)$$
$$\pi(\delta) = \mathcal{U}(0.01, 1)$$

$$\alpha_1 = 2/3$$

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noise

model=**noise**+mwspec_mwtemplate+lmcspec_lmctemplate

$$S_p(f) = N_p \left[1 + \left(\frac{2\,\mathrm{mHz}}{f}\right)^4\right] \mathrm{Hz}^{-1},$$

$$S_a(f) = \left[1 + \left(\frac{0.4\,\mathrm{mHz}}{f}\right)^2\right]\left[1 + \left(\frac{f}{8\,\mathrm{mHz}}\right)^4\right] \times \frac{N_a}{(2\pi f)^4} \mathrm{Hz}^{-1}$$

LISA Instrumental Noise spectral form as given in Amaro-Seone+17 and reproduced in Banagiri, AWC+21

Note: this is a **simple** model of the LISA instrumental noise. While it is commonly used in the literature at present, more complex treatments are in development and will need to be applied in the future. (See e.g. Littenberg+23, Hartwig23, Bayle+23, Novara+24)

_spatial: Fixed Anisotropic Templates

Anisotropies change how LISA responds to a SGWB as it moves through its orbit.

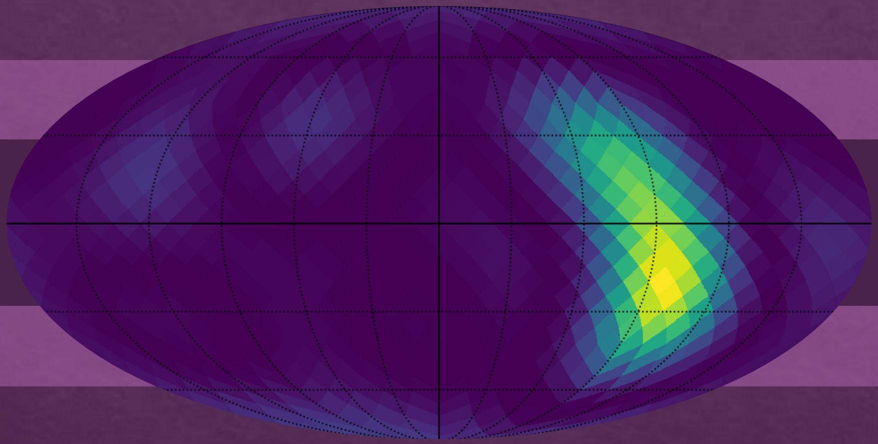
Different anisotropies → Different time-dependencies

(isotropy → no time-dependence)

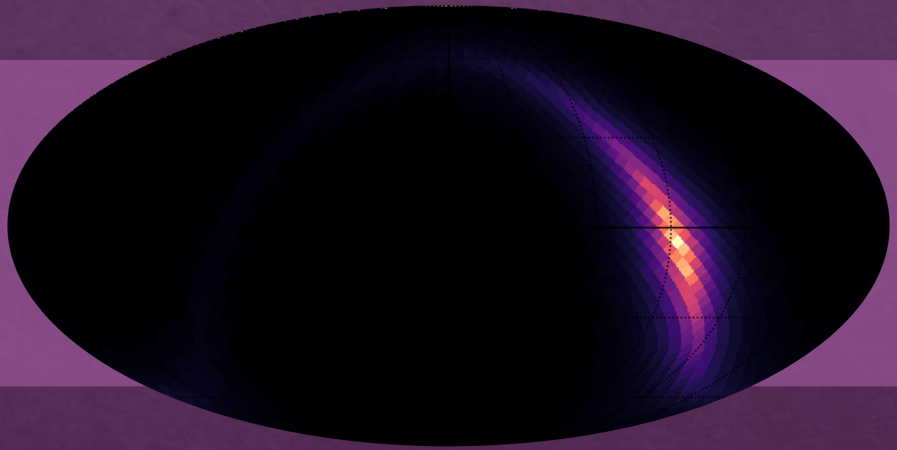
Healpix ecliptic projection

_spatial: Fixed Anisotropic Templates

Spherical Harmonic Basis Recovery



Pixel Basis Template



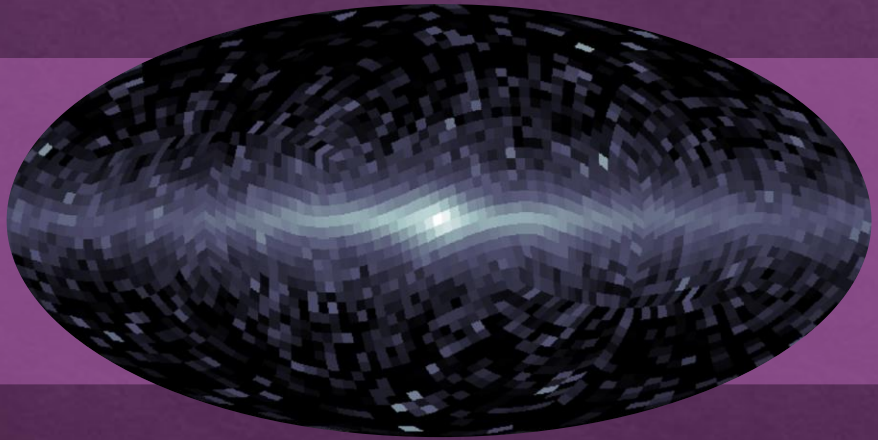
Instead of representing the SGWB spatial distribution on the sky in a spherical harmonic expansion, we do so with a **pixelated skymap** via Healpix (Gorski+05)

AWC+(submitted to PRD)

Healpix ecliptic projection

_spatial: Fixed Anisotropic Templates

Milky Way
(mwtemplate)



Large Magellanic Cloud
(lmctemplate)

Using MW and LMC population synthesis catalogues, we can create **population-derived anisotropic templates** for both the MW and LMC!

Healpix galactic projection

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Spectral Separation of the LMC SGWB and a Realistic Galactic Foreground

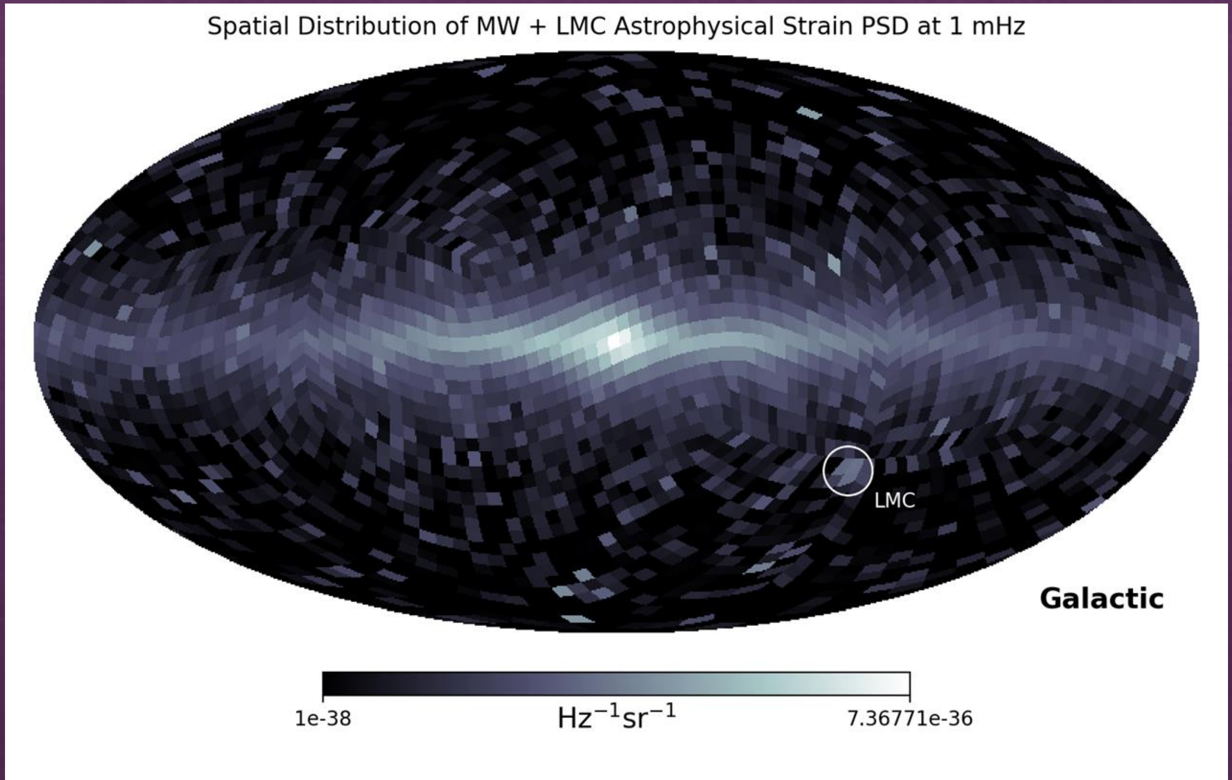
AWC+ in-prep)

Spectral Separation of the LMC SGWB and a Realistic Galactic Foreground

Simulation

Galaxy: realistic MW DWD population (Wilhelm+20), with SNR>7 binaries removed

LMC: realistic LMC DWD population (Keim+22), with SNR>7 binaries removed



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Spectral Separation of the LMC SGWB and a Realistic Galactic Foreground

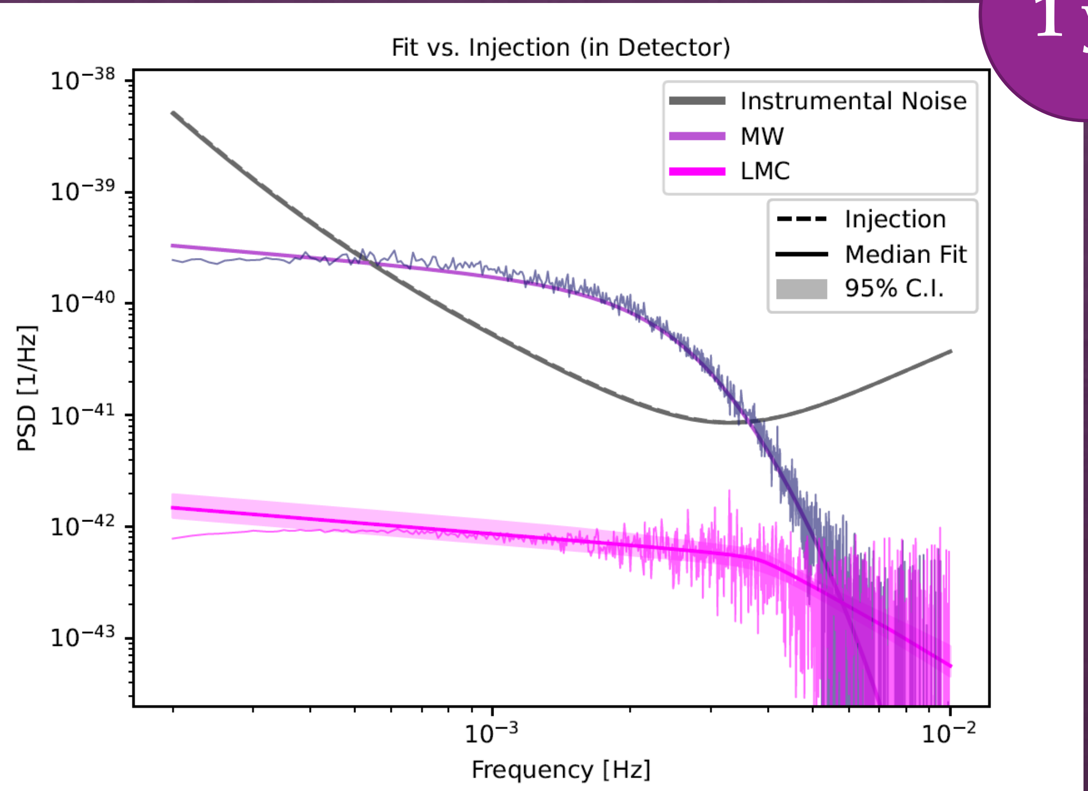
Simulation

Galaxy: realistic MW DWD ...tion (Wilhelm+20), with ...aries removed

Analysis

```
model=
 noise
+mwspec_mwtemplate
+lmcspec_lmctemplate
```

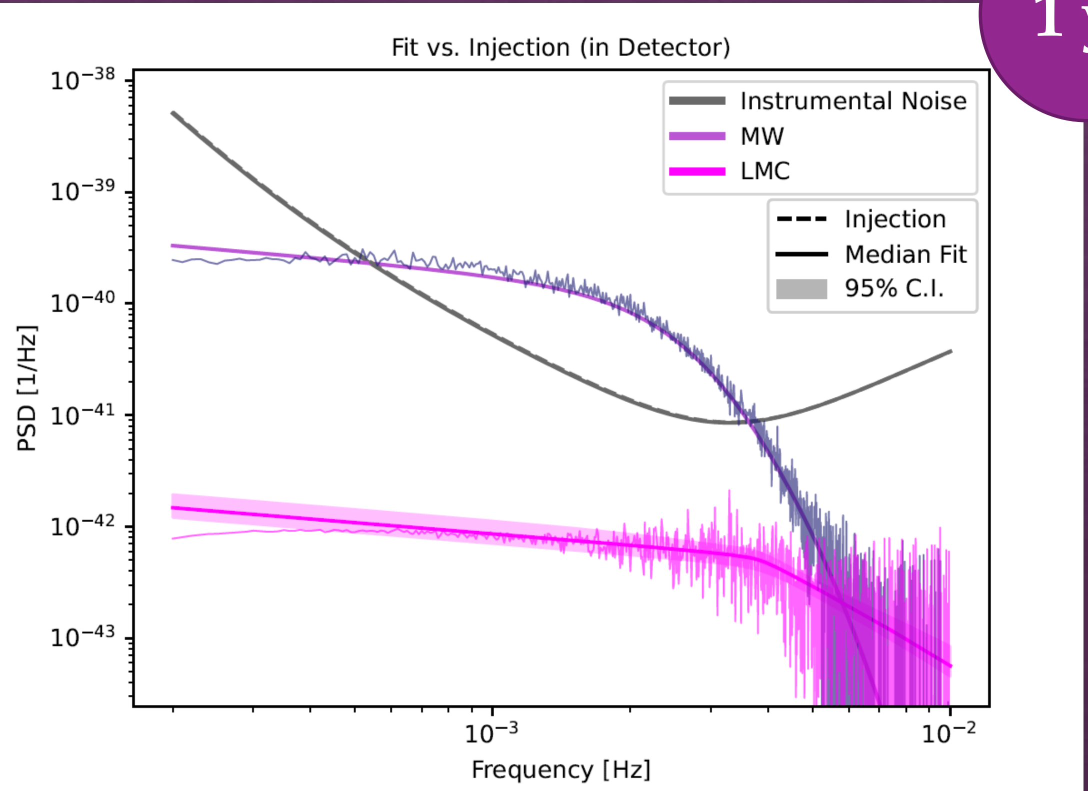
1 yr



Spectral Separation of the LMC SGWB and a Realistic Galactic Foreground

1 yr

This is the **first** demonstration of LISA's potential to perform spectral separation between two unresolved DWD populations' ASGWBs.



Thank you!

Summary

- The **Large Magellanic Cloud** (LMC) **will produce a significant astrophysical SGWB** in LISA
- **BLIP 2.0** is capable of performing flexible inference on any combination of isotropic and anisotropic SGWB in LISA
- **Templated anisotropic analyses** are powerful tools for **spectral separation**
- I have demonstrated a prototype analysis that can **separate** the LMC SGWB from the Galactic foreground
- This is the first demonstration of spectral separation between two unresolved white dwarf binary populations in LISA

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Acknowledgements: This material is based in part upon work supported by the National Science Foundation under Grant 1922512. The author acknowledges support by the LISA Preparatory Science Program under NASA grant 90NSSC19K0318. This work utilized computing resources provided by the Minnesota Supercomputing Institute (MSI) at the University of Minnesota.

Extra Slides

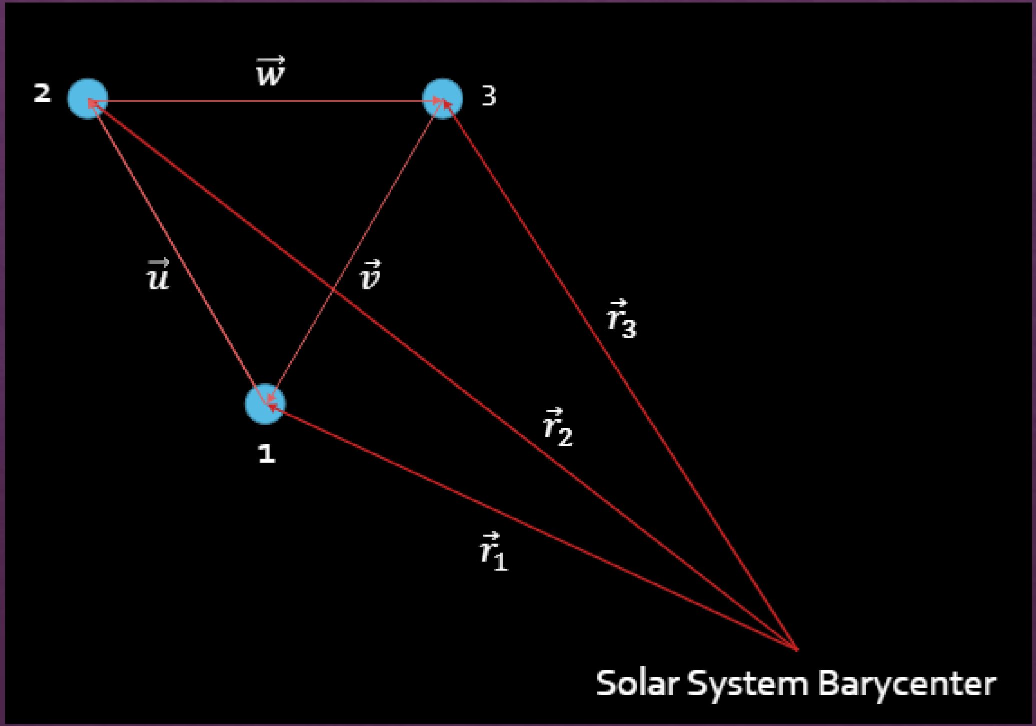
Time-dependent LISA Response Functions

$$D_I(f,t,\hat{n}) = \frac{1}{2}\left((\hat{u}\otimes\hat{u})\mathcal{T}(f,\hat{u}\cdot\hat{n}) - (\hat{v}\otimes\hat{v})\mathcal{T}(f,\hat{v}\cdot\hat{n})\right)\exp\left(2\pi i f\frac{\hat{n}\cdot\vec{r}_1}{c}\right),$$

$$\mathcal{T}(f,\hat{u}\cdot\hat{n}) = \frac{1}{2}\left[\operatorname{sinc}\left(\frac{f}{2f_*}(1-\hat{n}\cdot\hat{u})\right)\exp\left(-i\frac{f}{2f_*}(3+\hat{n}\cdot\hat{u})\right) + \operatorname{sinc}\left(\frac{f}{2f_*}(1+\hat{n}\cdot\hat{u})\right)\exp\left(-i\frac{f}{2f_*}(1+\hat{n}\cdot\hat{u})\right)\right],$$

$$F_I^A(f,t,\hat{n}) = D_I(f,t,\hat{n}) : e^A(\hat{n}).$$

$$\mathcal{R}_{IJ}(t,f|\vec{\theta}_S) = \frac{1}{2}\int \mathcal{P}(\hat{n}|\vec{\theta}_S)\left(\sum_A F_I^A(f,t,\hat{n})F_J^{A*}(f,t,\hat{n})\right)d^2n.$$



Cornish & Larson 10, Romano & Cornish 17

The BLIP Spherical Harmonic ASGWB Search

Infers the coefficients of a spherical harmonic expansion of the **square root of the power on the sky**

→ Mathematically ensures that the inferred power will be **real and non-negative in every direction.**

Sharan Banagiri

Post-doc at Monash

Note: this is a generic search, well-suited to ASGWBs where the spatial distribution is not known *a priori*

The BLIP Spherical Harmonic ASGWB Search

$$\Omega_{\rm GW}(f,\mathbf{n}) = \Omega(f)\mathcal{P}(\mathbf{n}).$$

Assume frequency and spatial dependence are separable (expected!)

Sharan Banagiri

Post-doc at Northwestern

The BLIP Spherical Harmonic ASGWB Search

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Assume frequency and spatial dependence are separable (expected!)

$$\mathcal{P}(\mathbf{n}) = \frac{1}{\sqrt{4\pi a_{0,0}}} \sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\mathbf{n}),$$

Standard spherical harmonic expansion of GW power on the sky. But $\mathcal{P}(\boldsymbol{n})$ must be non-negative everywhere on the sky…

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Applying this condition while sampling is **computationally expensive** and **ultimately ineffective**!

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Applying this condition while sampling is **computationally expensive** and **ultimately ineffective**!

$$\mathcal{S}(\mathbf{n}) = \left[\sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\mathbf{n})\right]^{1/2} = \sum_{\ell,m} b_{\ell,m} Y_{\ell,m}(\mathbf{n}).$$

Defining $\mathcal{S}(\boldsymbol{n})$ as the square root of the spherical harmonic expansion fulfills this condition as long as $\mathcal{S}(\boldsymbol{n})$ is real – or, equivalently, $b_{l,-m} = (-1)^m b^*_{l,m}$

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$$\sum_{L,M} a_{L,M} Y_{L,M} = \left(\sum_{\ell,m} b_{\ell,m} Y_{\ell,m}(\mathbf{n})\right)^2$$

We can then infer each $b_{l,m}$ up to some desired $l^b_{\rm max} = ½ l^a_{\rm max}$, quickly liaising between our $b_{l,m}$ parameterization and the power on the sky in the $a_{l,m}$s via **Clebsch-Gordon coefficients**

Clebsch-Gordon Expansion

$$\sum_{L,M} a_{L,M} Y_{L,M} = \left(\sum_{\ell,m} b_{\ell,m} Y_{\ell,m}(\mathbf{n}) \right)^2 .$$

$$\sum_{L,M} a_{L,M} Y_{L,M} = \sum_{\ell,m} \sum_{\ell',m'} b_{\ell,m} b_{\ell',m'} Y_{\ell,m}(\mathbf{n}) Y_{\ell',m'}(\mathbf{n}).$$

$$Y_{\ell,m}(\mathbf{n}) Y_{\ell',m'}(\mathbf{n}) = \sum_{L=L_{\min}}^{L_{\max}} \sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi(2L+1)}} \times C^{LM}_{\ell m,\ell' m'} C^{L0}_{\ell 0,\ell' 0} Y_{L,M}(\mathbf{n}).$$

$$\sum_{L,M} a_{L,M} Y_{L,M}(\mathbf{n}) = \sum_{L,M} \left(\sum_{\ell m} \sum_{\ell' m'} b_{\ell,m} b_{\ell',m'} \beta^{\ell m,\ell' m'}_{L,M} \right) \times Y_{L,M}(\mathbf{n}).$$

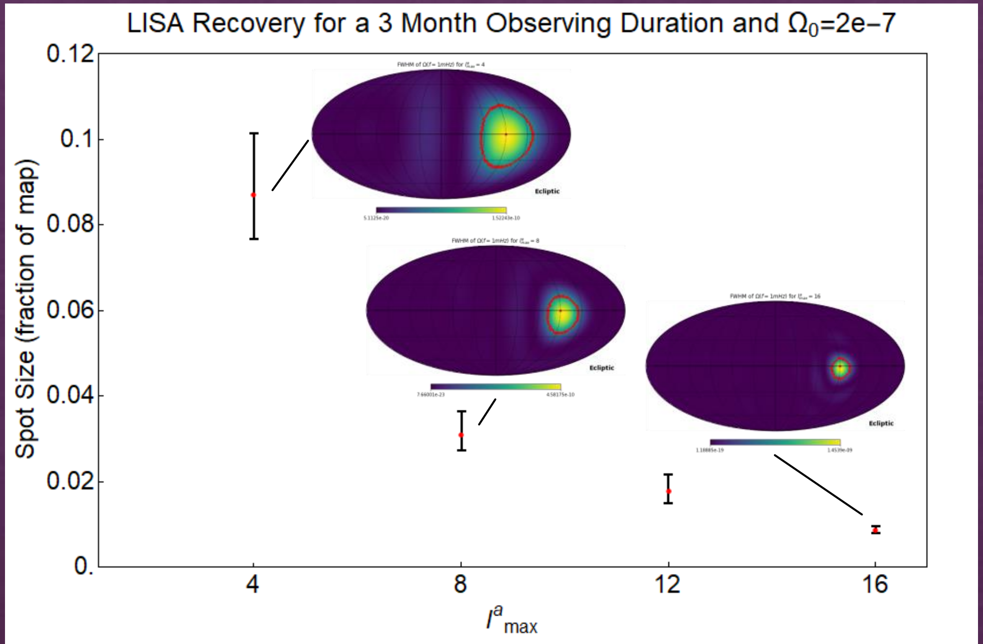
- $M = m + m'$
- $L_{\min} = \min(|\ell - \ell'|, |m + m'|)$ and $L_{\max} = \ell + \ell'$
- L is an integer

For compactness, let us define $\beta^{L,M}_{\ell m,\ell' m'}$ such that:

$$\beta^{L,M}_{\ell m,\ell' m'} = \sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi(2L+1)}} C^{LM}_{\ell m,\ell' m'} C^{L0}_{\ell 0,\ell' 0}, \quad (3.7)$$

when the selection rules are satisfied, but $\beta^{L,M}_{\ell m,\ell' m'} = 0$ otherwise.

Angular Resolution of the BLIP Spherical Harmonic ASGWB Analysis



Highest angular resolution achieved **exceeded the resolution of other, non-Bayesian ASGWB analyses** for similar signals ($\ell^a_{\max} = 16$ vs. $\ell^a_{\max} \leq 15$ for Contaldi+20*).

* Also surpassing Kudoh+05, Taruya+05, Breivik+20.

Bloom, AWC+(in-prep)

Original BLIP Likelihood

One SGWB signal + LISA instrumental noise

$$\mathcal{L} = \prod_{t,f} \frac{1}{2\pi T_{\rm seg} \det\left(C^M_{IJ}(t,f|\vec{\theta})\right)} \exp\left[-\frac{2}{T_{\rm seg}} \sum_{I,J} \tilde{d}^*_I(t,f) \left(C^M_{IJ}(t,f|\vec{\theta})\right)^{-1} \tilde{d}_J(t,f)\right]$$

BLIP Fourier-domain likelihood (Banagiri, AWC+21), based on the multi-dimensional complex Gaussian likelihood of Adams& Cornish 2010

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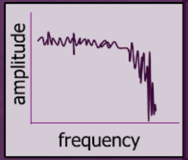
BLIP Fourier-domain likelihood (Banagiri, AWC+21), based on the multi-dimensional complex Gaussian likelihood of Adams& Cornish 2010

Modelling the Induced Covariance

$$C^M_{IJ}(t, f|\vec{\theta}) = S^n_{IJ}(f|\vec{\theta}_n) + \mathcal{R}_{IJ}(t, f|\vec{\theta}_S) S_{\mathrm{GW}}(f|\vec{\theta}_f)$$



Spectral dependence

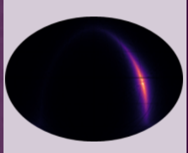
$S^n_{IJ}(f|\vec{\theta}_n)$ LISA instrumental noise PSD

$S_{\mathrm{GW}}(f|\vec{\theta}_f)$ The SGWB PSD

PSD: Power Spectral Density

Impact of Anisotropies

$$C^M_{IJ}(t, f|\vec{\theta}) = S^n_{IJ}(f|\vec{\theta}_n) + \mathcal{R}_{IJ}(t, f|\vec{\theta}_\mathcal{S}) S_{\mathrm{GW}}(f|\vec{\theta}_f)$$



Spatial dependence

$$\mathcal{R}_{IJ}(t, f|\vec{\theta}_\mathcal{S})$$

Time-dependent LISA response

$$= \int \mathcal{P}(\hat{n}|\vec{\theta}_\mathcal{S}) \mathcal{R}_{IJ}(t, f, \hat{n}) d^2 n$$

i.e., the effects of **detector geometry** convolved with the **model skymap** $\mathcal{P}(\hat{n}|\vec{\theta}_\mathcal{S})$

PSD: Power Spectral Density

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Simultaneous Inference Formalism

Arbitrary number and combination of SGWB signals + LISA instrumental noise

$$\mathcal{L}_{\mathrm{SI}} = \prod_{t,f} \left[\frac{1}{2\pi T_{\mathrm{seg}} \det\left(C^{\mathrm{SI}}_{I,J}(t,f|M_{1\ldots N};\vec{\theta}) \right)} \times \exp\left[-\frac{2}{T_{\mathrm{seg}}} \sum_{I,J} \tilde{d}^*_I(t,f) \left(C^{\mathrm{SI}}_{I,J}(t,f|M_{1\ldots N};\vec{\theta}) \right)^{-1} \tilde{d}_J(t,f) \right] \right]$$

Extension of the BLIP likelihood to multiple SGWBs: the simultaneous inference covariance C^{SI}_{IJ} will be the **sum of all component covariances.**

$$C^{\mathrm{SI}}_{I,J}(t,f|M_{1\ldots N};\vec{\theta}) = \sum_{k=1}^{N} C^{M_k}_{I,J}(t,f|\vec{\theta}_k)$$

Simultaneous Inference Formalism

Arbitrary number and combination of SGWB signals + LISA instrumental noise

$$p(\vec{\theta}|d) \propto \mathcal{L}(d|C(M_{1\ldots k};\vec{\theta}))\pi(\vec{\theta})$$

Extension of the BLIP likelihood to multiple SGWBs with respective SGWB models M_k for $k \in [1 \ldots N]$ with parameters $\vec{\theta}_k$. The total covariance will be the **sum of all model covariances:**

$$C_{\mathrm{total}}(M_{1\ldots k};\vec{\theta}) = C_{\mathrm{noise}}(\vec{\theta}_{\mathrm{noise}}) + \sum_{k=1}^{N} C_k(\vec{\theta}_k)$$

AWC+(in-prep)

An Aside on Spectral Representations

$S_{\mathrm{GW}}(f|\vec{\theta}_f)$ & $\Omega_{\mathrm{GW}}(f|\vec{\theta}_f)$ are related by

$$\Omega_{\mathrm{GW}}(f) = \frac{2\pi^2}{3H_0^2} f^3 S_{\mathrm{GW}}(f).$$

The SGWB PSD

The dimensionless GW energy density

PSD: Power Spectral Density