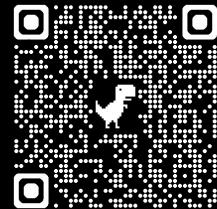


Constraining EMRI Population using Hierarchical Bayesian Inference

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5 Nov 2024

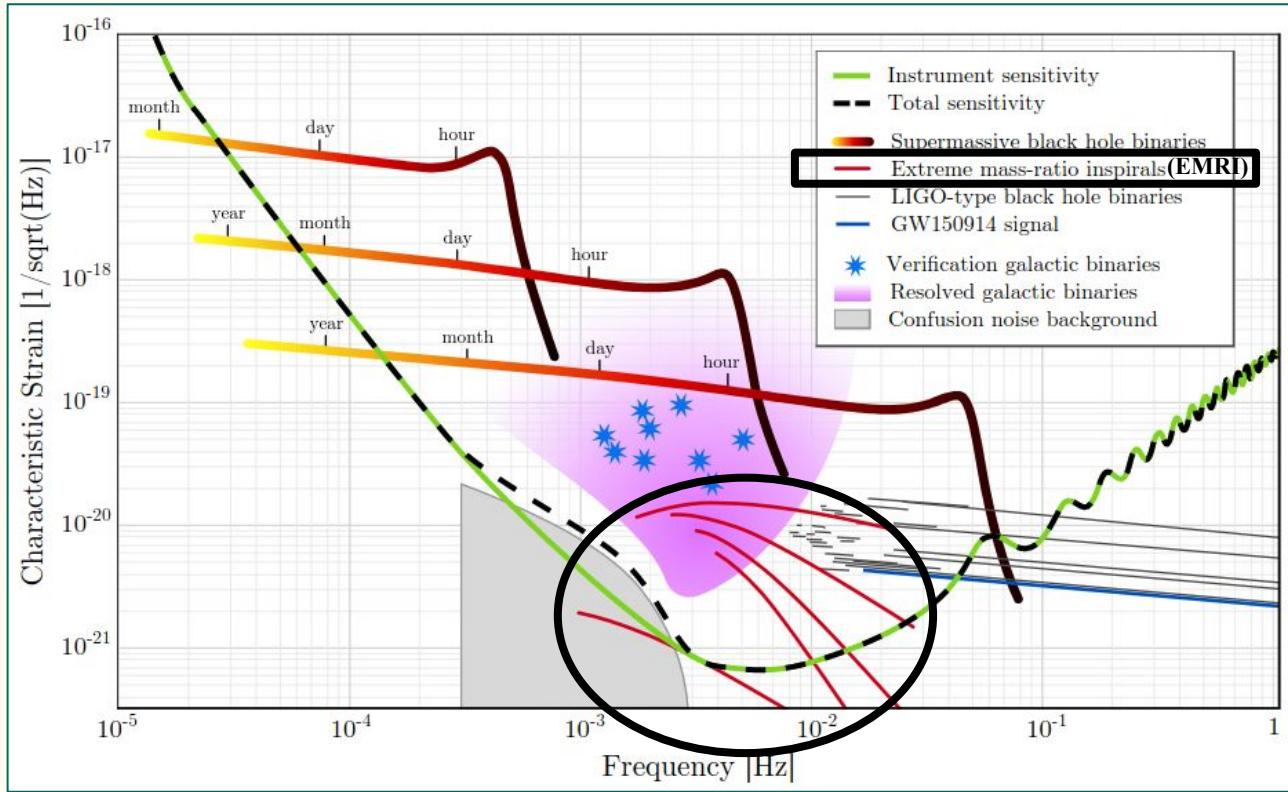


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LISA and its sources



Amaro-Seoane *et al.* arXiv:1702.00786

What is an EMRI ?

Inspiral of compact objects with
large mass ratio

complex dynamics \Rightarrow rich in information

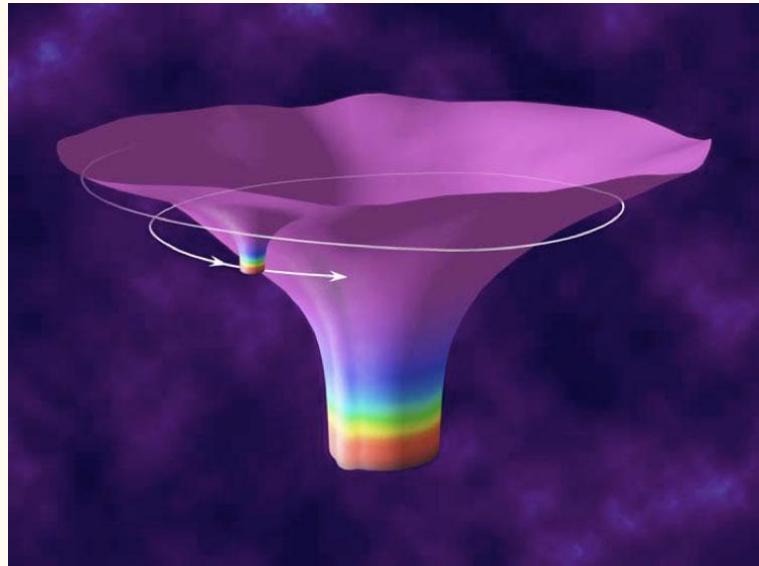
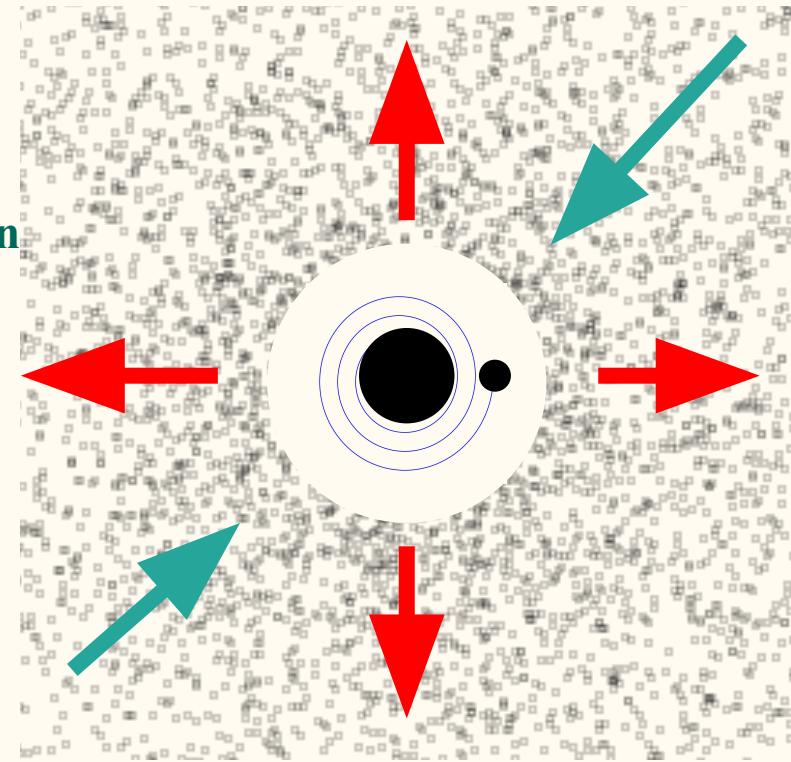


Image Credit: NASA

Cusp erosion

Mass segregation

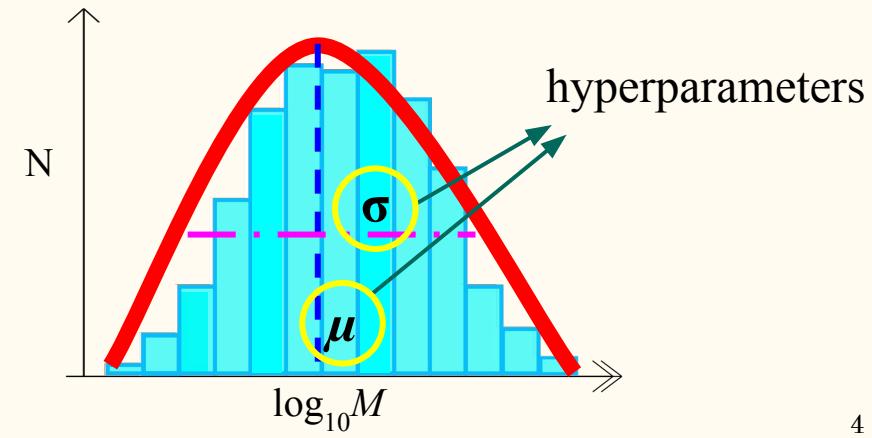
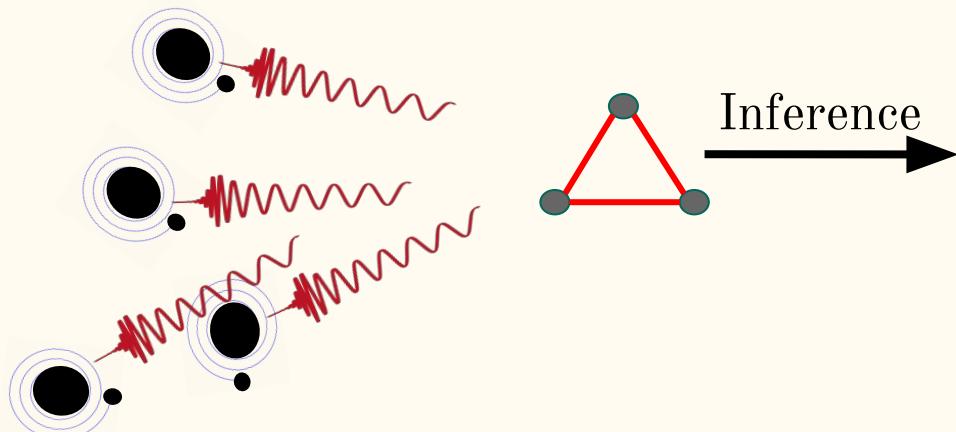


Astrophysical processes
affecting formation

Constraining the astrophysical parameters

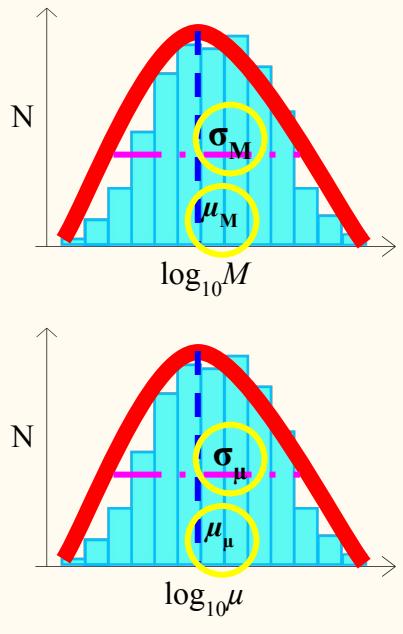
EMRI
detections

Astrophysical
processes

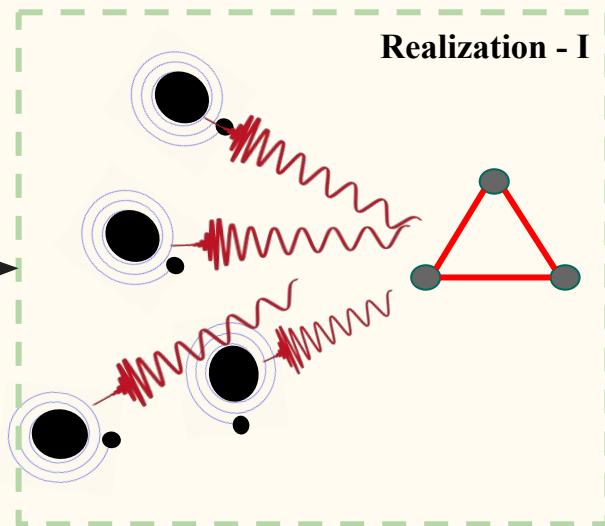


Our analysis

Astrophysical
processes



EMRI
detections

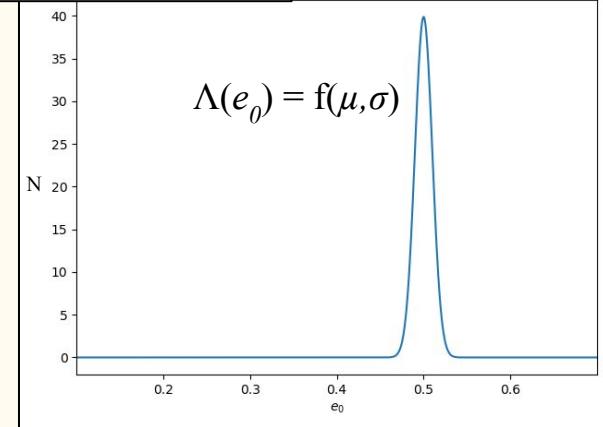
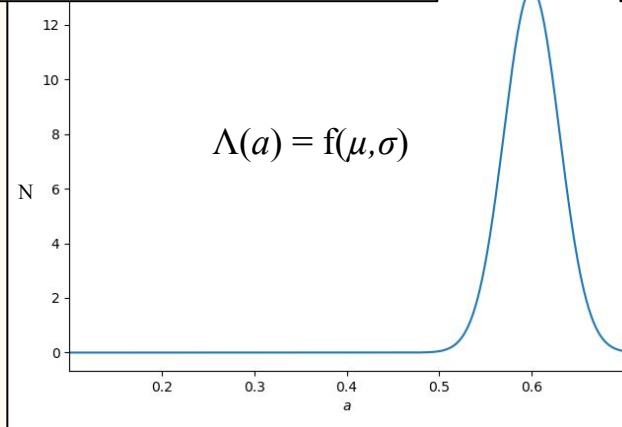
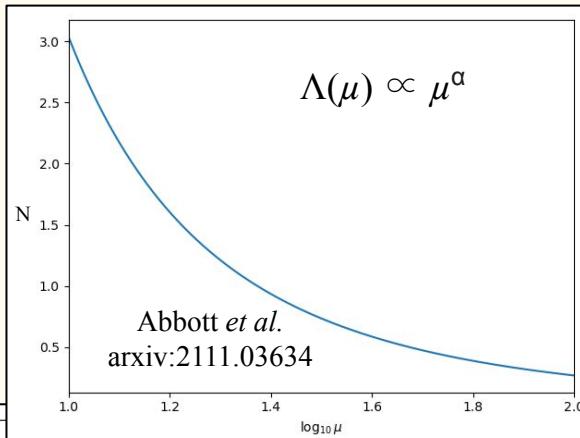
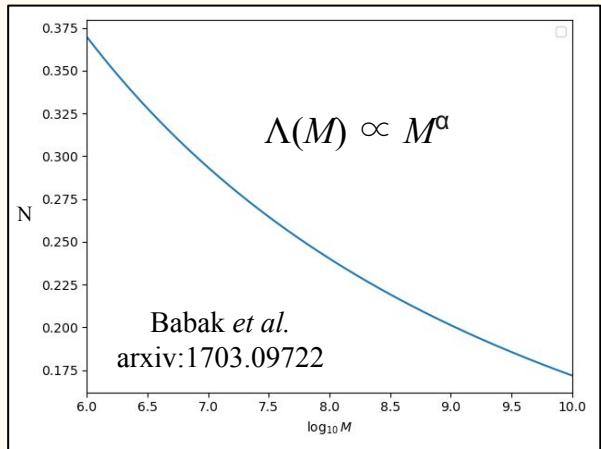


Inference
→
 $\text{SNR} > \rho_{\text{th}}$

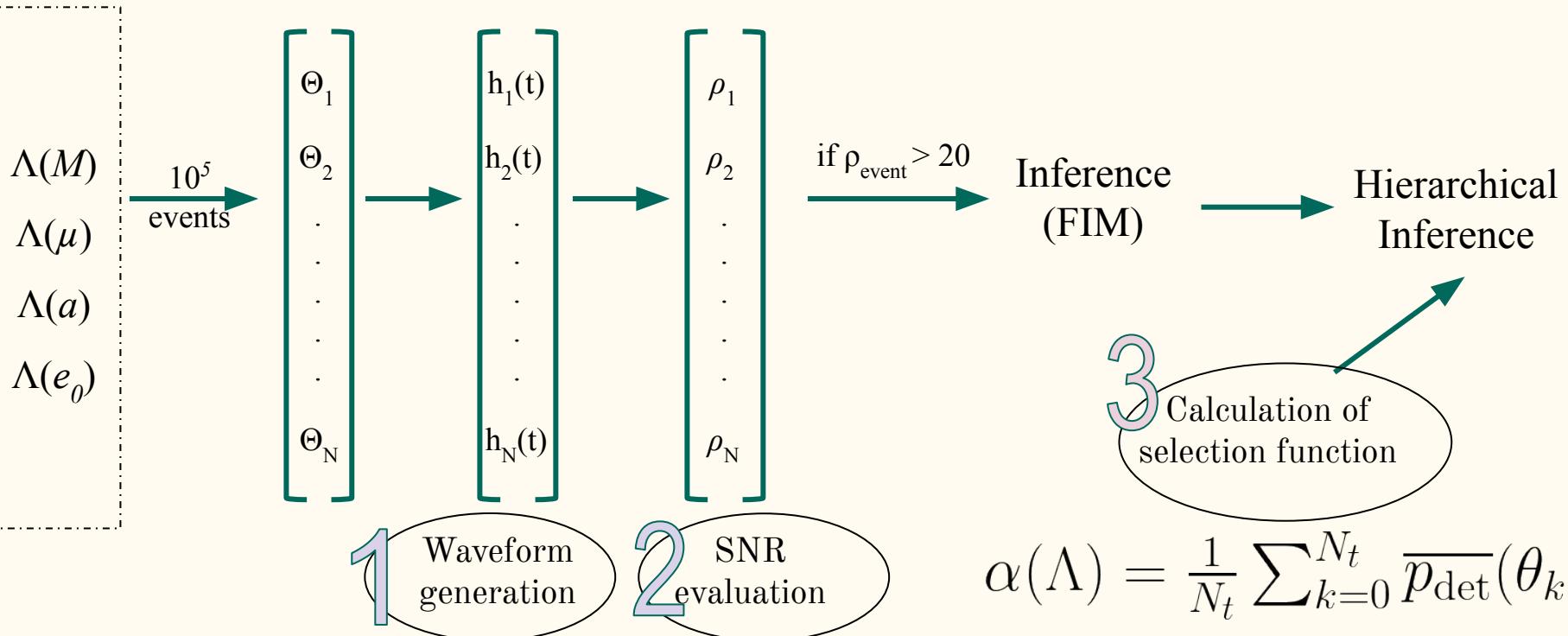
Hierarchical
Inference –
estimating
hyperparams

σ_M
 μ_M
 σ_μ
 μ_μ

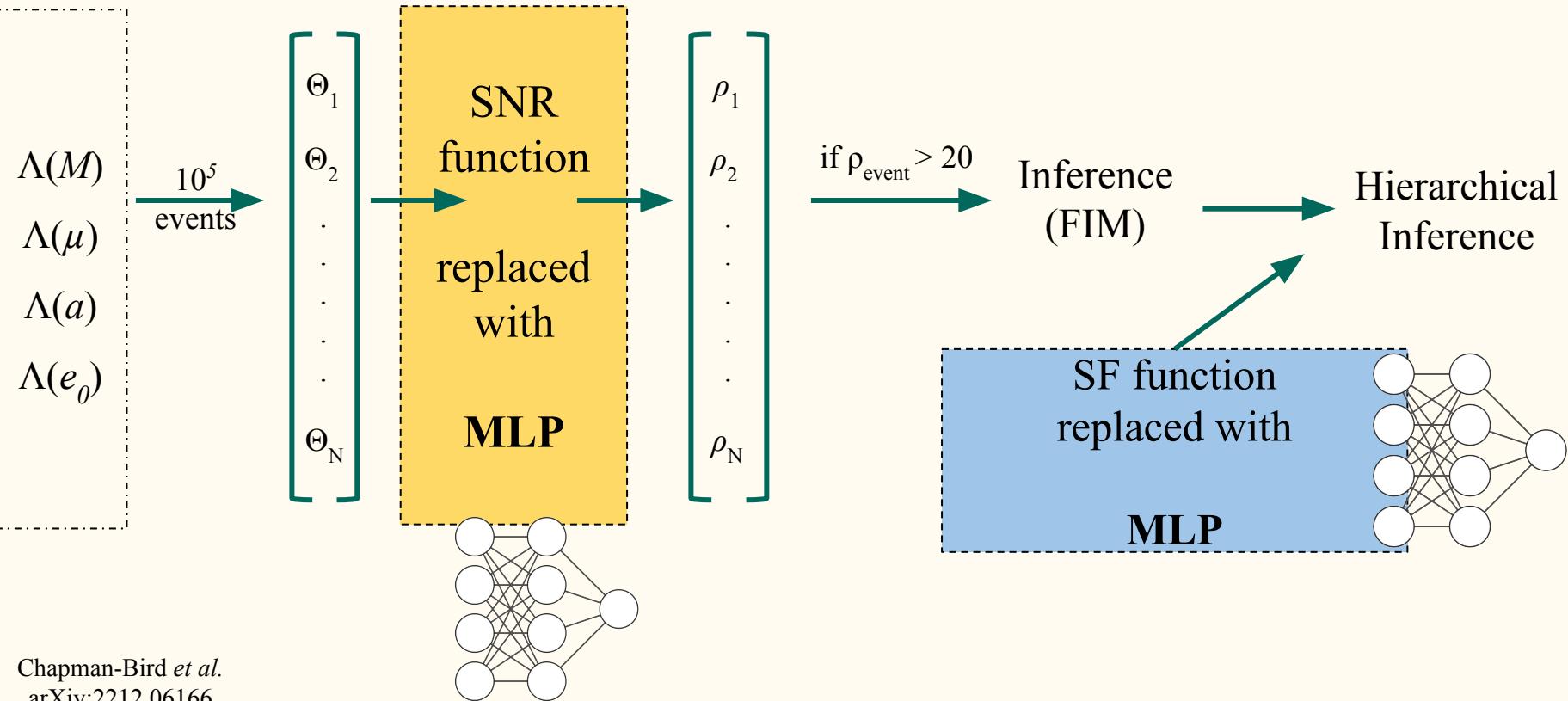
Our analysis – distributions



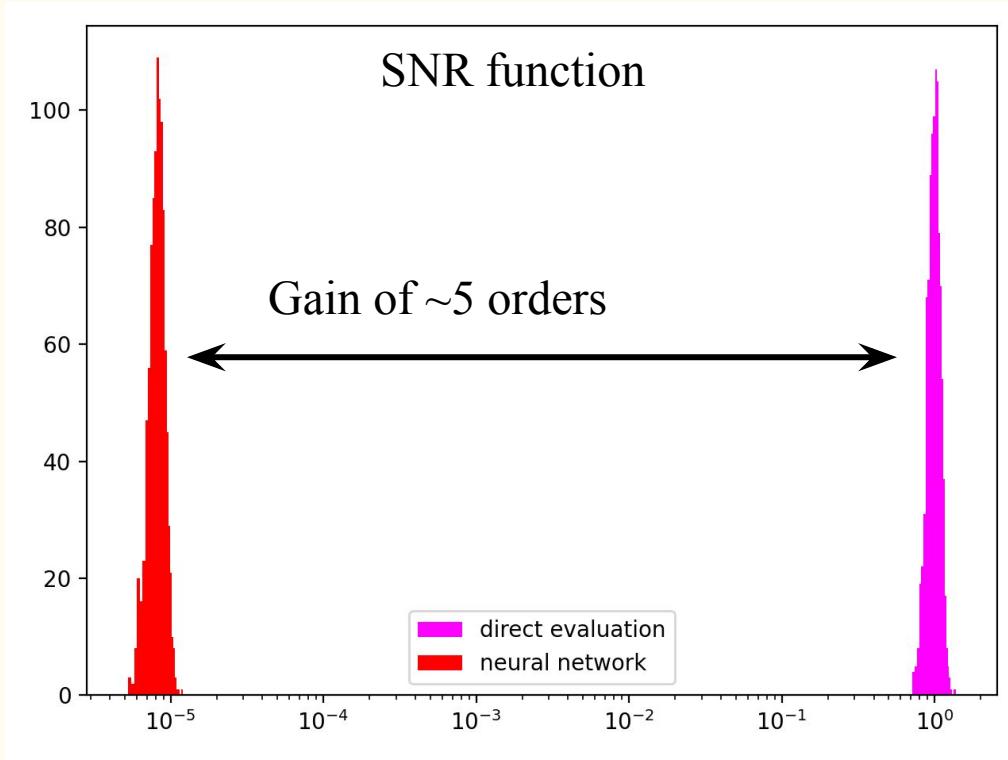
Our analysis



Our analysis – speed up



Our analysis – speed up

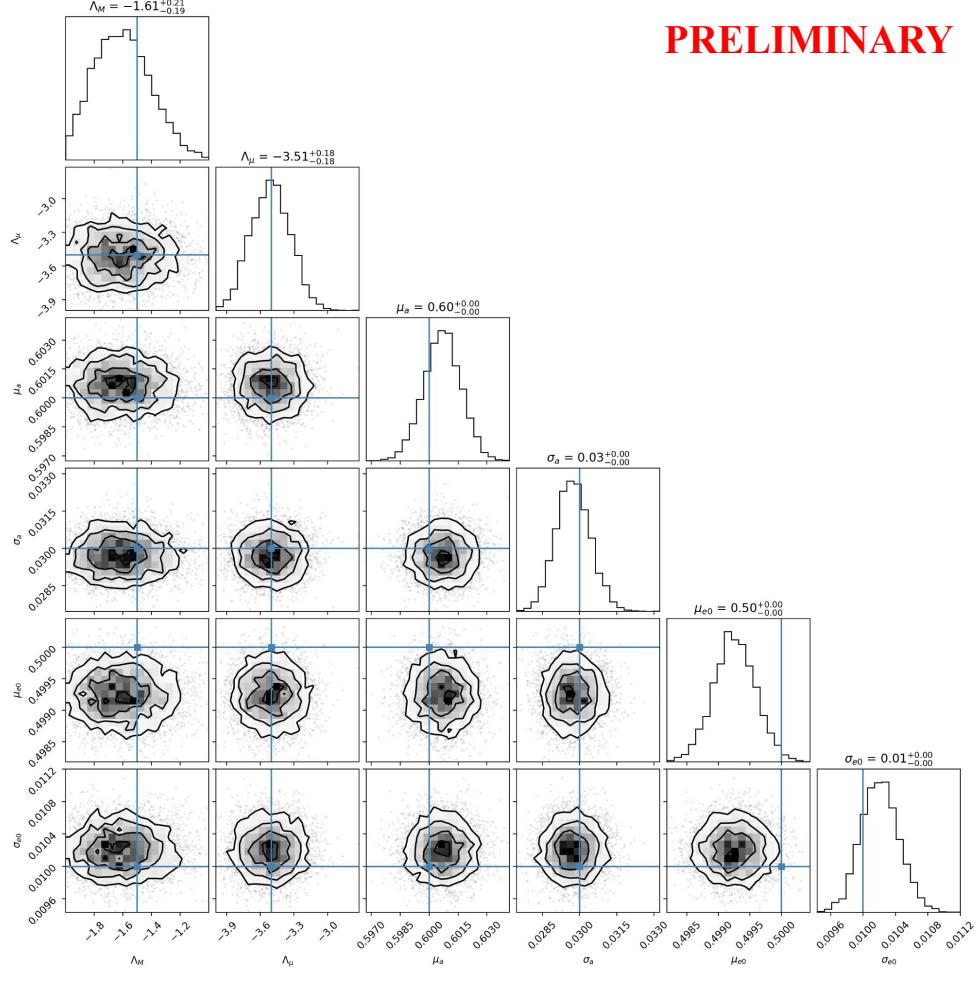


Gain in SF

Direct evaluation
 10^4 events \Rightarrow 1000s \sim 16 mins

Neural network
 10^4 events \Rightarrow .1s

PRELIMINARY



Results

- Slope of ***mass function of MBH and CO*** are constrained within **15%** and **5%**. **Spin & eccentricity** are within **5%**.
- Inference on $\sim 2.10^2$ events out of 10^4 injections for SNR threshold of 20.
- We are able to speed up the
 - SNR analysis
 - SF evaluation
 using neural networks.

Conclusion

- With multiple EMRI detections, it will be possible to constrain EMRI population parameters.
- Formation mechanism is affected by various astrophysical processes which are hard to infer with the current detections.
- Using MLP it is possible to speed up the SNR evaluation upto 5 orders of magnitude and consequently likelihood evaluation.
- With our method it is possible to constrain the population parameters and consequently study the formation mechanism.

END

Our analysis – *hierarchical inference*

Prior

Likelihood function

$$p(\lambda \mid d_i) = \pi(\Lambda) \prod_{i=1}^{N_{\text{obs}}} \frac{\frac{1}{S_i} \sum_{j=1}^{S_i} \frac{p_{\text{pop}}(\theta_i^j \mid \Lambda)}{\pi(\theta)}}{\int d\theta p_{\text{det}}(\theta) p_{\text{pop}}(\theta \mid \Lambda)} \times e^{-N_{\text{det}}} (N_{\text{det}})^{N_{\text{obs}}}$$

selection function

$$\alpha(\Lambda) = \frac{1}{N_t} \sum_{k=0}^{N_t} \overline{p_{\text{det}}}(\theta_k)$$

A diagram illustrating the hierarchical inference process. It shows the joint probability $p(\lambda \mid d_i)$ as a product of a prior $\pi(\Lambda)$ (enclosed in a green box) and a likelihood function (enclosed in a blue box). The likelihood function contains a selection function term, which is highlighted with a red oval and connected by a dashed teal arrow to the selection function equation below.