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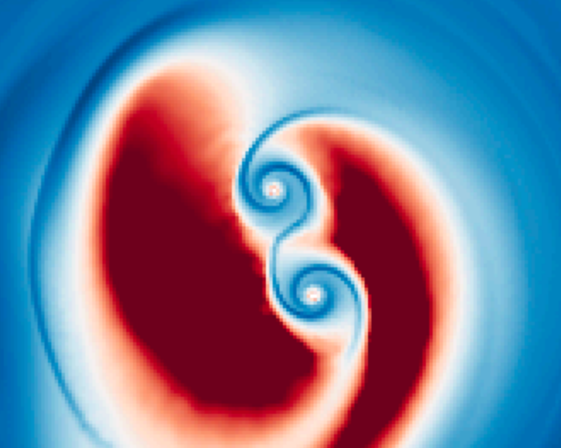


# General relativistic meshless hydrodynamics for massive black hole binary simulations

LISA Astrophysics WG meeting - Garching, 5-7 November 2024

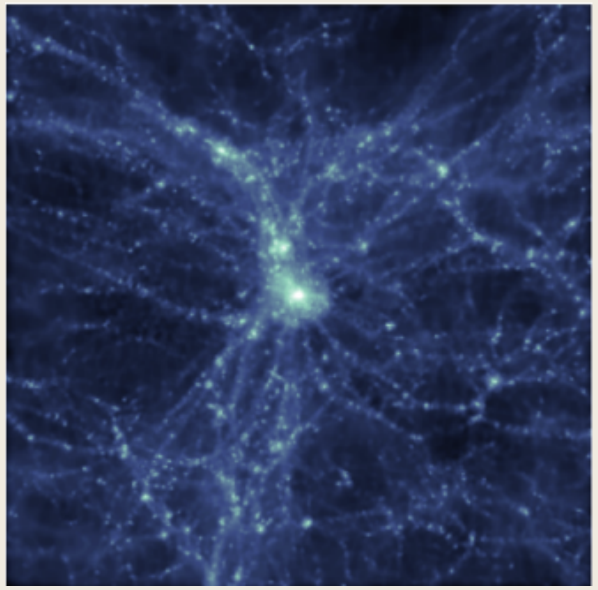
In collaboration with: G. Fedrigo, A. Franchini, M. Bonetti, F. Cattorini


Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

# Black hole binary evolution from pairing to merger

## Path to coalescence

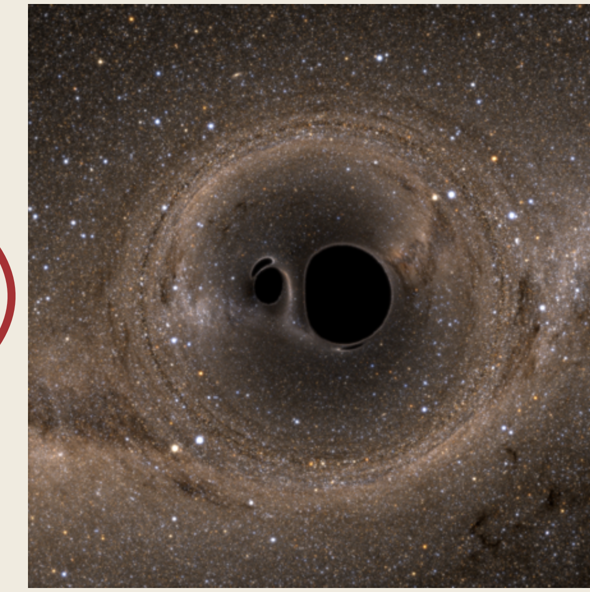
AGN EM detection



... and in between?

=> PAIRING => STELLAR/GAS HARDENING (CBD) => DECOUPLING =>

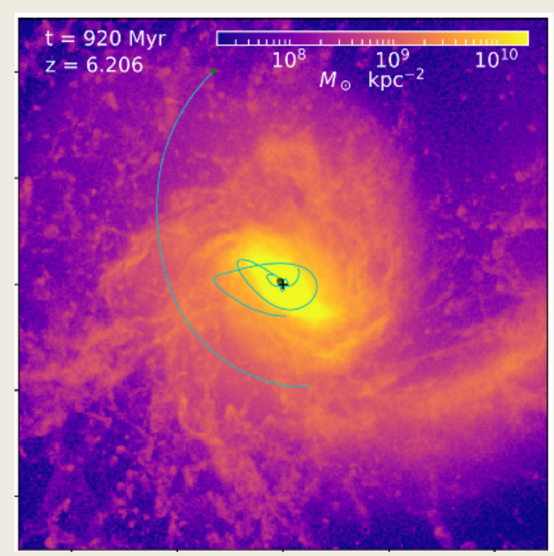
GW detection



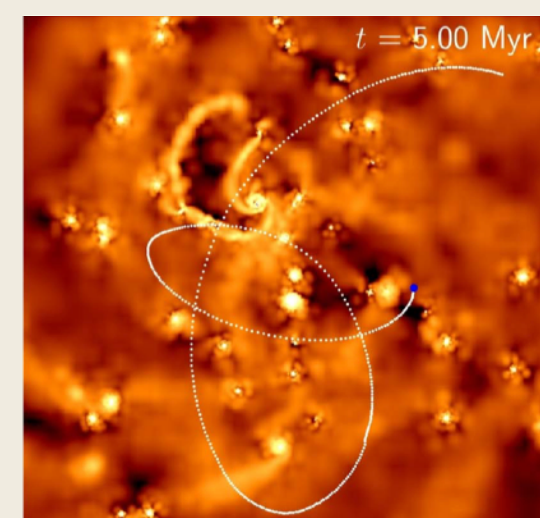
...however...

interaction with clumps, another MBH, etc.

Bortolas+20



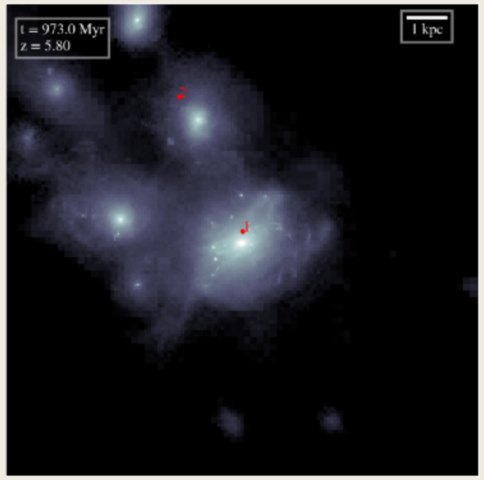

Sousa Lima+17



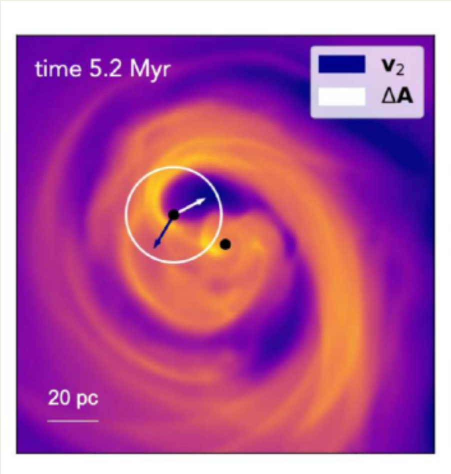
# Consistent modelling of the different stages

## $10^{-3}$-$10^{4}$ pc (Newtonian)

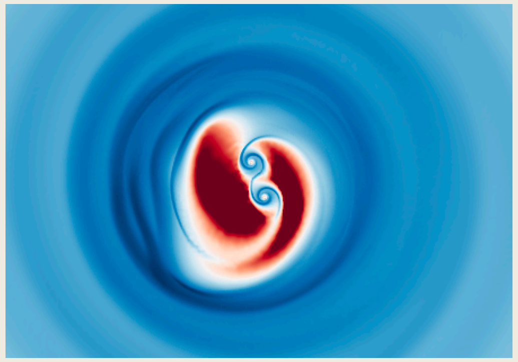
Pfister+19




Bollati, Lupi+2022

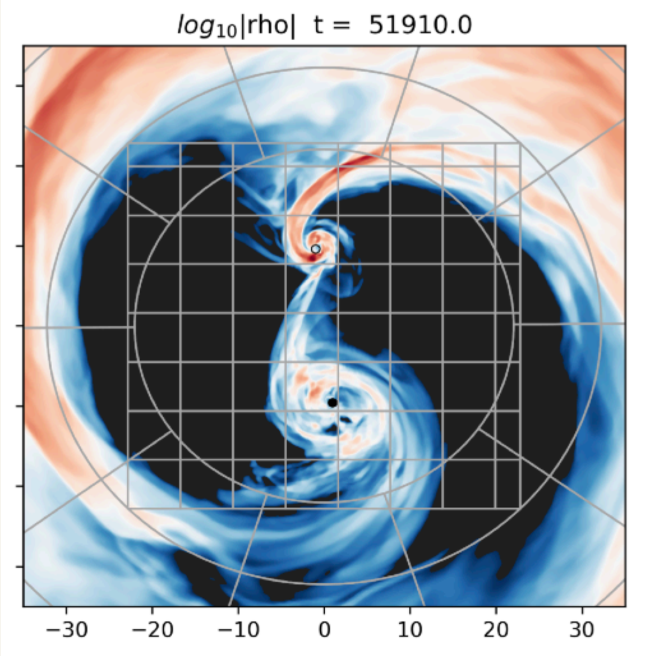



Franchini, Lupi+2022

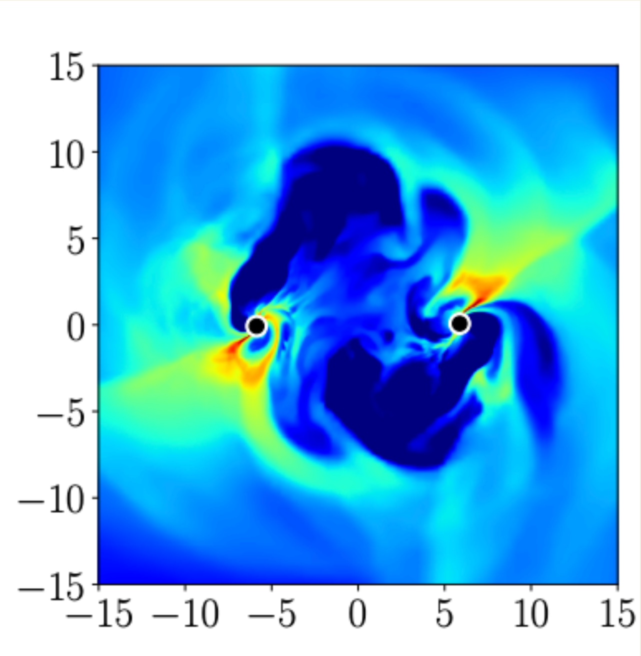


## pre/post merger (GRMHD-NR)

Avara+24




Fedrigo + 2024



# Consistent modelling of the different stages

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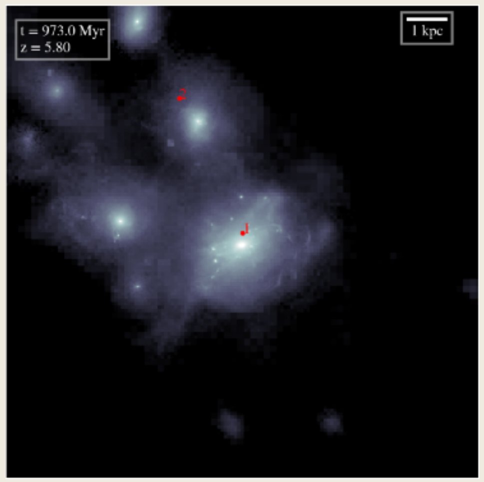
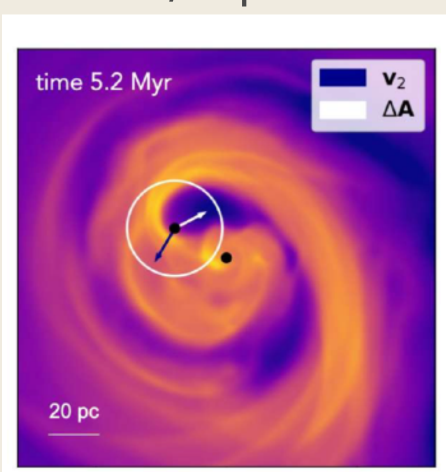
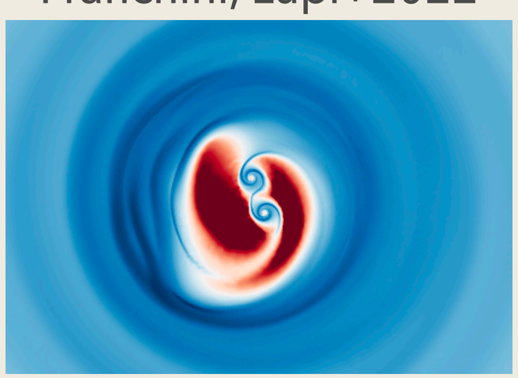



Bollati, Lupi+2022

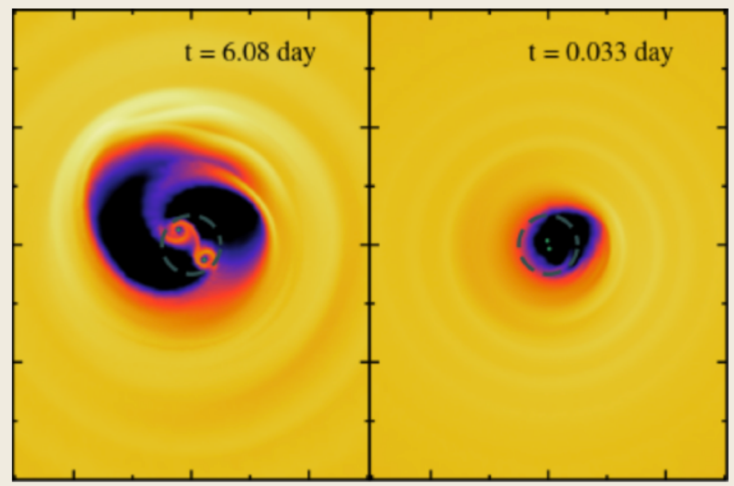



Franchini, Lupi+2022

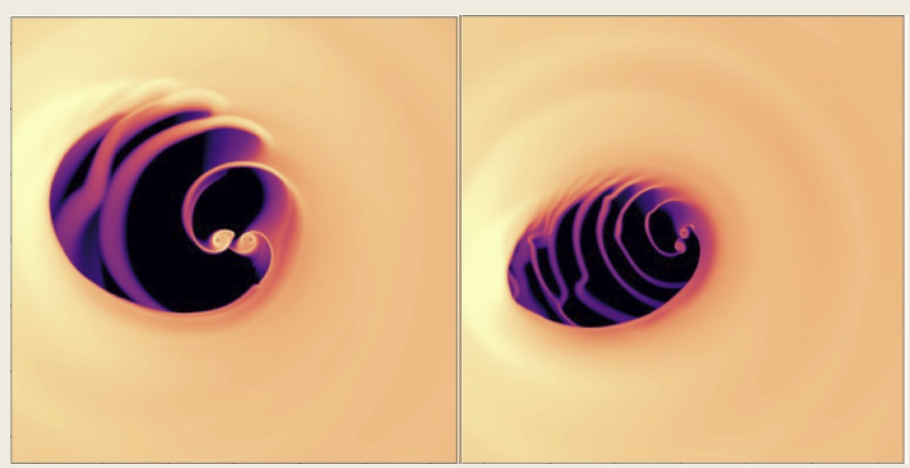


## ~100-1000$R_g$ (Post-Newtonian)

Franchini, Bonetti, Lupi+2023




Krauth+23



## pre/post merger (GRMHD-NR)

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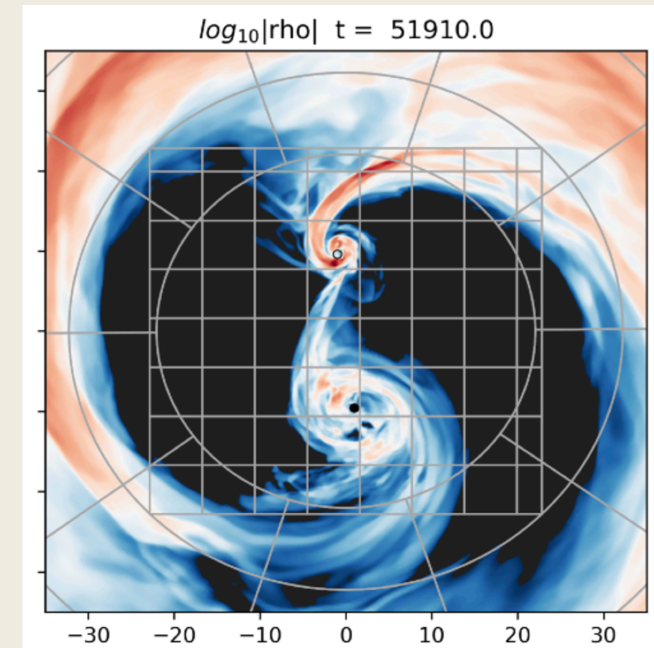
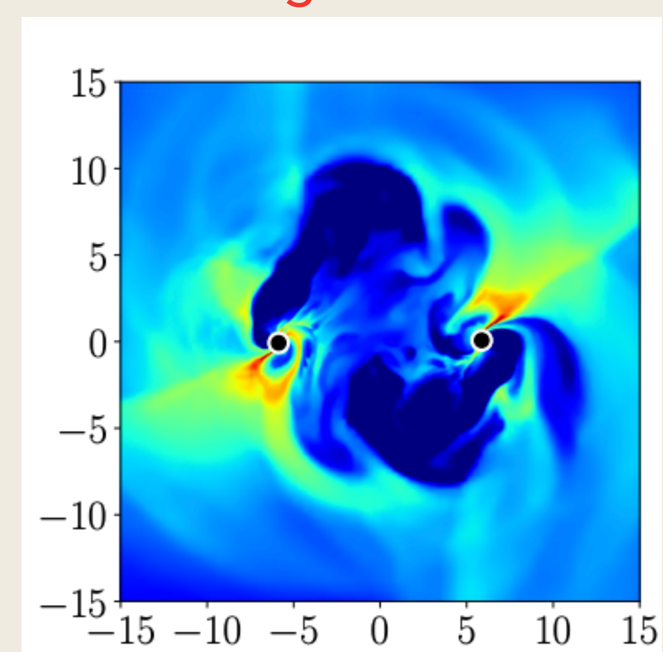
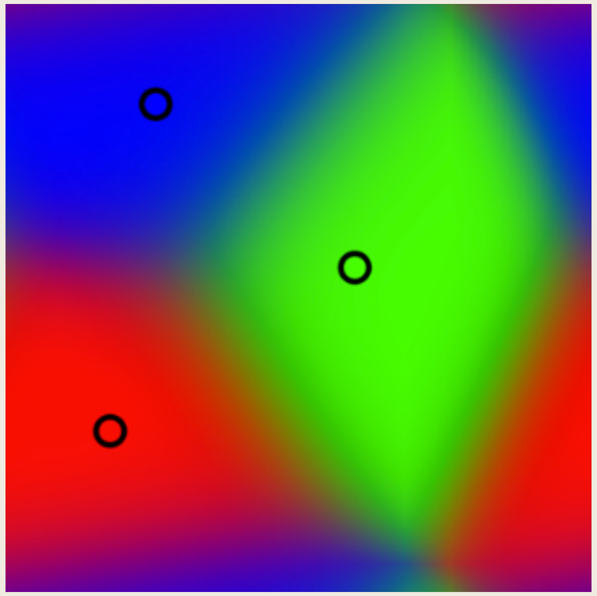



Fedrigo + 2024



# GIZMO-GR: meshless relativistic hydrodynamics

(Lupi 2023)



We extended the meshless conservative scheme of the Newtonian hydrodynamic code GIZMO (Hopkins 2015) to incorporate GR hydrodynamics with stationary metrics.

Why GIZMO?

- Unstructured "mesh" => better conservation of mass, angular momentum, and energy
- Naturally moving/adapting mesh => Intrinsic adaptive resolution and lower diffusion
- Finite volume formulation => Better treatment of shock (no artificial viscosity/conduction needed)

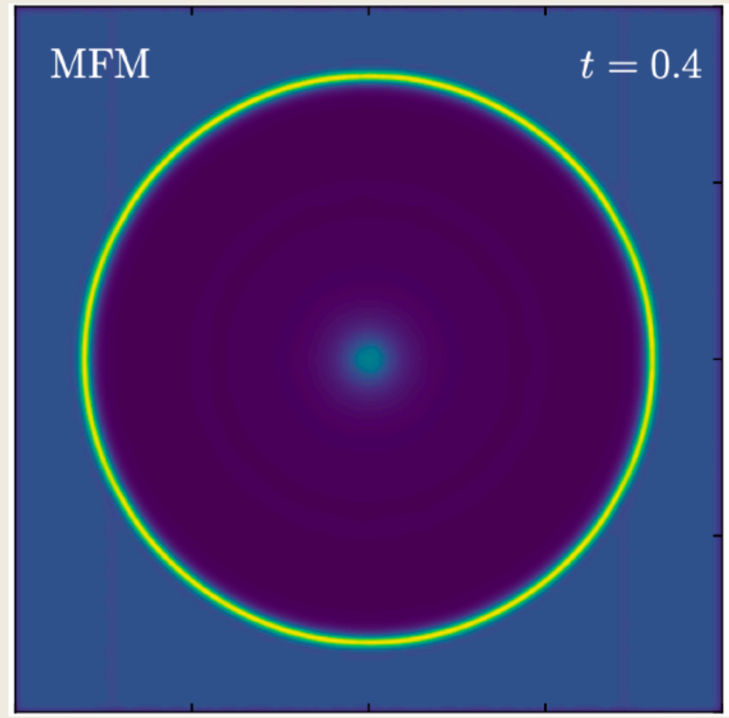
$$\frac{d(V_i \mathcal{U}_i)}{dt} + \sum_j (\bar{\mathcal{F}} - \mathbf{w}_{ij}\bar{\mathcal{U}}) \cdot \mathbf{A}_{ij} = V_i \mathbf{S}_i$$

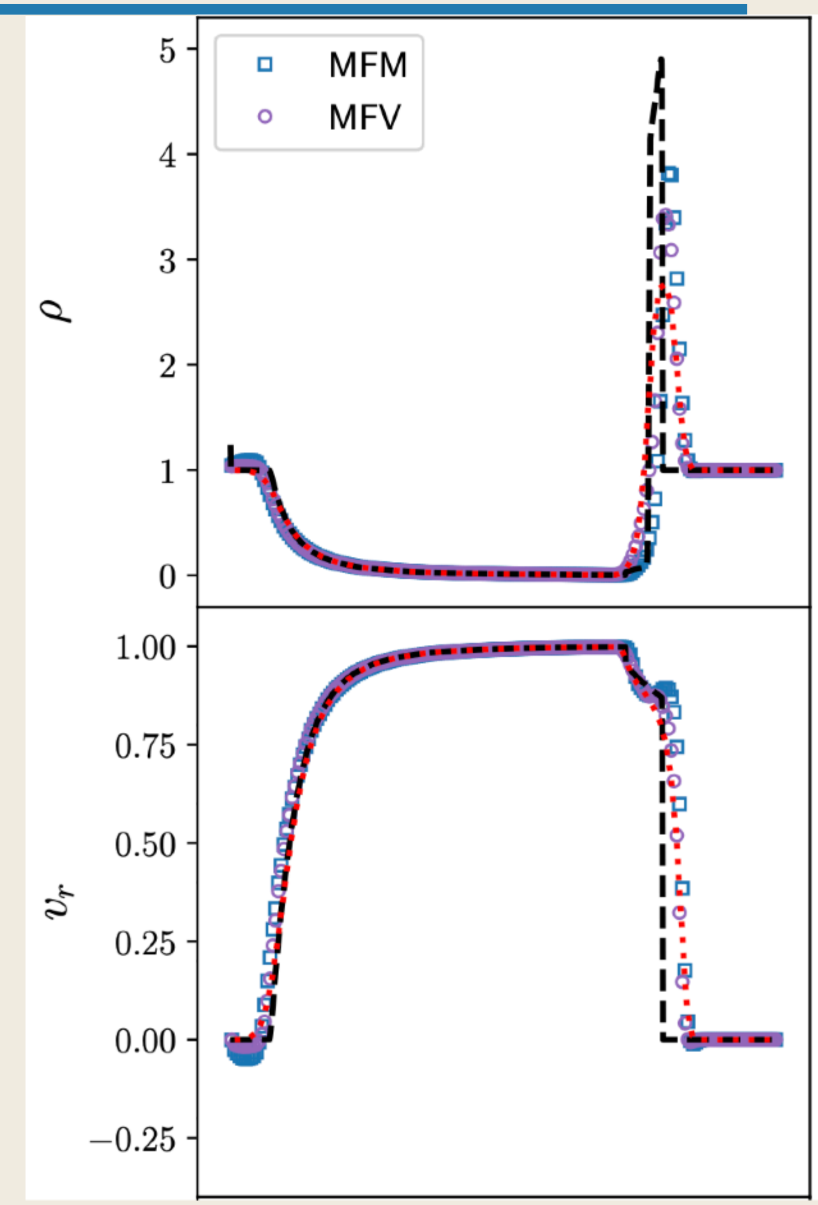
$$\mathcal{U} = \begin{cases} D \equiv \sqrt{\gamma}\rho W \\ S_j \equiv \sqrt{\gamma}\rho h W^2 v_j \\ \tau \equiv \sqrt{\gamma}(\rho h W^2 - P - \rho W) \end{cases} \qquad \mathcal{F} = \begin{cases} D\tilde{\mathbf{v}} \\ S_j \tilde{\mathbf{v}} + \alpha\sqrt{\gamma} P \hat{\mathbf{n}} \\ \tau\tilde{\mathbf{v}} + \alpha\sqrt{\gamma} P \mathbf{v}, \end{cases}$$

# GIZMO-GR: meshless relativistic hydrodynamics

(Lupi 2023)

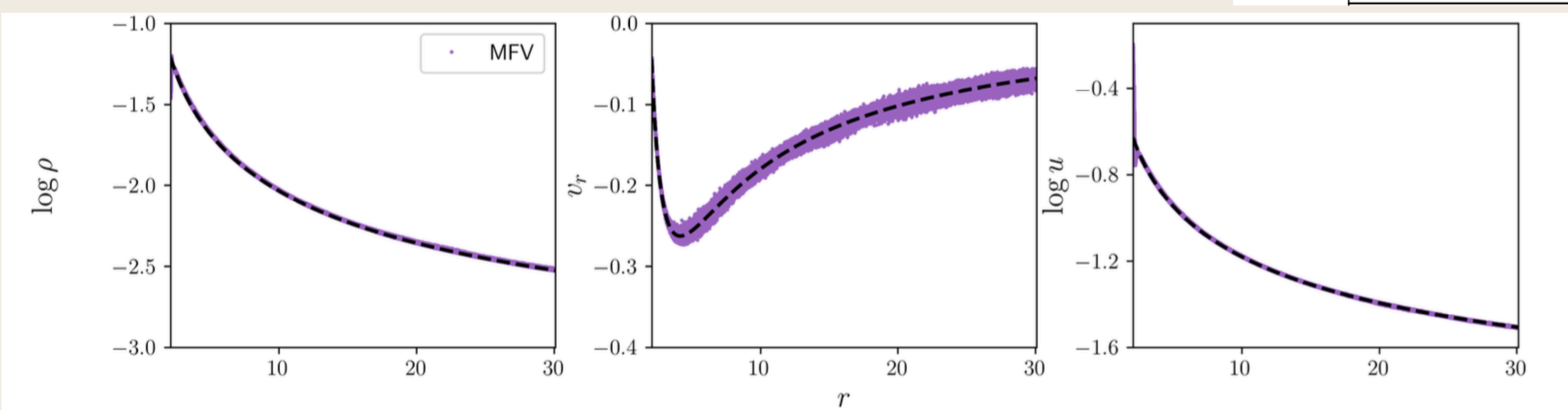
Relativistic BW




The code showed very good accuracy even at relatively low resolution compared to fixed-grid and SPH codes

Bondi solution



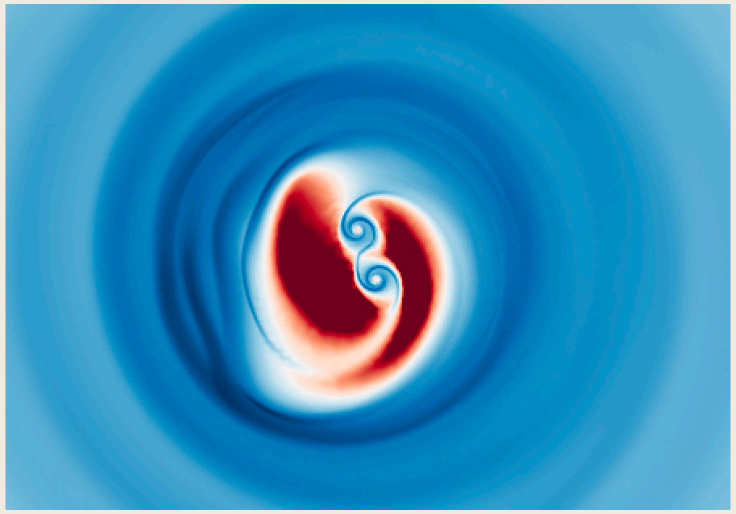
# Black hole binaries from Newtonian to relativistic scales

A novel approach (Fedrigo, Lupi et al. in prep.)

By exploiting the double nature of GIZMO (Newtonian and relativistic), we can naturally extend simulations from Newtonian scales (e.g. Franchini, Lupi+2022) down to the relativistic regime.

- Tabulated (CBwaves) or live binary evolution (including PN corrections up to 2.5, see Franchini, Bonetti, Lupi+24)
- Analytic binary metric in Kerr-Schild coordinates (Combi & Ressler 2024)
- Adaptive refinement around the two BHs (user-defined)
- Gas removal inside the horizon (accretion or excision)

***Goal (work in progress): constrain the evolution of a MBHB binary from hundreds of gravitational radii down to the last few orbits (6M) surrounded by a 'thin' CBD***



# Black hole binaries from Newtonian to relativistic scales

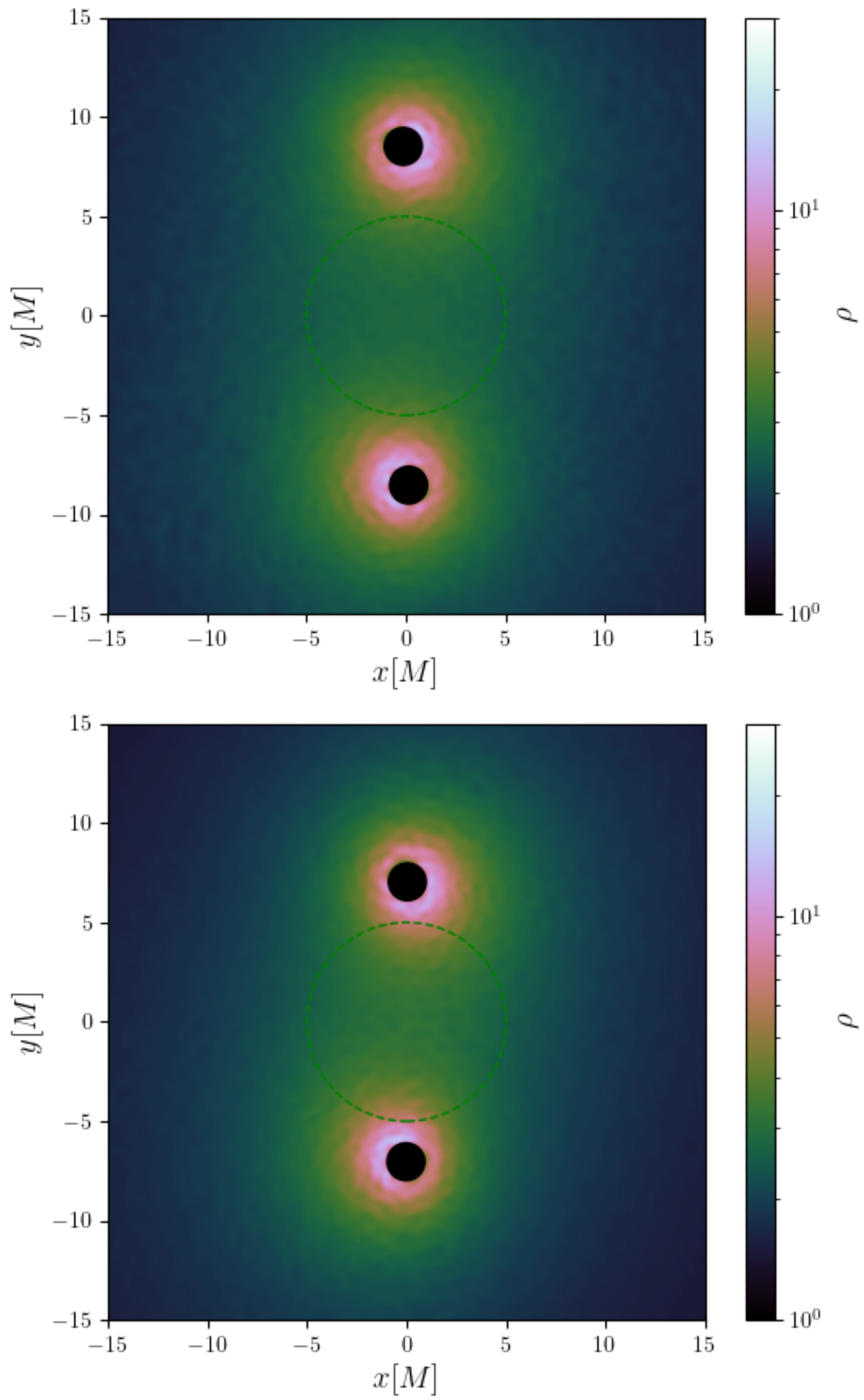
A novel approach (Fedrigo, Lupi+in prep.)

Validation test: Bondi-like accretion - max resolution ~0.6M at horizon

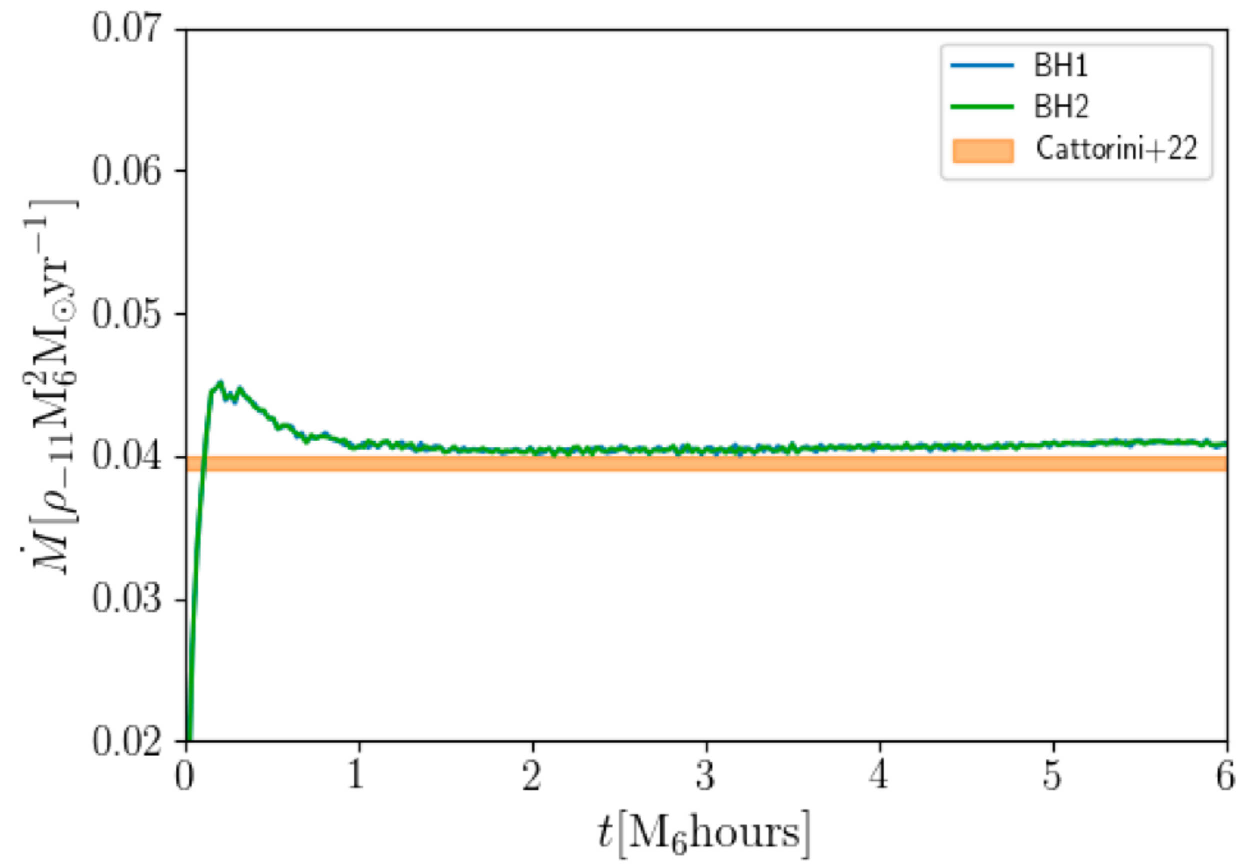
Homogeneous cloud initially at rest in the binary metric at 16M ($\Gamma = 4/3$)

IC

$t=10t_{orb}$




BH accretion rate

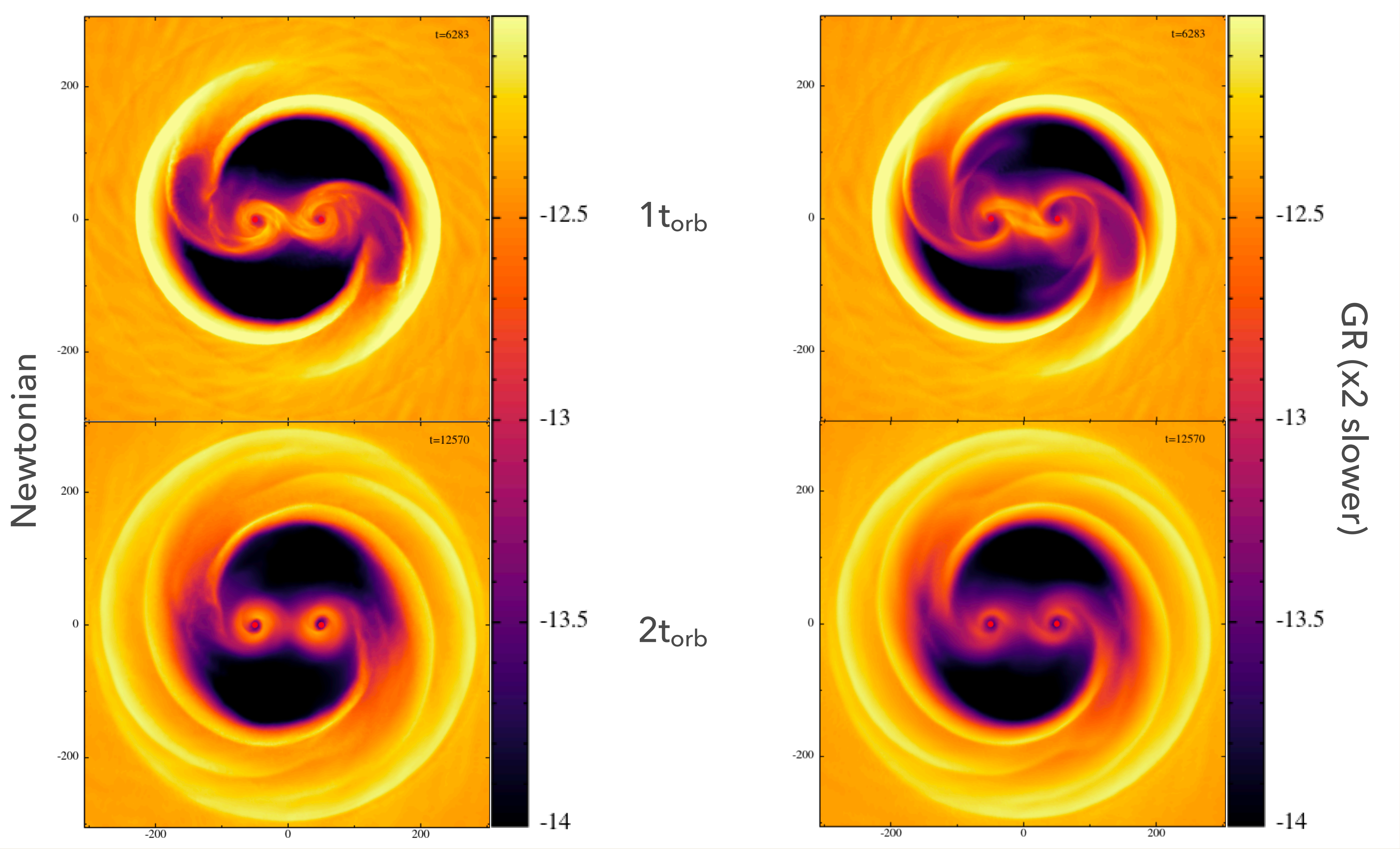


# Black hole binaries from Newtonian to relativistic scales

A novel approach (Fedrigo, Lupi+in prep.)

Inviscid CBD around an equal mass non-spinning BH binary ($\Gamma = 4/3$, no gas self gravity)

Max resolution ~1M at horizon

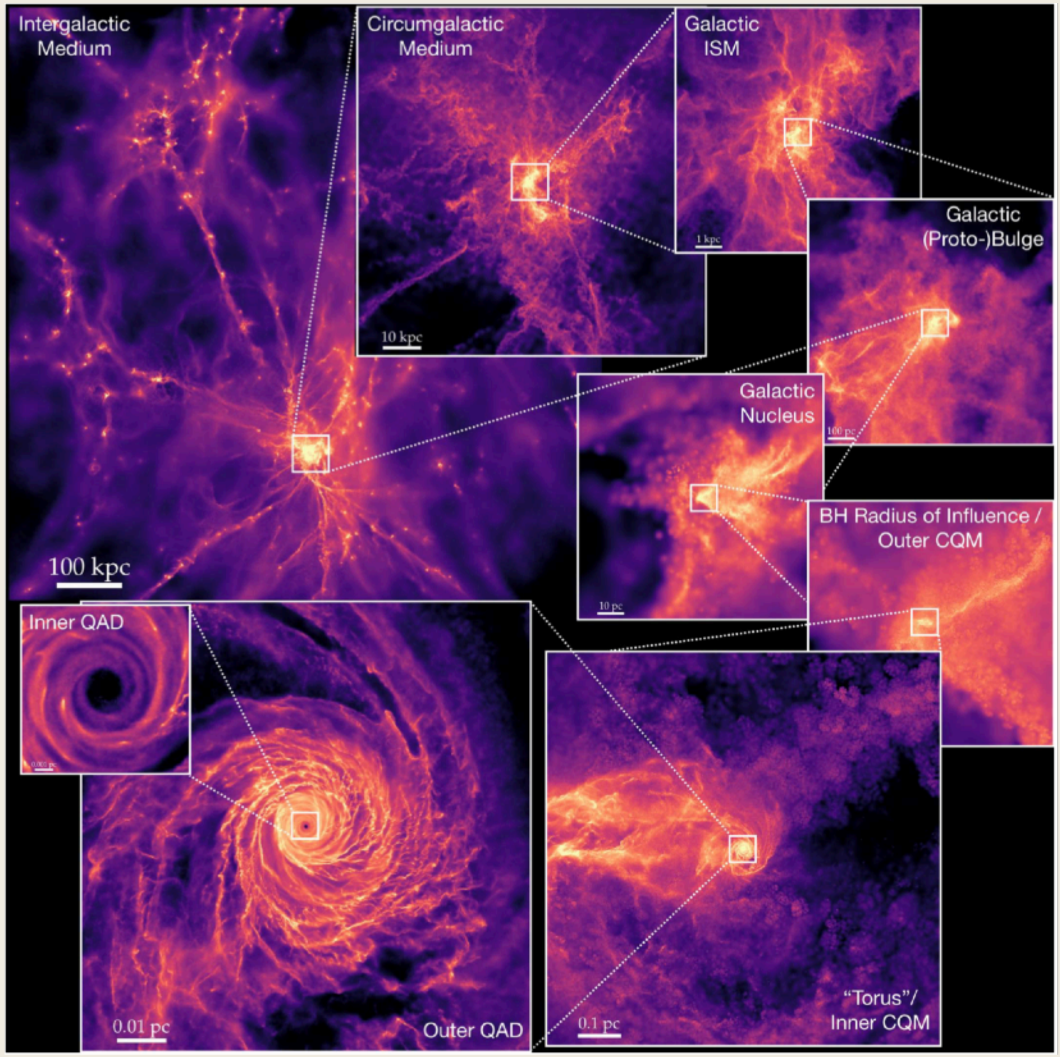


# Disc magnetisation: GIZMO-GR(MHD)

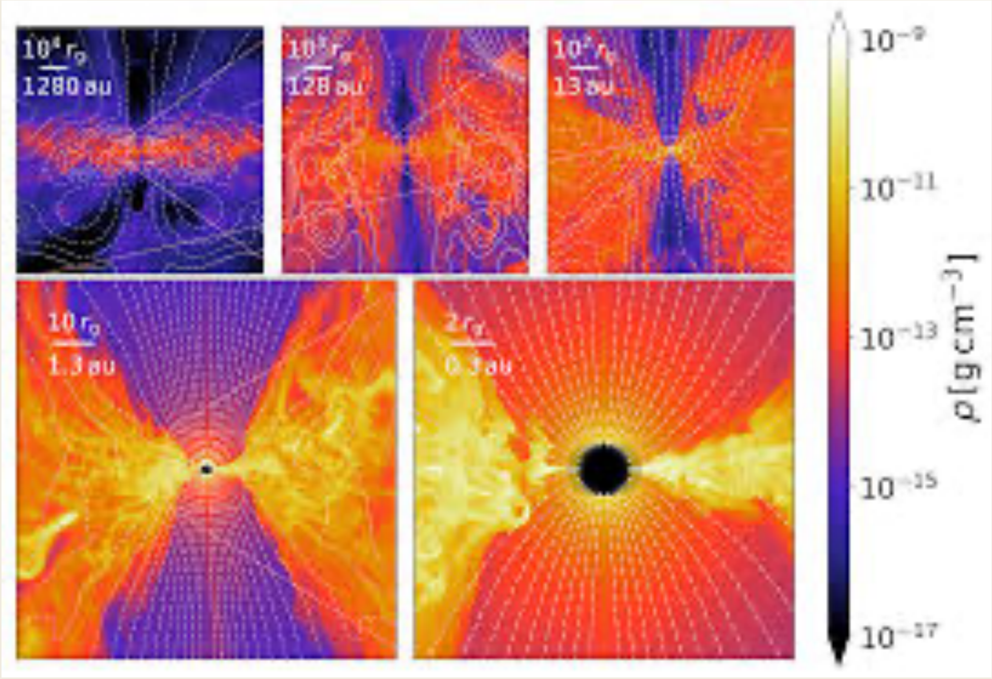
(Fedrigo & Lupi in prep.)

Although the properties of CBDs are not easy to constrain, it is well known that magnetic fields play a crucial role in accretion discs.

Hopkins+23 (Forge'd in Fire)




Kaaz+24 (H-AMR Forge'd in Fire)




**Strongly magnetised accretion disc**

# Disc magnetisation: GIZMO-GR(MHD)

(Fedrigo & Lupi in prep.)

In order to improve our numerical scheme, we are now extending GIZMO to GRMHD.

This will allow us to consistently extend Forge'd in FIRE down to the horizon scales (without remapping), but also to explore the evolution of magnetised CBDs around MBHBs.

$$\mathbf{U} = \begin{cases} D \equiv \sqrt{\gamma}\rho W, \\ S_j \equiv \sqrt{\gamma}(\rho h^* W^2 v_j - \alpha b^0 b_j), \\ \tau \equiv \sqrt{\gamma}(\rho h^* W^2 - P^* - (\alpha b^0)^2) - D, \\ \mathcal{B}^j = \sqrt{\gamma} B^j, \end{cases}$$

$$\mathbf{F}^i = \begin{cases} D\tilde{v}^i, \\ S_j \tilde{v}^i + \alpha\sqrt{\gamma} P^* \delta^i_j - \alpha b_j \mathcal{B}^i / W, \\ \tau \tilde{v}^i + \alpha\sqrt{\gamma} P^* v^i - \alpha^2 b^0 \mathcal{B}^i / W, \\ \mathcal{B}^j \tilde{v}^i - \mathcal{B}^i \tilde{v}^j, \end{cases}$$

$$b^0 = \frac{W B^i v_i}{\alpha},$$

$$b^i = \frac{B^i}{W} + W(B^j v_j)\left(v^i - \frac{\beta^i}{\alpha}\right),$$

$$b^2 = \frac{B^i B_i}{W^2} + (B^j v_j)^2,$$

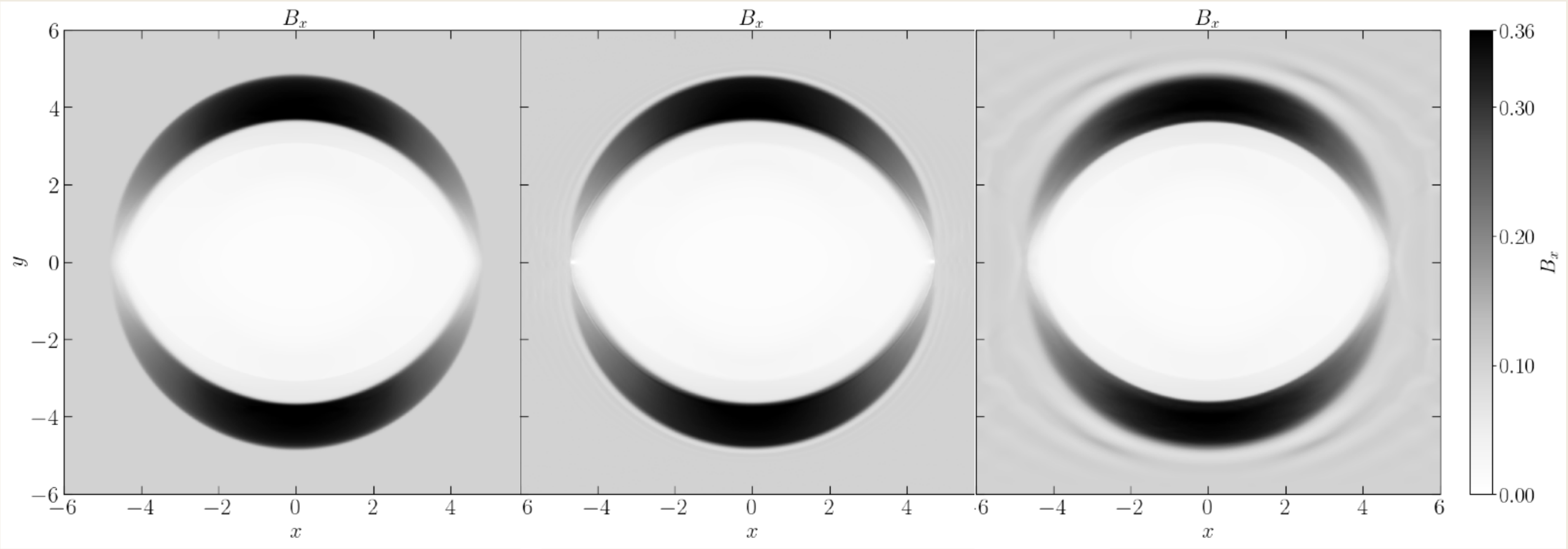
**Complications:**

- $\nabla_i \mathscr{B}^i = 0$ **=> Powell-like (8-waves) + Dedner divergence cleaning**
- tight connection between B and the hydrodynamic quantities => **an accurate prediction of the cell volume change over time is required, unlike in common fixed-grid codes**

# Disc magnetisation: GIZMO-GR(MHD)

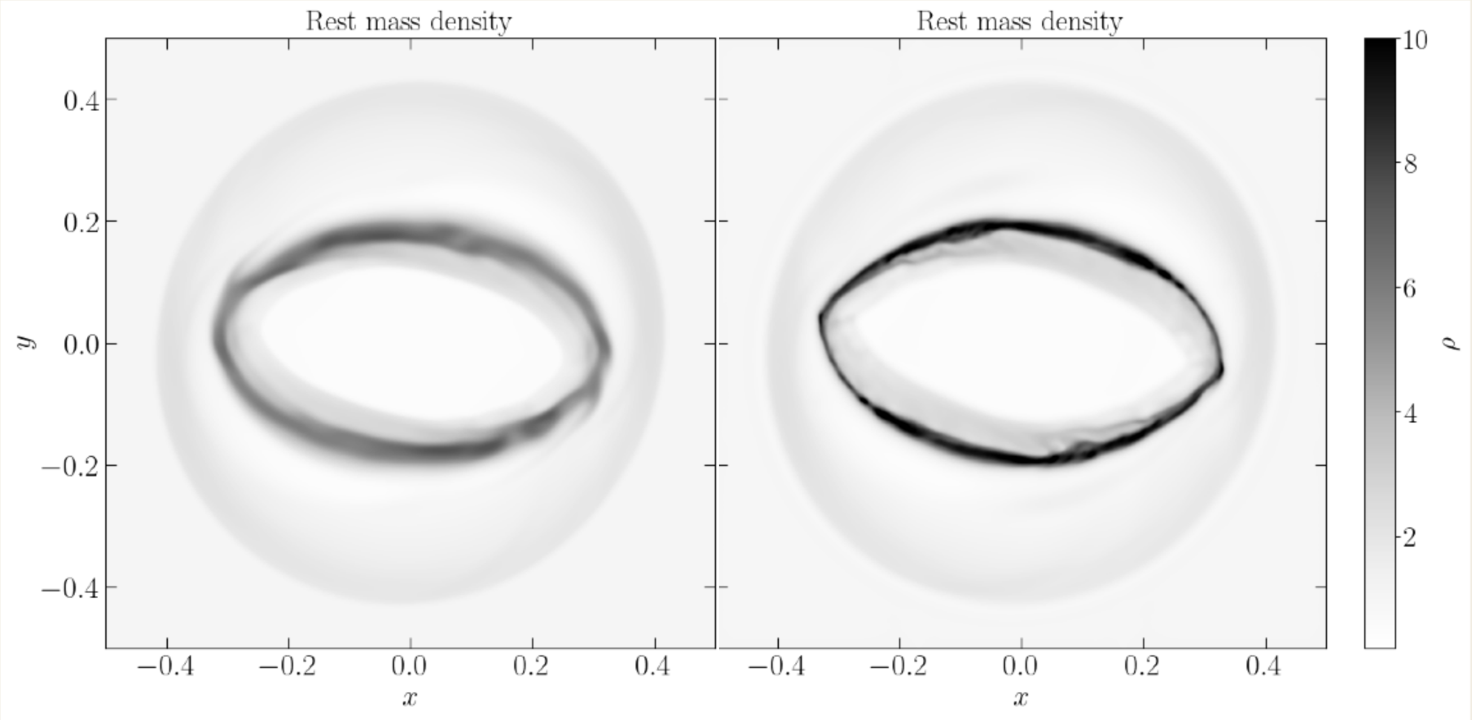
(Fedrigo & Lupi in prep.)

Fixed grid | MFV - eq. Vol. | MFM - eq. mass




Cylindrical BW

Fixed grid | MFM - eq. mass




Magnetic rotor

# Conclusions

- We extended the extensively used meshless code GIZMO to general relativistic hydrodynamics, demonstrating its potential.

- By incorporating an analytic metric for a MBHB binary, we showed that the code well captures the basic Bondi-like accretion on the two BHs prior to merger, consistent with NR results by Cattorini+22

  ➡ We are now in the position to apply this model to CBDs to study the transition from the Newtonian to the relativistic regime.

- We have incorporated magnetic field evolution in GIZMO-GR, which will allow us to consistently follow accretion disc formation and evolution from galaxy scales down to the horizon, all with the same code.