# CYCLOSTATIONARY PROCESSES IN LISA

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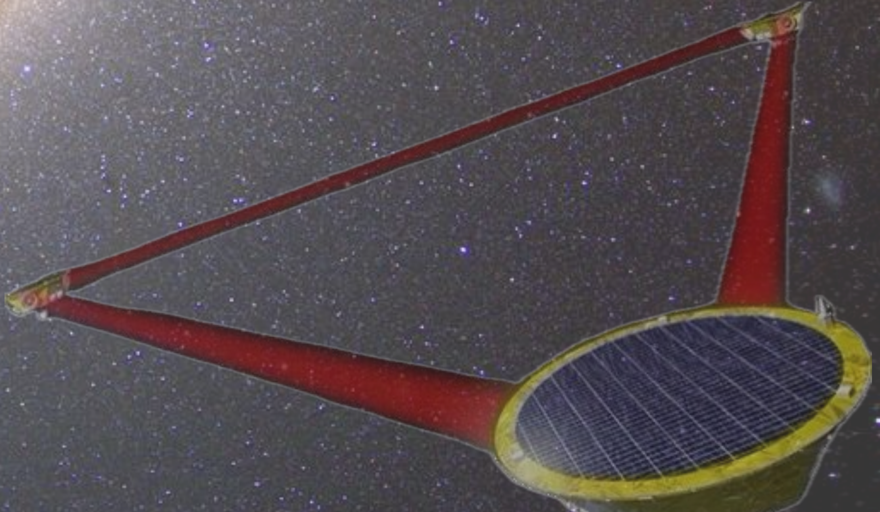
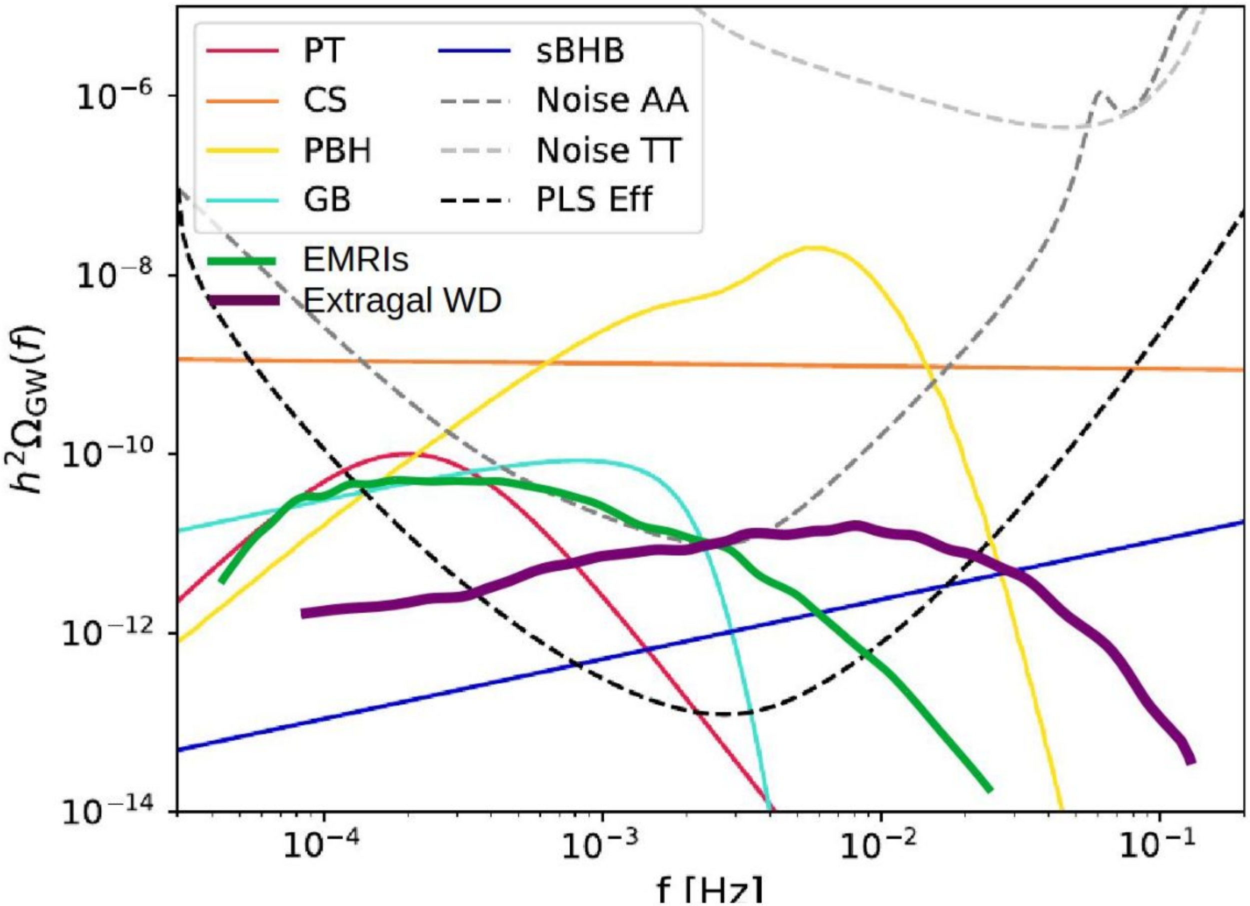
**Co-Authors**: R. Buscicchio, A. Klein, V. Korol, A. Sesana, F. Haardt
(2410.08274)

LISA AstroWG, Munich
6/11/24






Cosmo:
Caprini+24
Auclair+19
Bartolo+19

Astro:
Nelemans 09
Babak+23
Pozzoli+23
Hofman+24

# SEARCHING BACKGROUND IN LISA -CHALLENGES

NOISE SIGNAL

$$\Sigma(f, f') = \Sigma_n(f, f') + \Sigma_{\mathrm{GW}}(f, f')$$

# SEARCHING BACKGROUND IN LISA -CHALLENGES

$$\Sigma(f, f') = \Sigma_n(f, f') + \Sigma_{\rm GW}(f, f')$$

- Non-stationarity, Anisotropy, Non-Gaussianity
- Overlapping signals
- Uncertainties in the Models (both Astro&Cosmo)

# SEARCHING BACKGROUND IN LISA -CHALLENGES

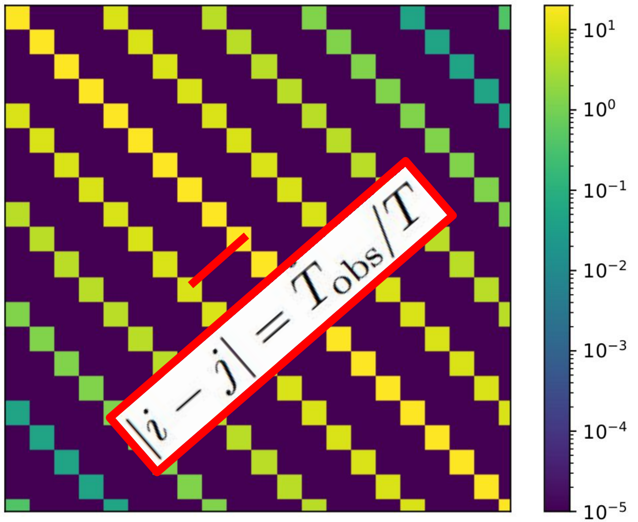
$$\Sigma(f, f') = \Sigma_n(f, f') + \Sigma_{\mathrm{GW}}(f, f')$$

TODAY

- Non-stationarity, Anisotropy, Non-Gaussianity
- Overlapping signals
- Uncertainties in the Models (both Astro&Cosmo)

# CYCLOSTATIONARY PROCESSES

Cyclostationary processes are stochastic processes whose statistical properties are periodic in time

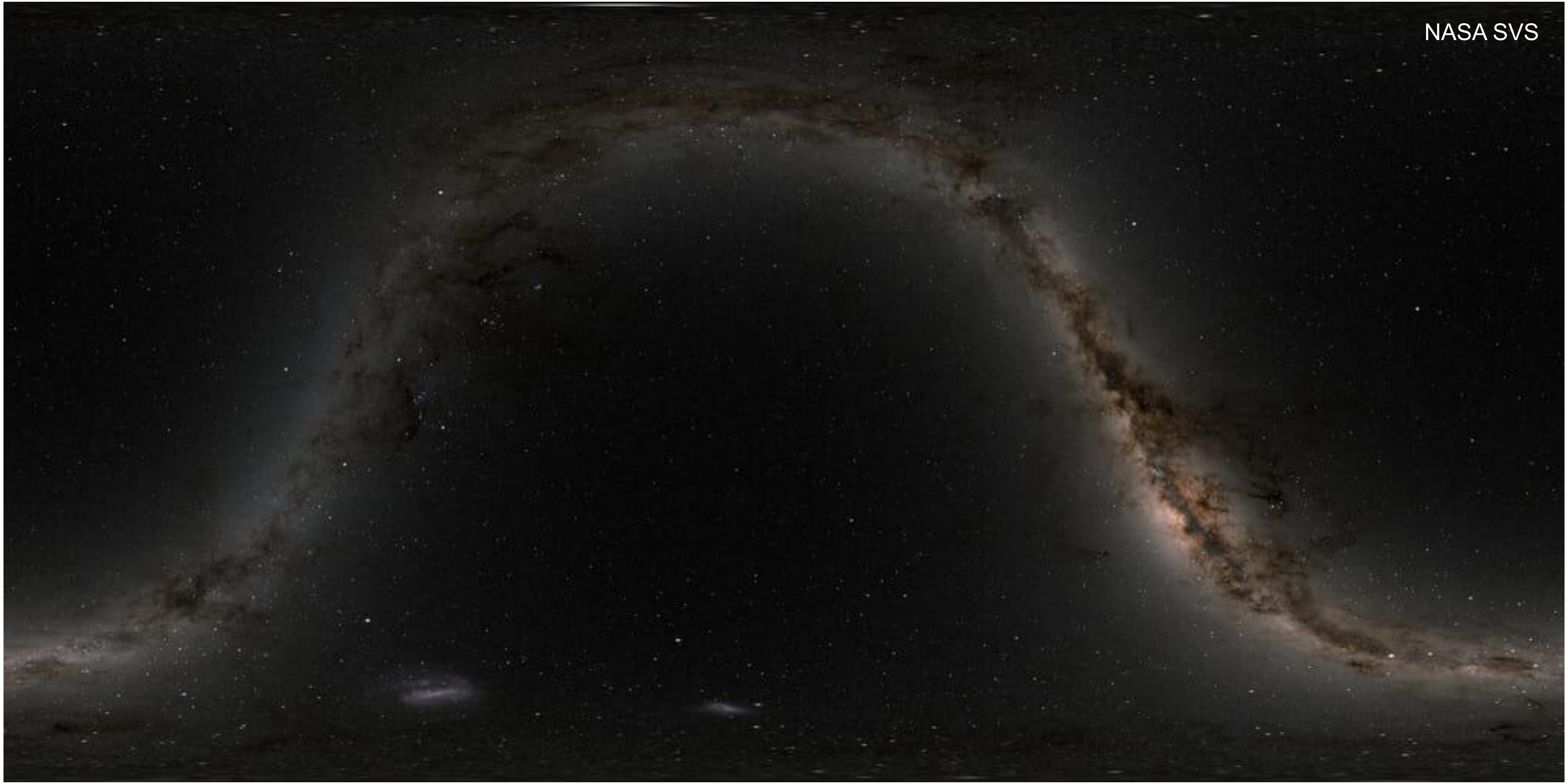



$$E\left[X(t)\right] = m(t) = m(t+T)$$
$$E\left[X(t')X(t)\right] = \Sigma(t',t) = \Sigma(t'+T, t+T)$$

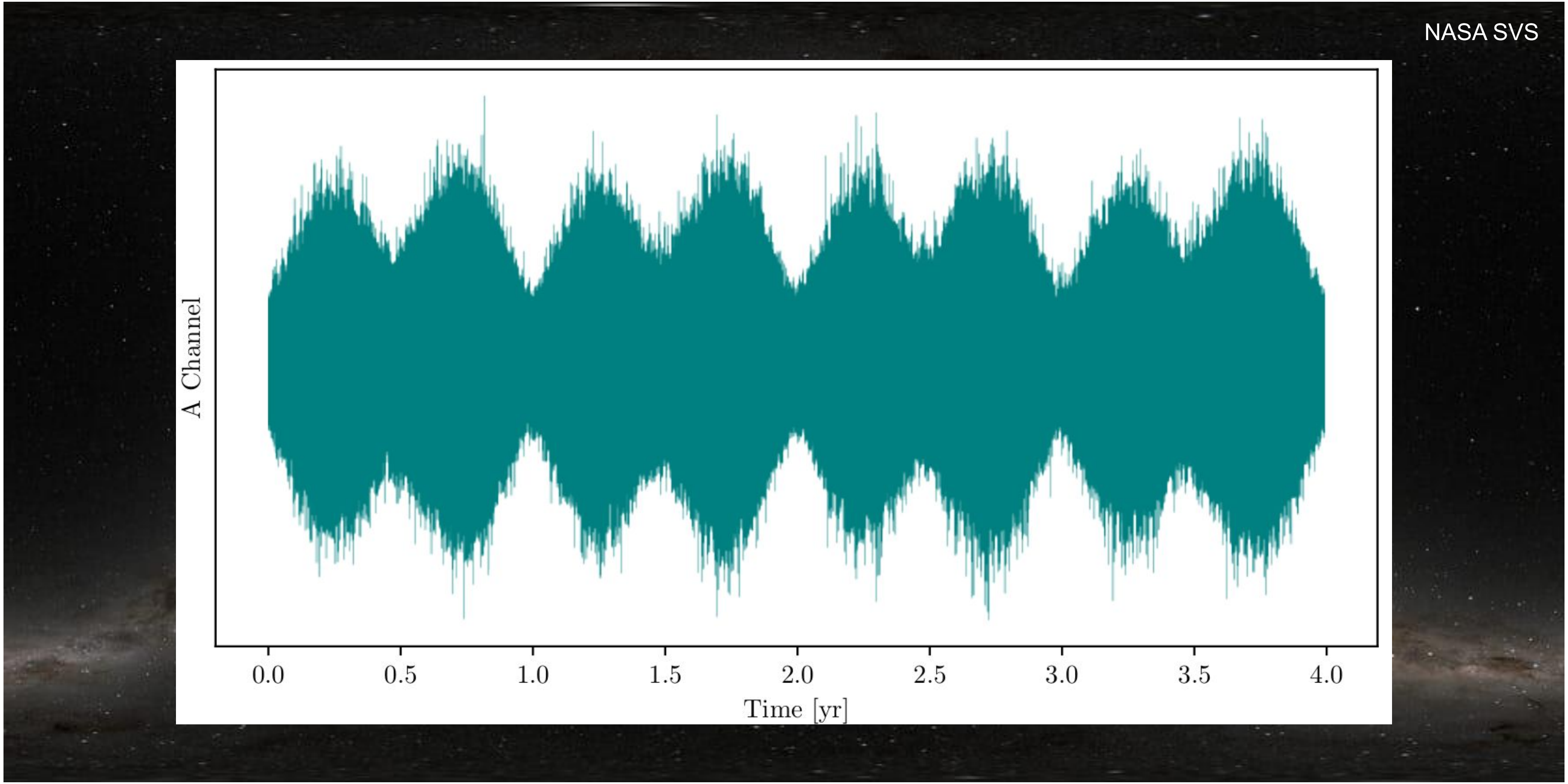
$$B(t,\tau) = \Sigma(t',t)$$
$$B(t,\tau) = \sum_{n=-\infty}^{+\infty} B_n(\tau) e^{2\pi i \frac{nt}{T}}$$

$$\longrightarrow \quad C(f,f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f'+f}{2}\right)\delta\left(f-f'+\frac{n}{T}\right)$$

# CYCLOSTATIONARITY IN LISA



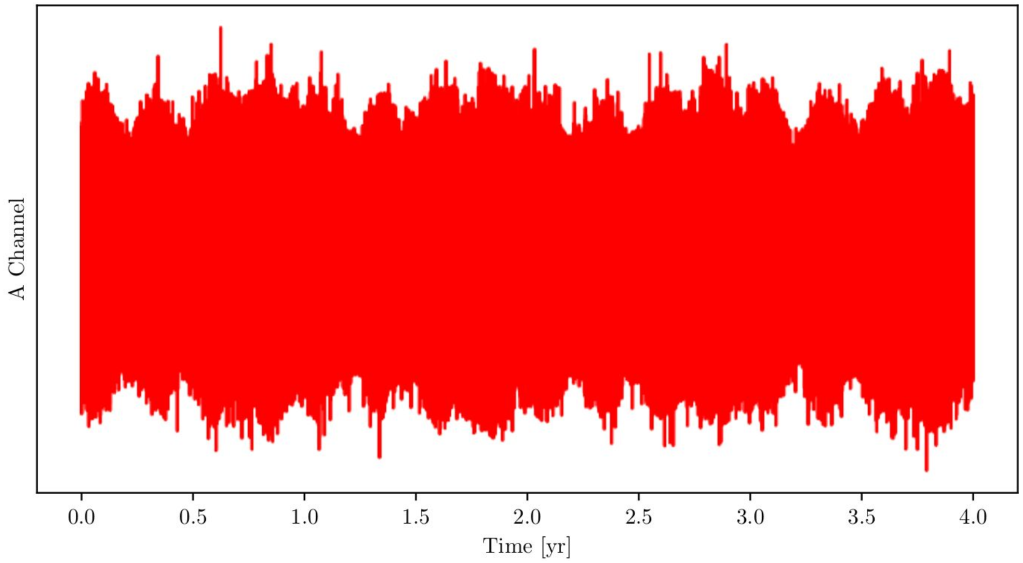
# CYCLOSTATIONARITY IN LISA



# CYCLOSTATIONARITY IN LISA

Unresolved DWDs in Milky Way Satellite (e.g., LMC, SMC, Sagittarius,...) and in nearby Galaxies (e.g., Andromeda) contribute to a SGWB

LMC



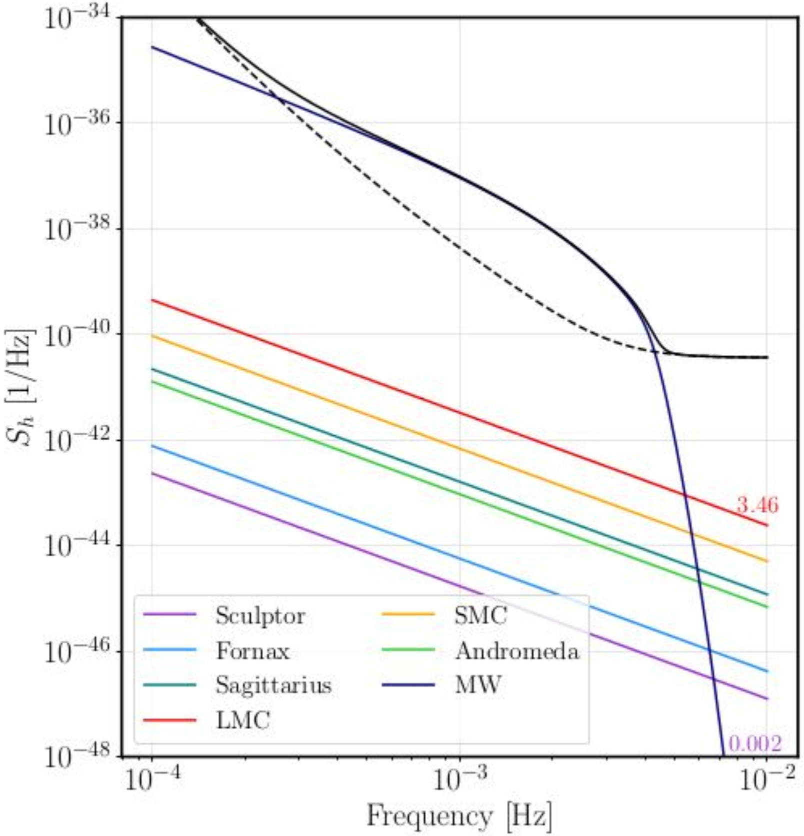
$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h \left( \frac{f' + f}{2} \right) \delta \left( f - f' + \frac{n}{T} \right)$$

ASTROPHYSICAL SPECTRUM

$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f' + f}{2}\right) \delta\left(f - f' + \frac{n}{T}\right)$$

Fourier coefficient of
MODULATION

# ASTROPHYSICAL SPECTRUM




$$C(f,f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f'+f}{2}\right)\delta\left(f - f' + \frac{n}{T}\right)$$

## Milky Way Foreground

(Karnesis+21)

$$S_h(f) = \frac{A}{2} f^{-7/3} e^{-(f/f_1)^{\alpha_{\rm MW}}}\left(1 + \tanh\left(\frac{f_{\rm knee} - f}{f_2}\right)\right)$$

## Satellite Background

Integrate Waveform Inspiral over…

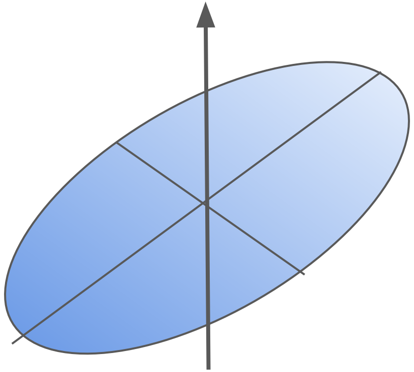
$$S_h(f) = A_{\rm sat}\left(\frac{f}{10^{-3.5}{\rm Hz}}\right)^{\gamma}$$

$$\gamma = -(9 + 3\alpha)/3$$

- Chirp mass
- Orbital Frequency
- Luminosity distance

# MODULATION

**We provide an analytical prescription to compute the modulation, relating it to the properties of the distribution!!!**

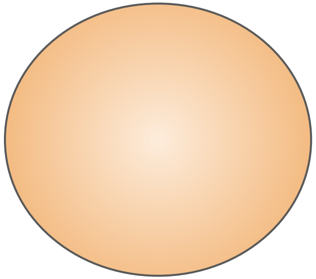


**Milky Way Modulation Parameters:**

- Center Coordinates of distribution $\lambda, \sin\beta$
- Rotation Angle $\psi$
- Gaussian Variances (Sizes of distribution) $\sigma_1, \sigma_2$

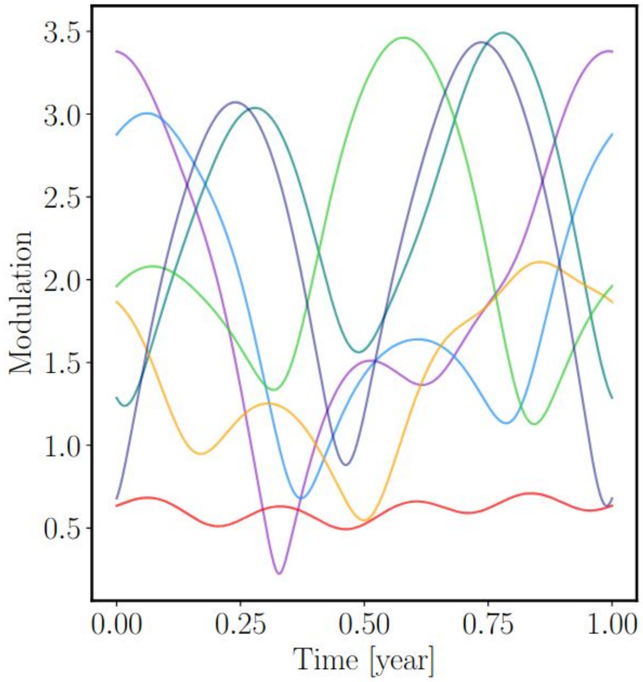
**Satellite Modulation Parameters:**

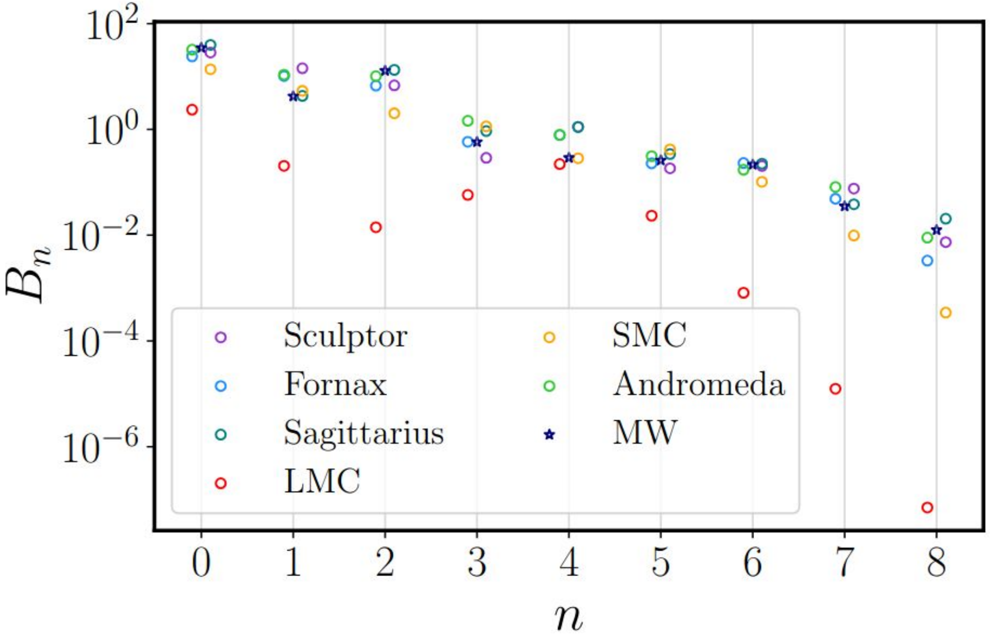
- Center Coordinates of distribution $\lambda, \sin\beta$
- Gaussian Variance (Size of distribution) $\sigma$



$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f'+f}{2}\right)\delta\left(f - f' + \frac{n}{T}\right)$$

## Fourier Coefficient of Modulation

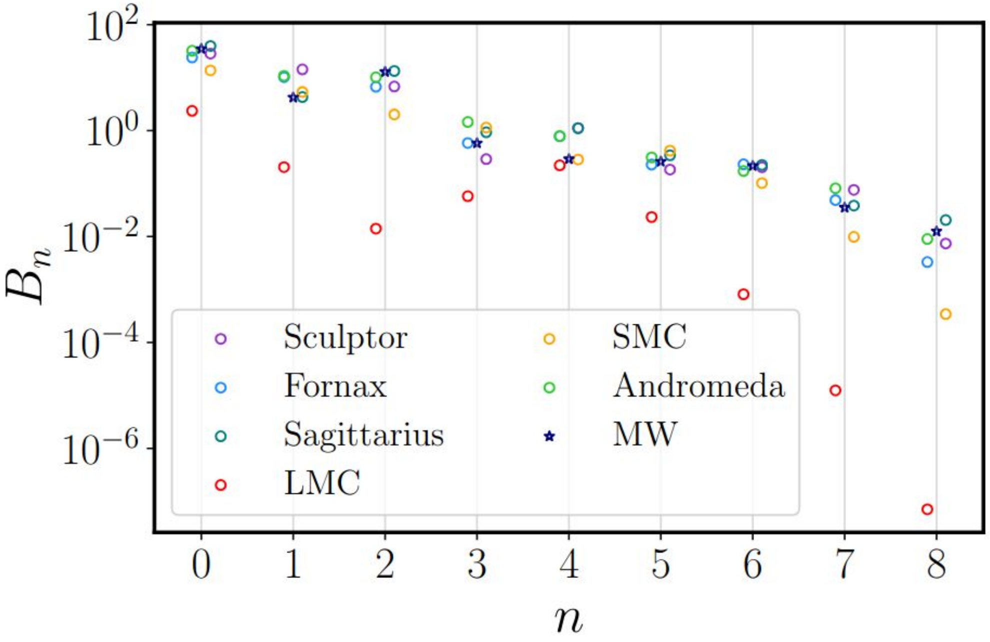


$$C(f, f') = \sum_{n=-8}^{n=8} B_n S_h\left(\frac{f' + f}{2}\right) \delta\left(f - f' + \frac{n}{T}\right)$$

Fourier Coefficient of Modulation

The modulation is **primarily** influenced by **latitude**, while the impact of **size** is a **secondary effect.**



# CYCLOSTATIONARY MODEL

## Likelihood

$$\log \mathcal{L}(\tilde{d}|\boldsymbol{\theta} = \{\boldsymbol{\theta}_{\mathrm{MW}}, \boldsymbol{\theta}_{\mathrm{sat}}, \boldsymbol{\theta}_{\mathrm{n}}\}) \propto -\sum_{i=A,E} \frac{1}{2}\log(\det\,[\Sigma_{\mathrm{d}}]_{\mathrm{i}}) + \frac{1}{2}\tilde{\mathrm{d}}_{\mathrm{i}}^{\mathrm{T}}\,[\Sigma_{\mathrm{d}}]_{\mathrm{i}}^{-1}\,\tilde{\mathrm{d}}_{\mathrm{i}}$$

$$[\Sigma_{\mathrm{d}}]_i = (\Sigma_{\mathrm{MW}}(\boldsymbol{\theta}_{\mathrm{MW}}) + \Sigma_{\mathrm{sat}}(\boldsymbol{\theta}_{\mathrm{sat}}) + \Sigma_{\mathrm{n}}(\boldsymbol{\theta}_{\mathrm{n}}))_i$$

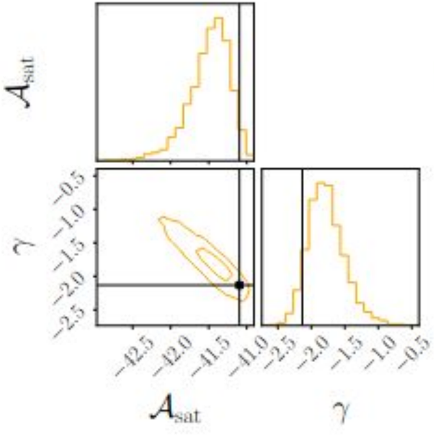
## Parameter

Spectrum | Modulation

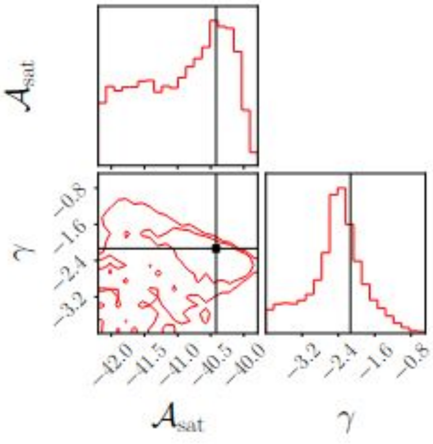
- $\boldsymbol{\theta}_{\mathrm{MW}} = \{\mathcal{A}_{\mathrm{MW}}, \alpha, f_{\mathrm{knee}}, f_2, f_1, \lambda, \sin\beta, \sigma_1, \sigma_2, \psi\}$
- $\boldsymbol{\theta}_{\mathrm{sat}} = \{\mathcal{A}_{\mathrm{sat}}, \gamma, \lambda, \sin\beta, \sigma\};$
- $\boldsymbol{\theta}_n = \{\mathcal{P}_{\mathrm{tm}}, \mathcal{P}_{\mathrm{oms}}\}.$

# RESULTS - Satellite + Noise

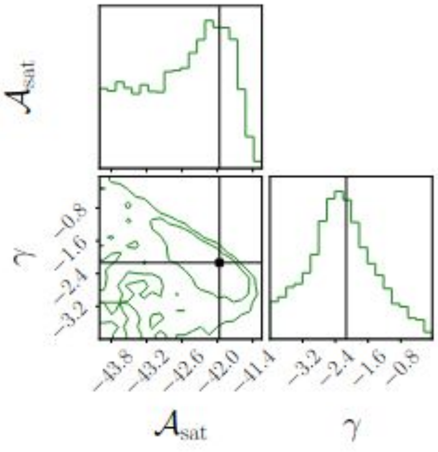
The detectability of satellite SGWB depends on the interplay between the spectrum and modulation.



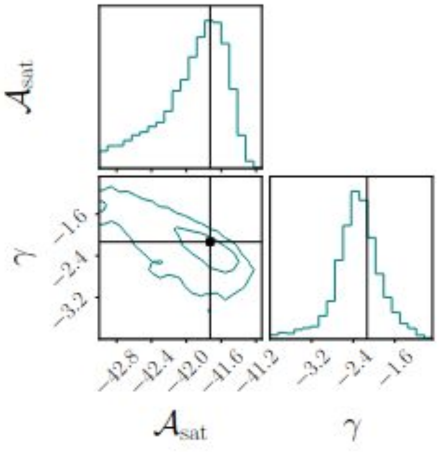

SMC




LMC

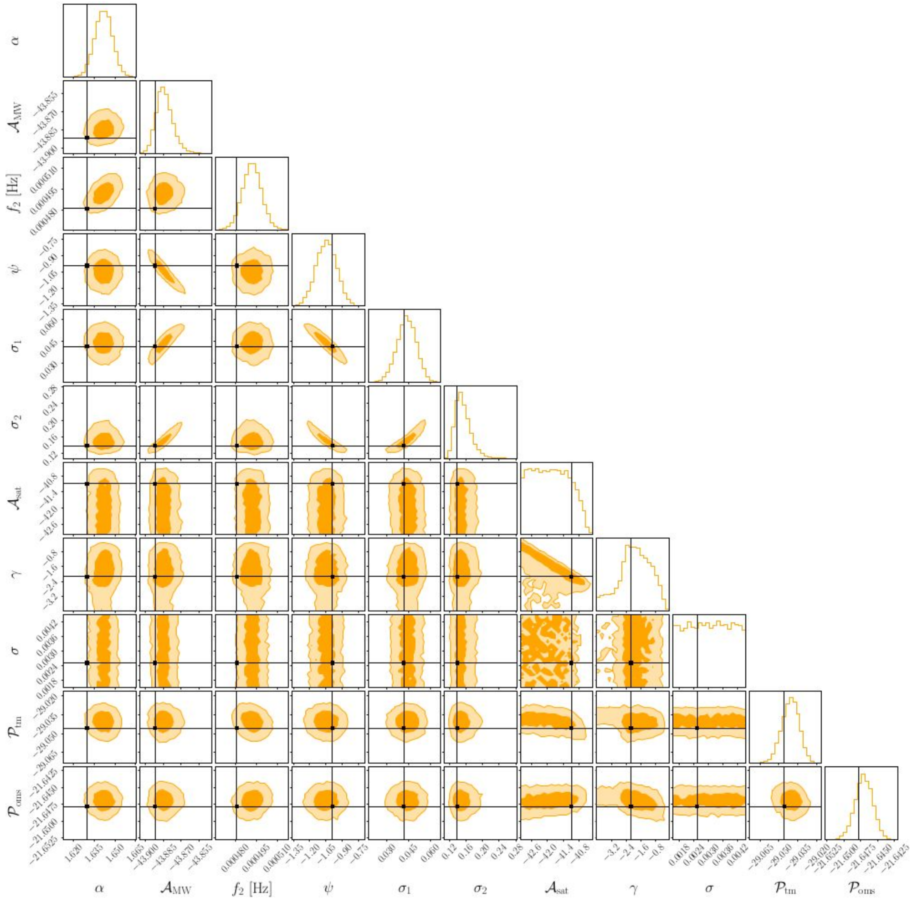



Andromeda

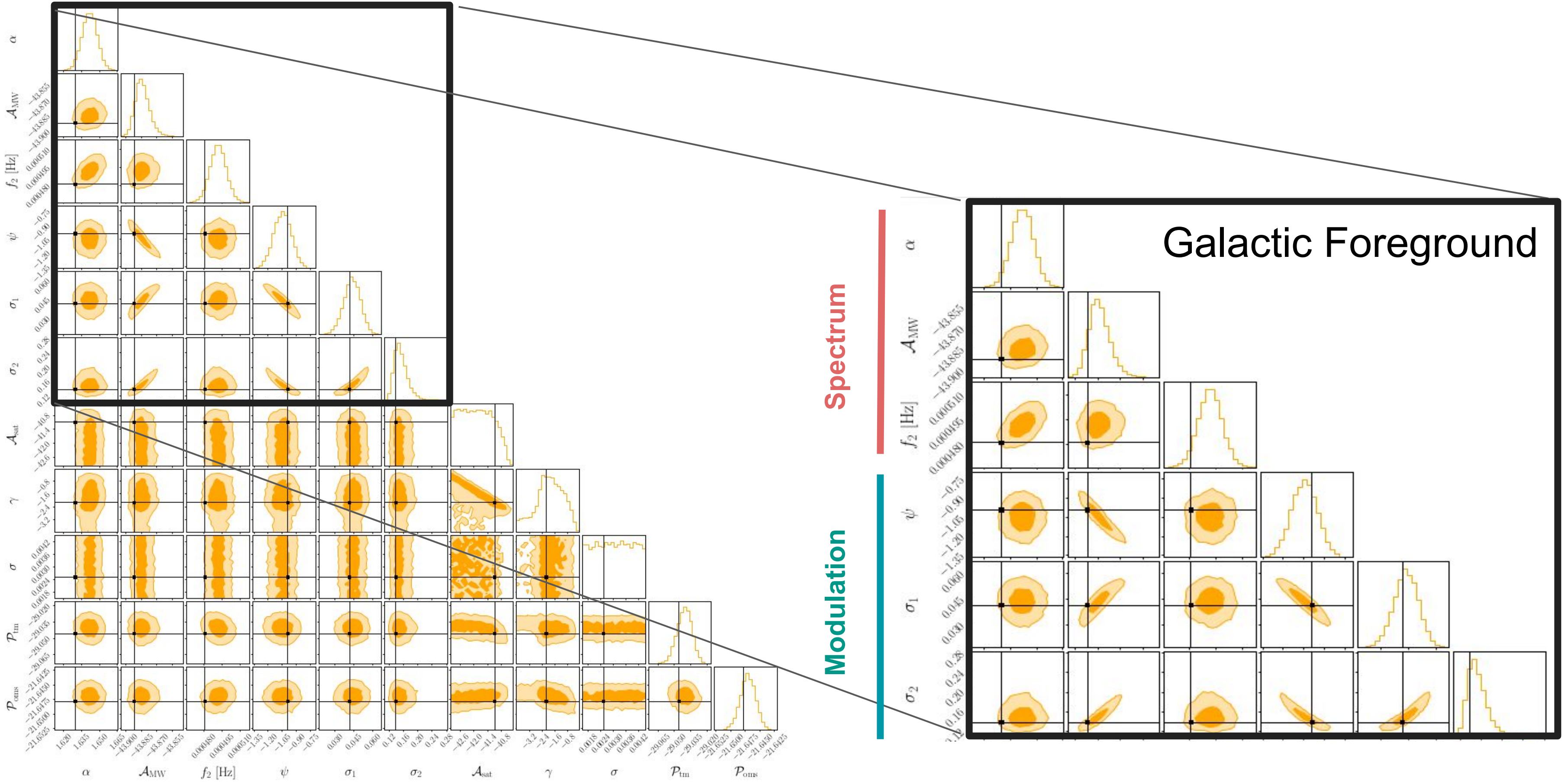



Sagittarius

# RESULTS - Satellite + Noise + MW



# RESULTS - Satellite + Noise + MW



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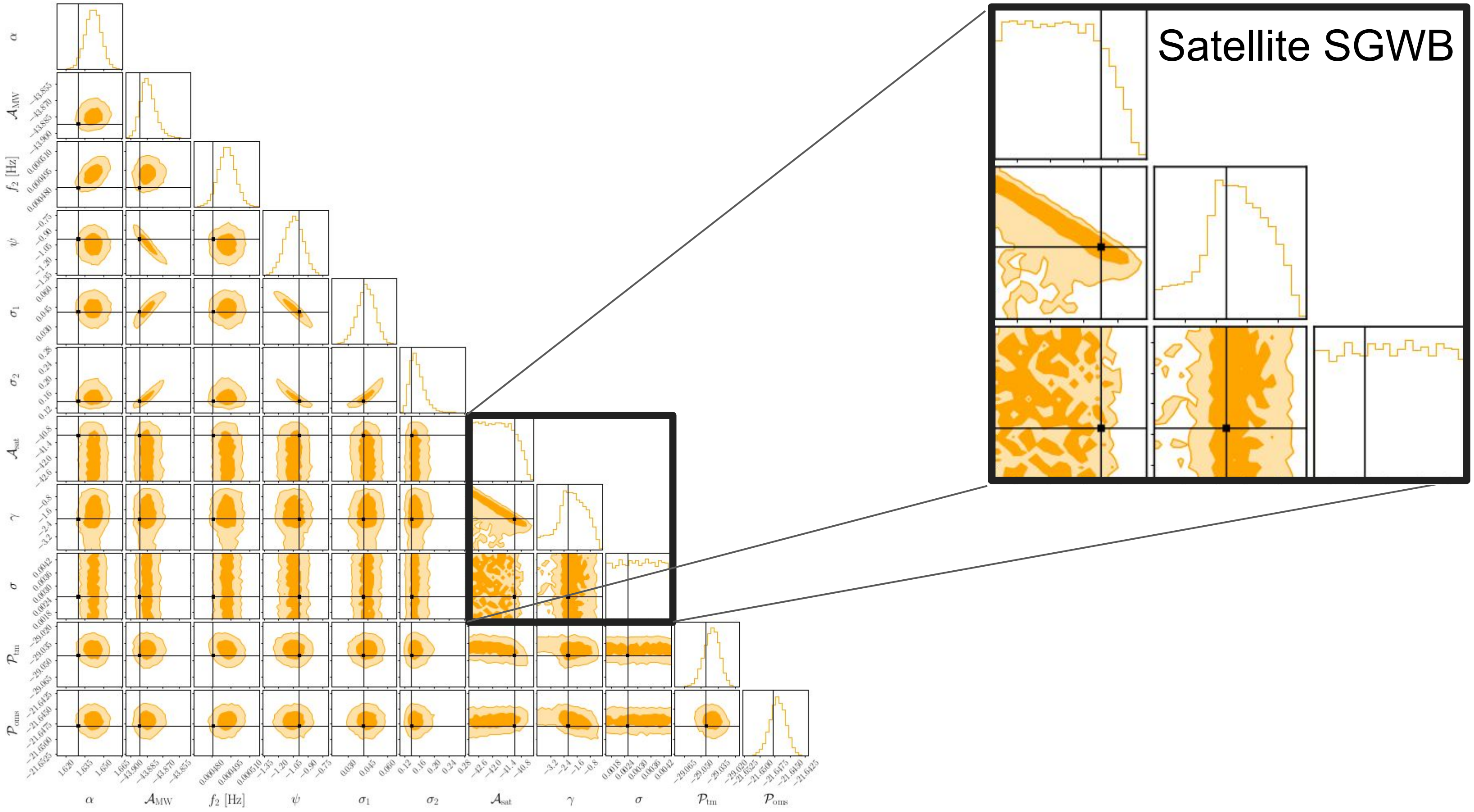
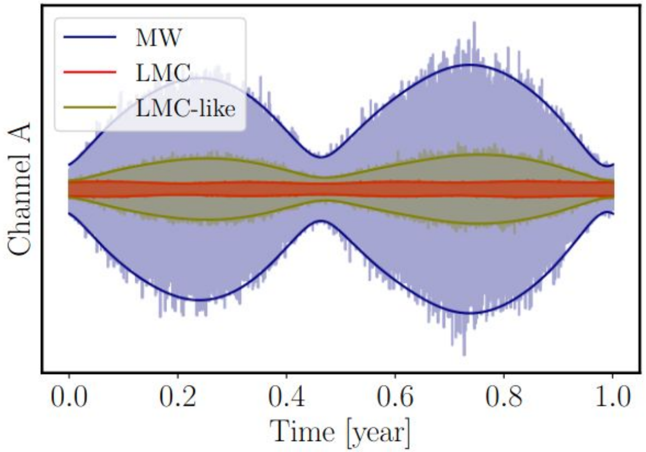
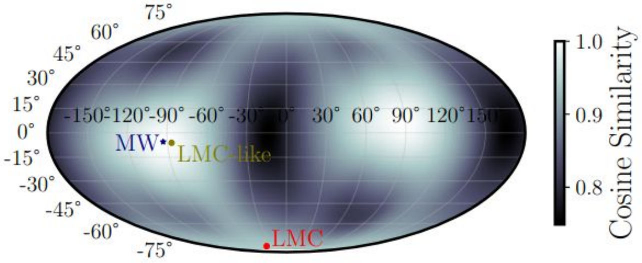


# RESULTS - Hidden Satellite



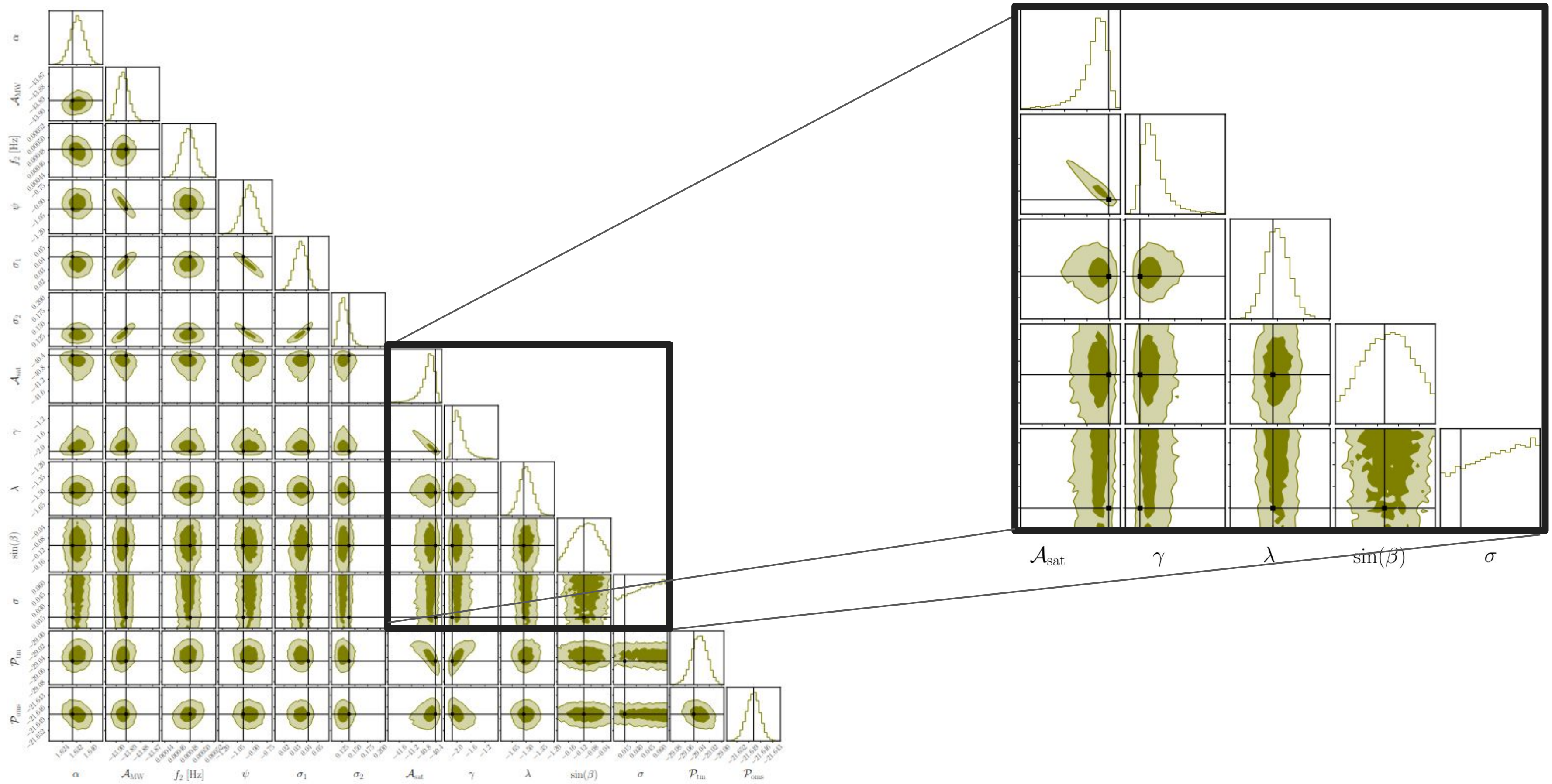

Unlike EM radiation, GW are not obscure by gas and dust

Thus, LISA has the potential to observe beyond the galactic plane (Zone of Avoidance)

We consider an **LMC-like satellite behind the Milky Way** (i.e. same Astrophysical spectrum -> same total mass and distance)

Are we able to observe it?

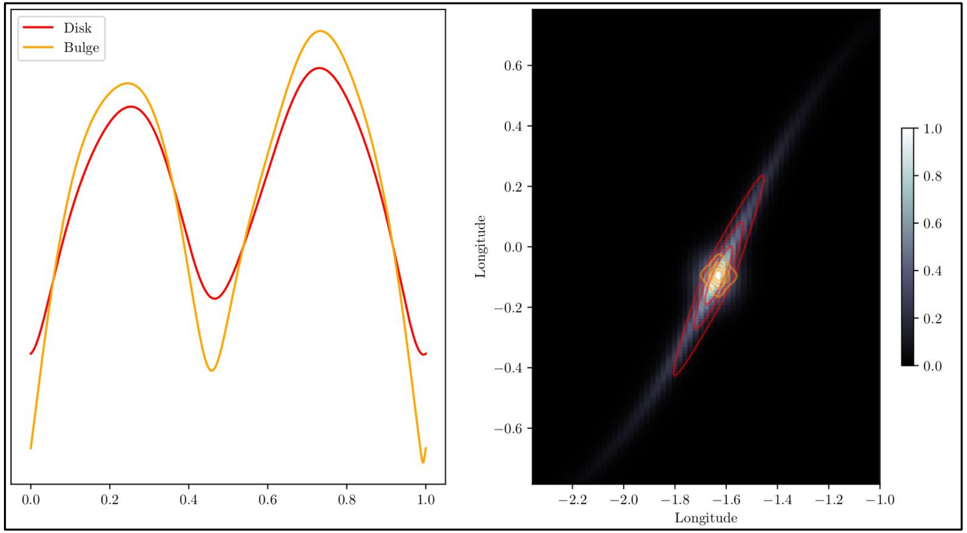
# RESULTS - Hidden Satellite



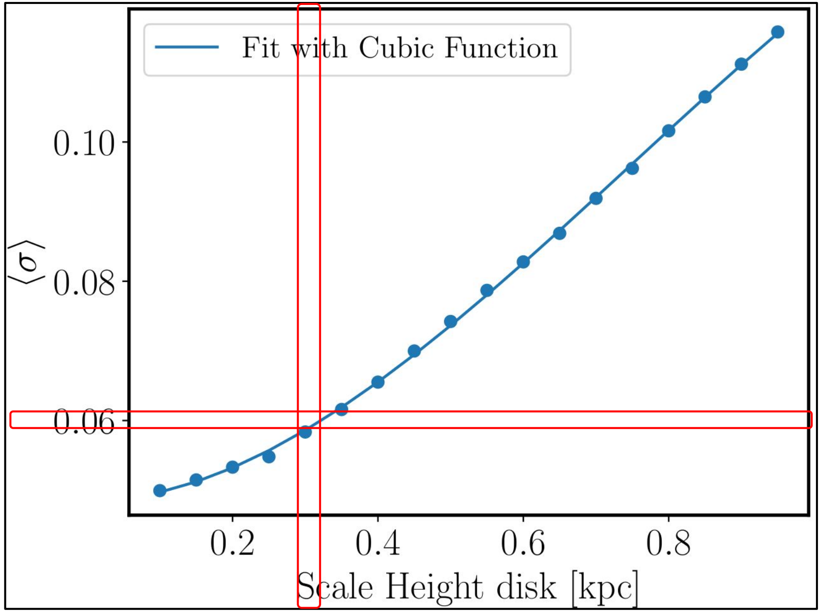
# CONCLUSION

- We introduce a novel method to address anisotropy from astrophysical SGWB.
- Detection of MW satellite strongly depends on the interplay between the spectrum and modulation.
- We could have access to Zone of Avoidance with LISA behind Milky Way
- Study Milky Way Morphology and structure with DWD foreground

# CONCLUSION



Disk
Bulge
0.0
0.2
0.4
0.6
0.8
1.0
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Longitude
−2.2
−2.0
−1.8
−1.6
−1.4
−1.2
−1.0
Longitude
1.0
0.8
0.6
0.4
0.2
0.0

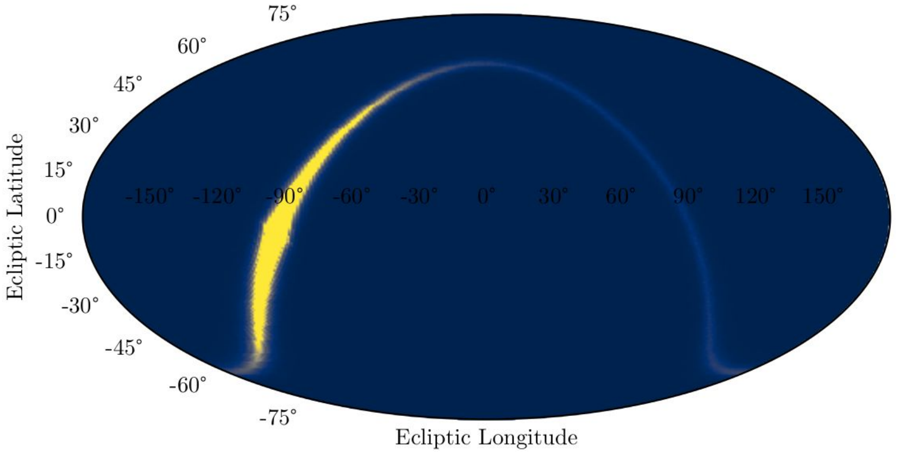



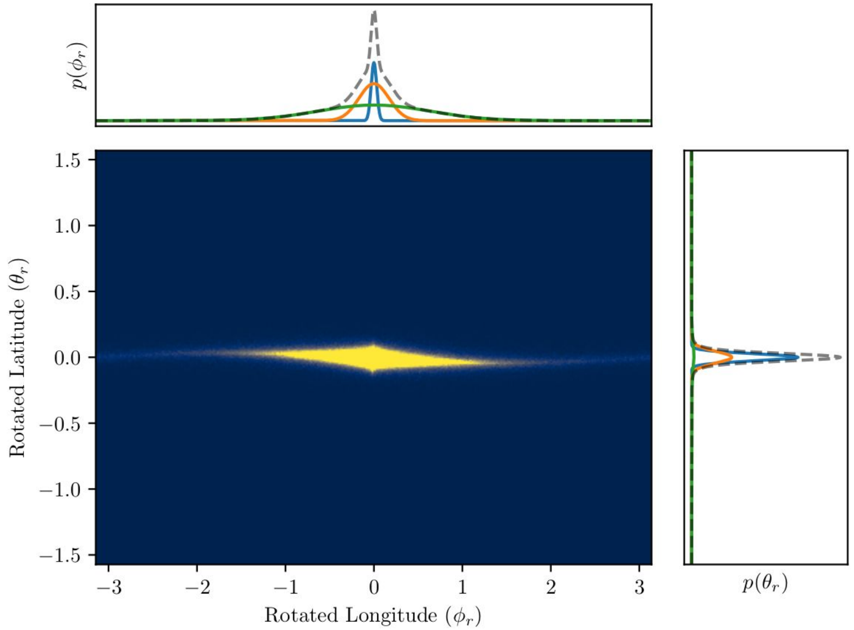
Fit with Cubic Function
0.10
0.08
0.06
⟨σ⟩
0.2
0.4
0.6
0.8
Scale Height disk [kpc]

# BACKUP SLIDES

# MODULATION

We have to average the time domain signal in LISA over the probability distribution of the sources in the sky



# MODULATION

We have to average the time domain signal in LISA over the probability distribution of the sources in the sky

The problem reduces to resolve integral like

$$\int_{\mathcal{R}} d\theta_r \int_{\mathcal{R}} d\phi_r p\left(\theta_r\right) p\left(\phi_r\right) e^{im\theta_r} e^{in\phi_r} = \varphi_{\theta_r}(m)\varphi_{\phi_r}(n)$$

The solution is well-know for a large set of probability distribution, and it is called
**CHARACTERISTIC FUNCTION**

# ASTROPHYSICAL SPECTRUM

Korol+22

Amplitude of GW Inspiral

$$S_h(f) = \int d\mathcal{M}_c p(\mathcal{M}_c) \int df_s p(f_s) \delta(f - f_s) \frac{(G\mathcal{M}_c)^{10/3}}{(c^4 D)^2} (\pi f_s)^{4/3}$$

**Satellite Background**

$$S_h(f) = A_{\rm sat} \left( \frac{f}{10^{-3.5} \rm Hz} \right)^{\gamma}$$

$$\gamma = -(9 + 3\alpha)/3$$

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DWDs in a satellite have all the same distance

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Due to Fourier Transform of cos
In Inspiral waveform

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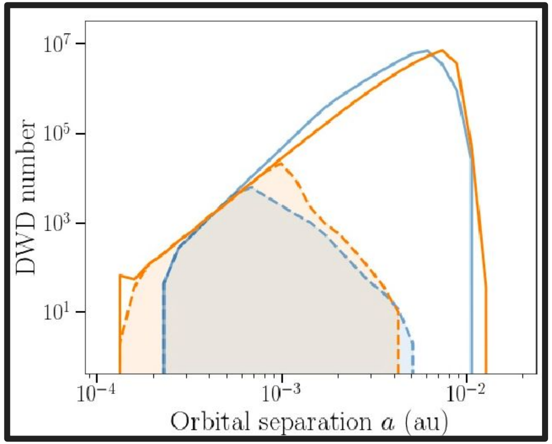
Maoz+18 Binary Separation distribution is a power law with slope α + 4

$$\alpha \approx -1.3$$

Based on spectroscopic observation

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Korol+22

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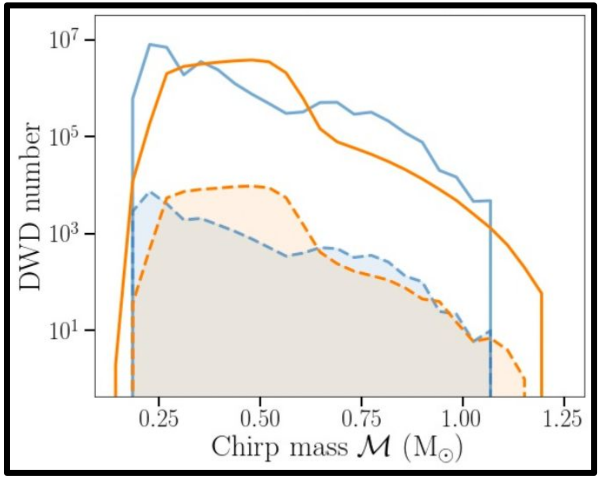
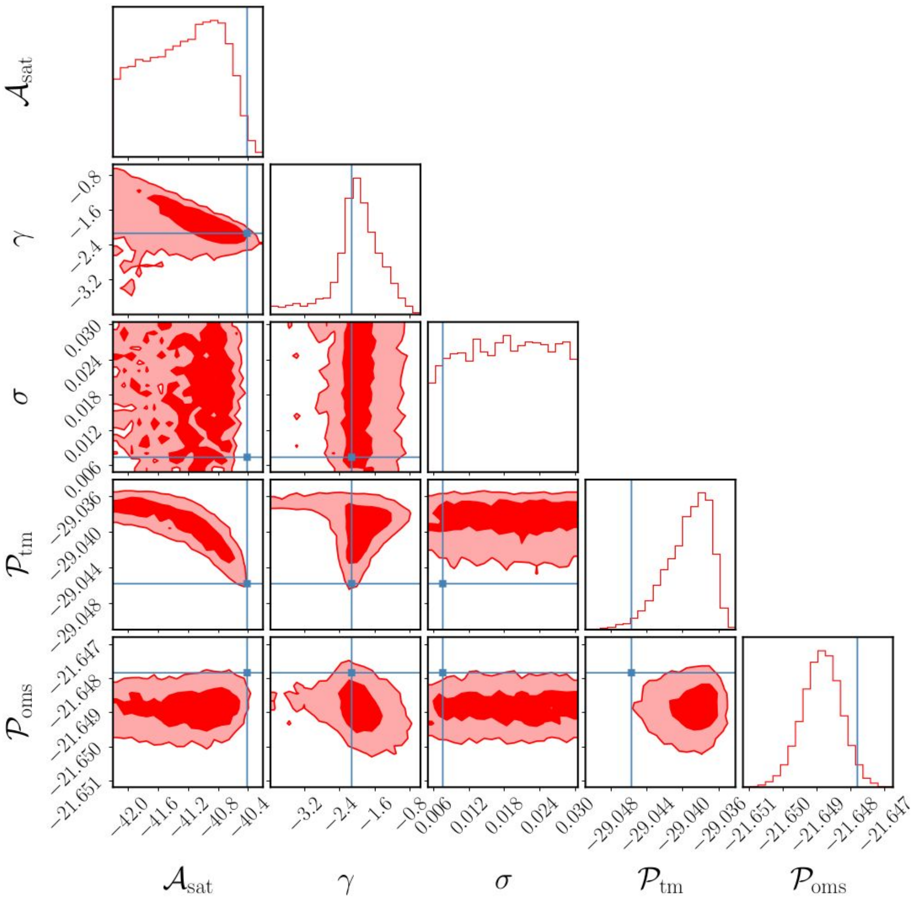
Primiray Mass m1: Gaussian Mixture based on SDSS spectroscopic observation (Kepler+15)
Secondary Mass m2: Flat distribution [0.15 M⊚, m1]



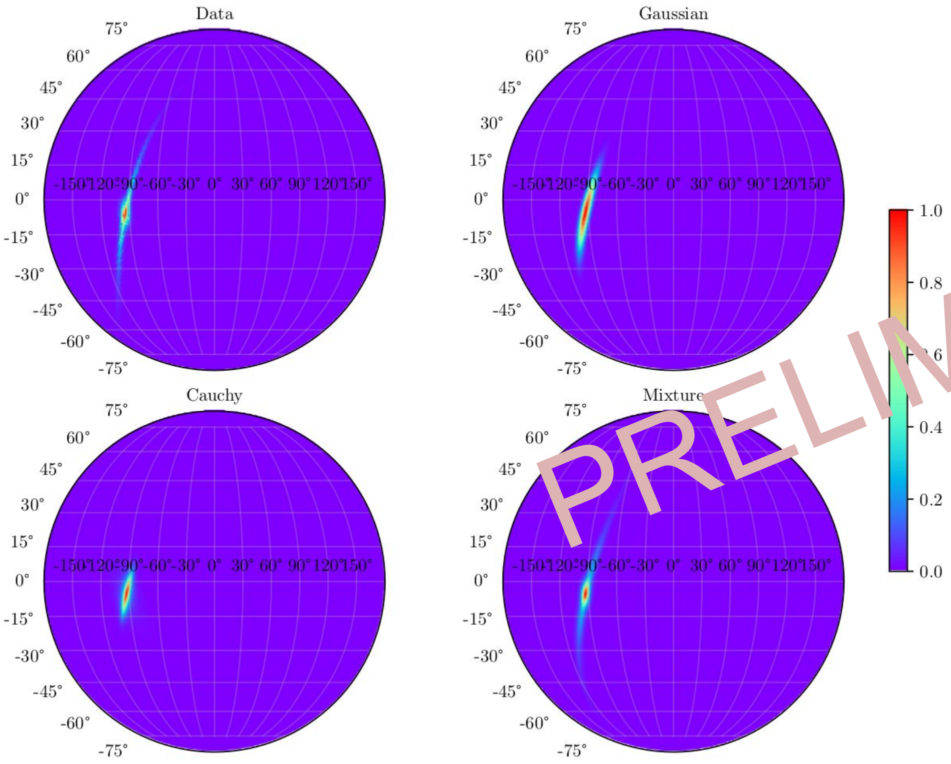
# RESULTS - Satellite (Realistic) + Noise

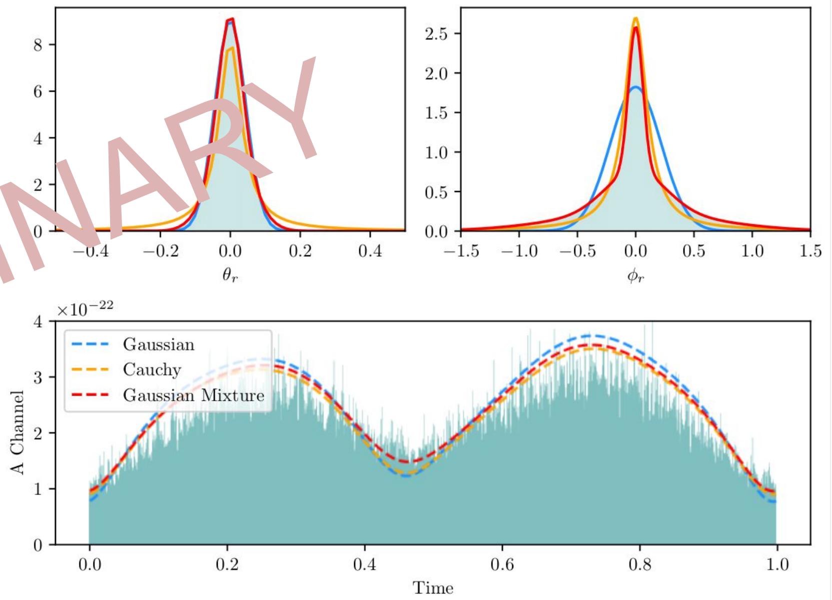



LMC from catalog generated with Stellar Population Synthesis code (Korol+24)

We fix the sky position of LMC in the modulation model

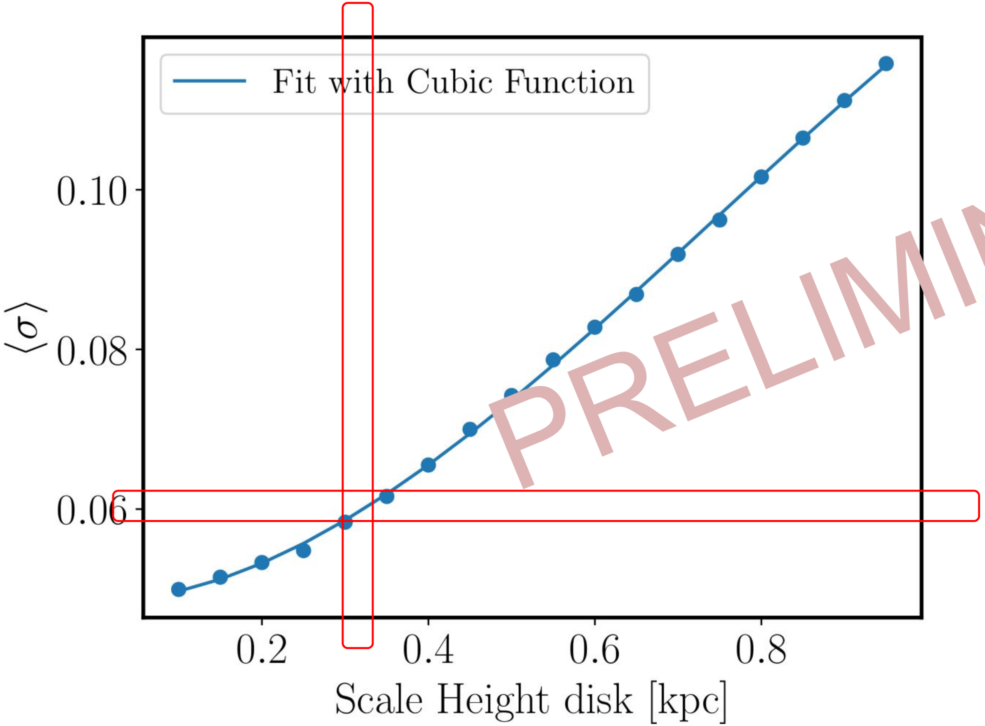
# WHAT's NEXT




**Pozzoli,** Buscicchio, Klein in prep

# WHAT's NEXT




$$\rho(r)\rho(z) \propto \exp\left(-r/r_h\right)\exp\left(-z/z_h\right)$$

**Pozzoli,** Buscicchio, Klein in prep