Parity-Odd Power Spectra: Concise Statistics for Cosmological Parity Violation

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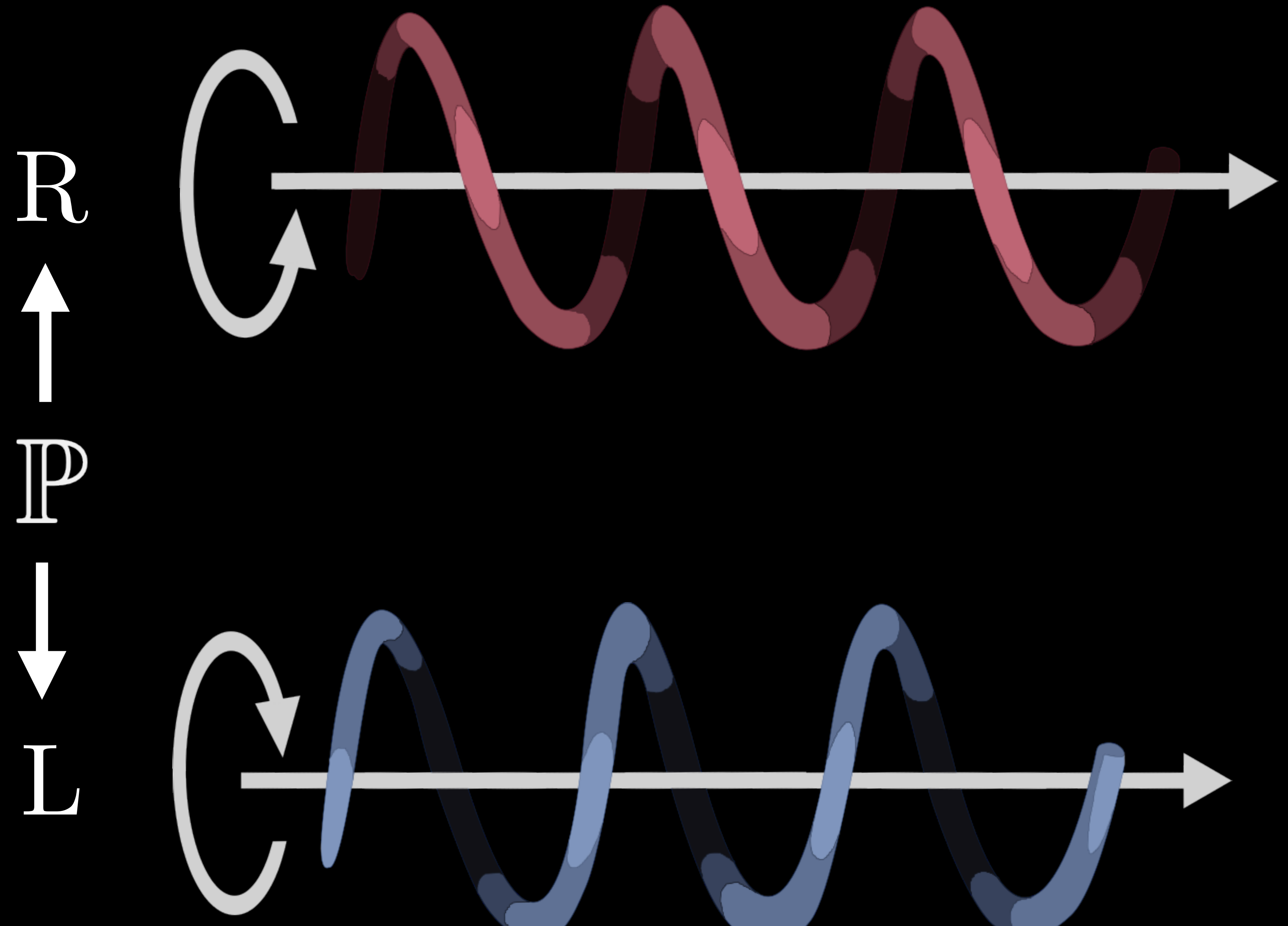
FÜR ASTROPHYSIK

Parity Violation

Spacetime transform:

$$\mathbb{P} : \mathbf{x} \to -\mathbf{x}$$

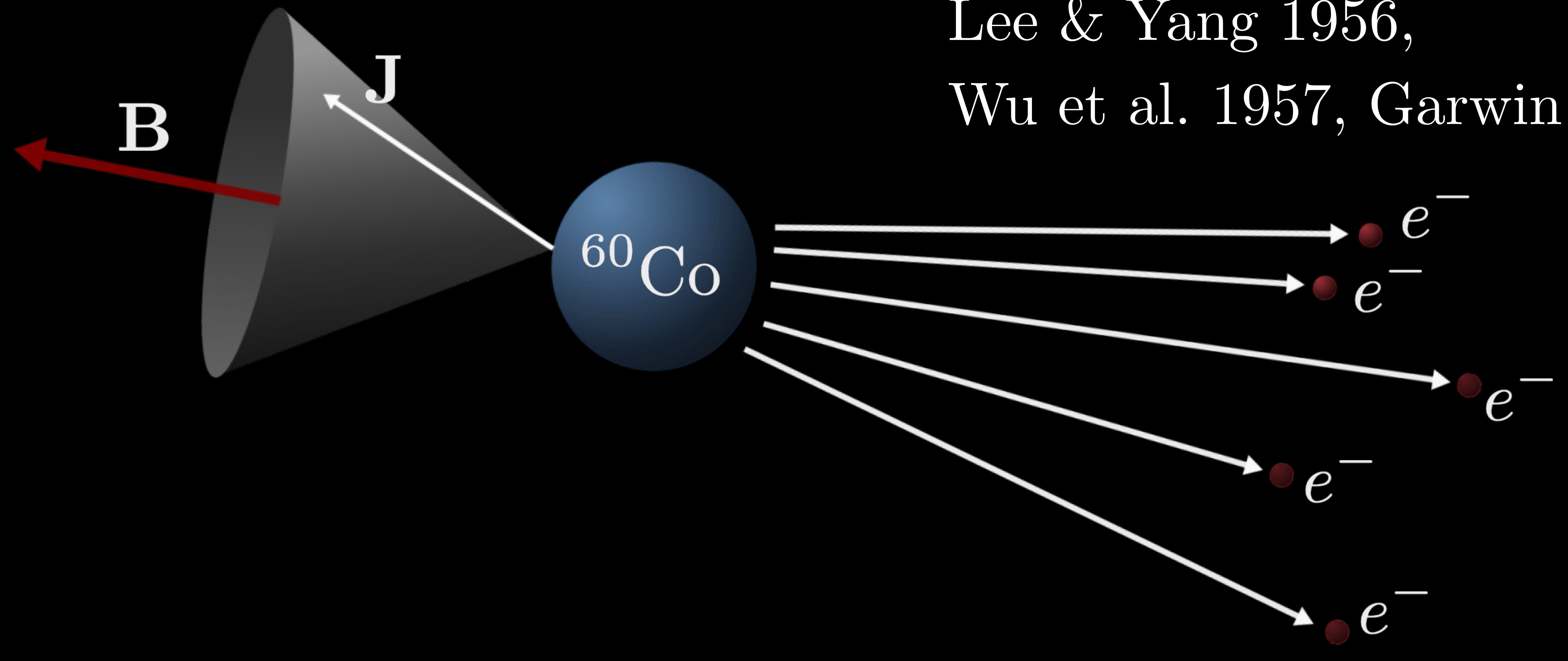
Distinguishes right/left helicities



Known to be violated on small scales:

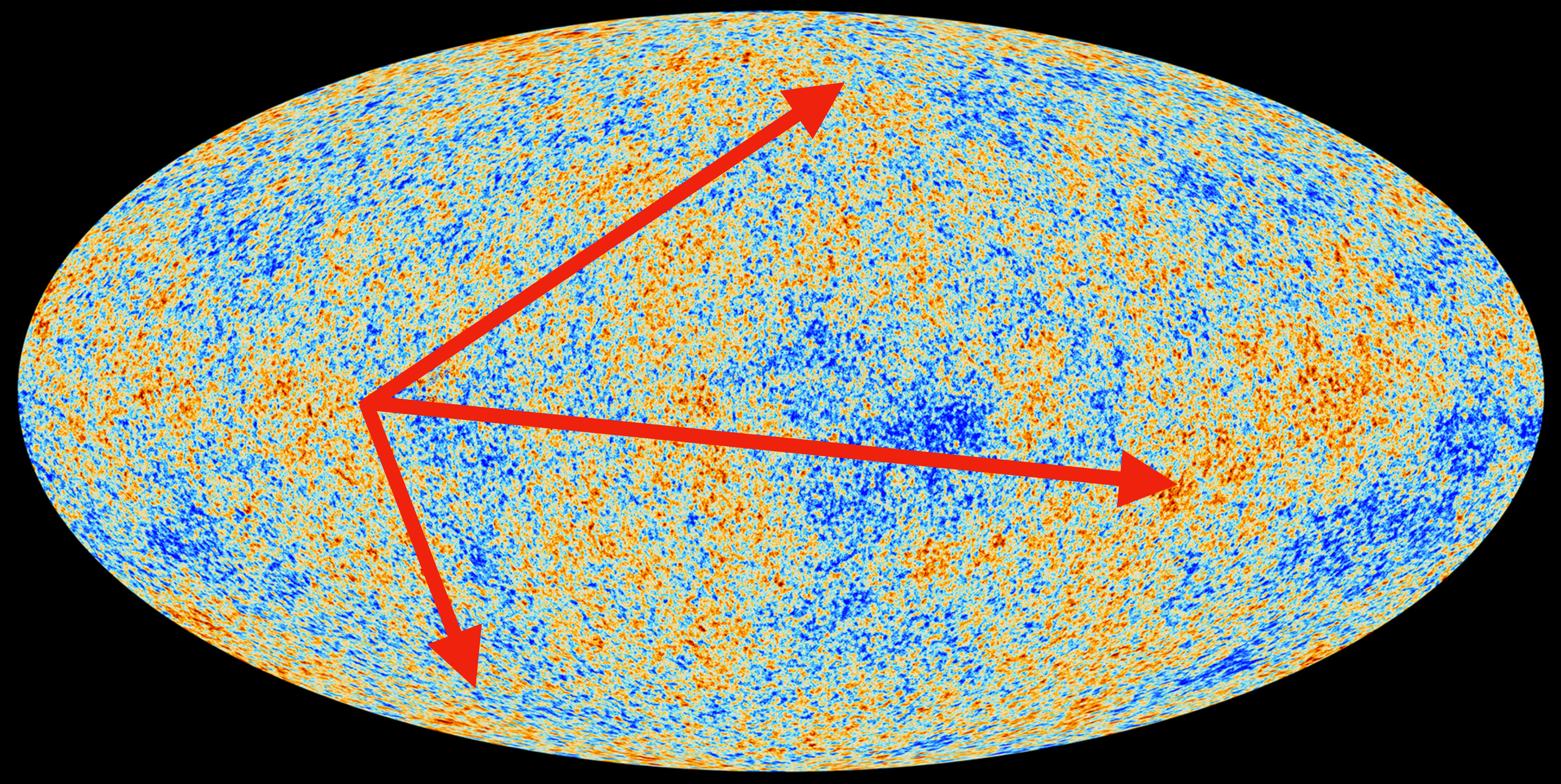
- Weak nuclear force

Lee & Yang 1956,
Wu et al. 1957, Garwin et al. 1957



- Spontenously broken in biology
  - Chiral molecules
  - Organism morphology

Primordial Parity Violation



Planck 2018

From inflation:

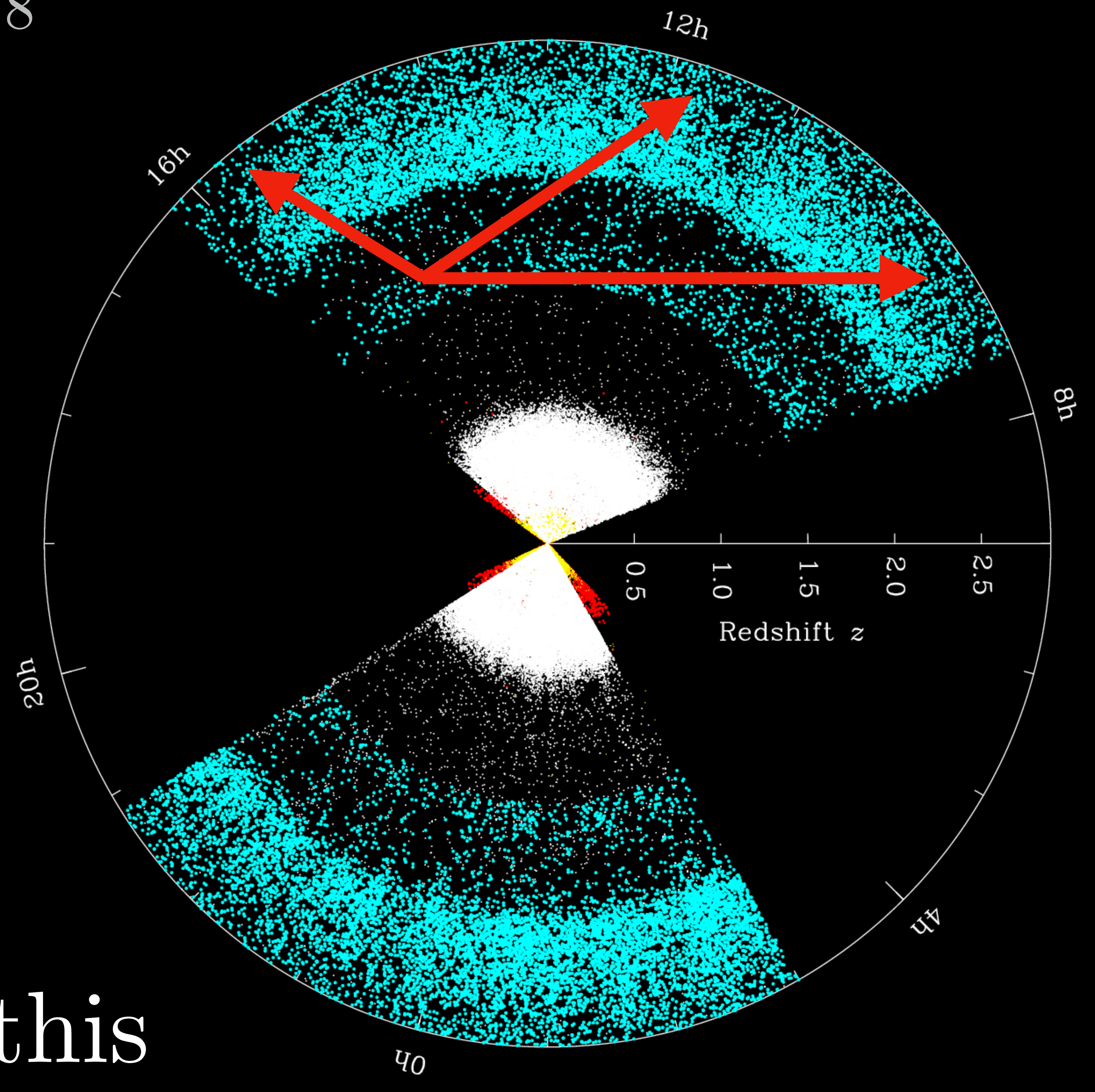
- Axion-like fields

  Sorbo 2011, Barnaby 2012, Shiraishi 2016

- Non-standard vaccua
- Interactions with massive spinning particles

  Cabass, Jazayeri, Pajer, Stefanyszyn 2023

  Cabass, Ivanov, Philcox 2023



BOSS

Signatures imprint on CMB and LSS

Lue 1999, Cahn 2021, Philcox 2022, Hou 2022, Philcox, Shiraishi 2023

Need to measure (at least) the 4-point statistics to probe this

Parity in the Trispectrum

Really $4! = 24$
Tetrahedra

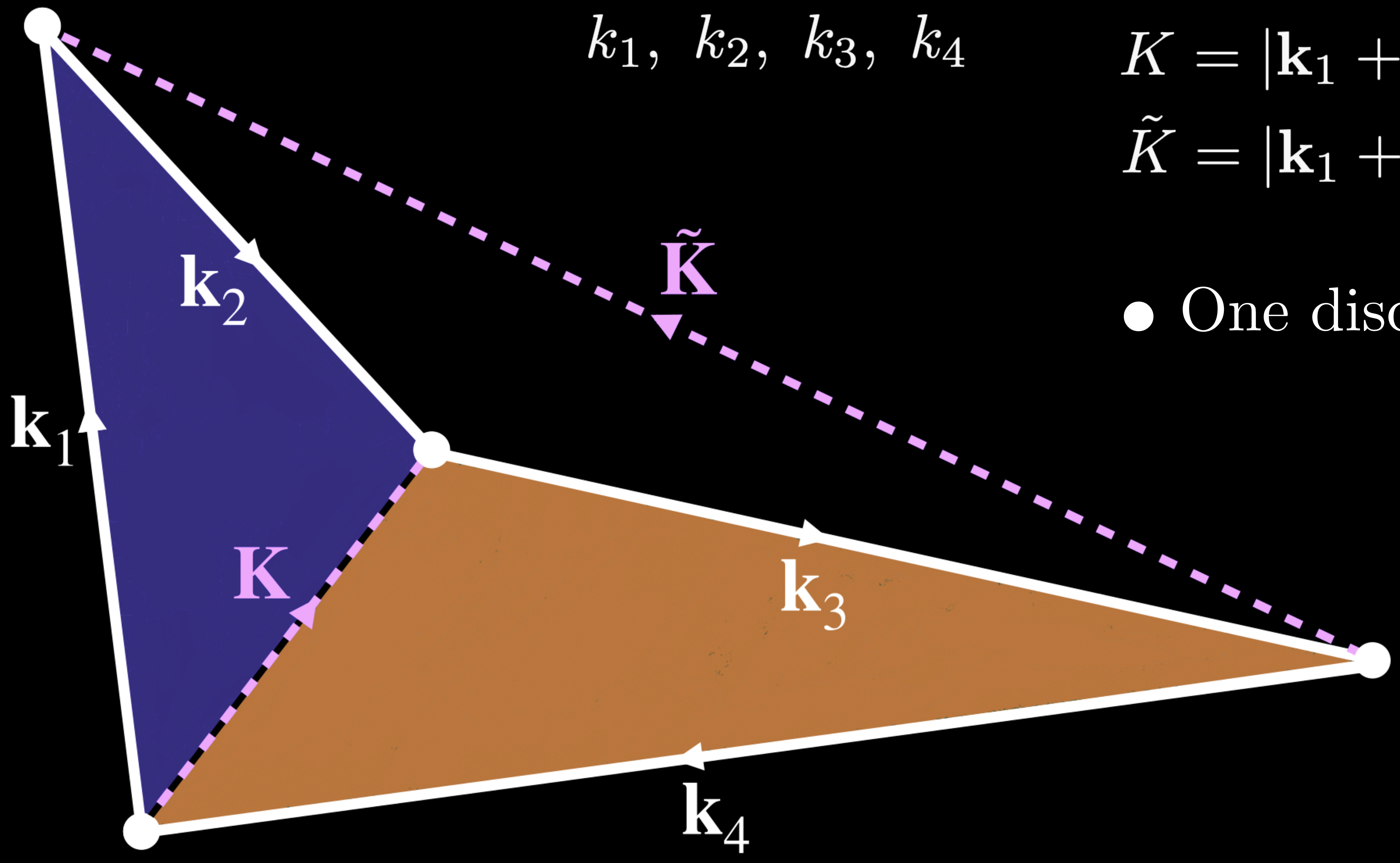
- Trispectrum is a function of wave-vector tetrahedra

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle = (2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- Six continuous degrees of freedom

$$k_1,\ k_2,\ k_3,\ k_4$$

$$K = |\mathbf{k}_1+\mathbf{k}_2| = |\mathbf{k}_3+\mathbf{k}_4|$$
$$\tilde{K} = |\mathbf{k}_1+\mathbf{k}_4| = |\mathbf{k}_2+\mathbf{k}_3|$$



- One discrete degree of freedom (helicity)

$$\mathrm{sign}\Big(\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)\Big)$$

- Imaginary trispectrum is parity-odd

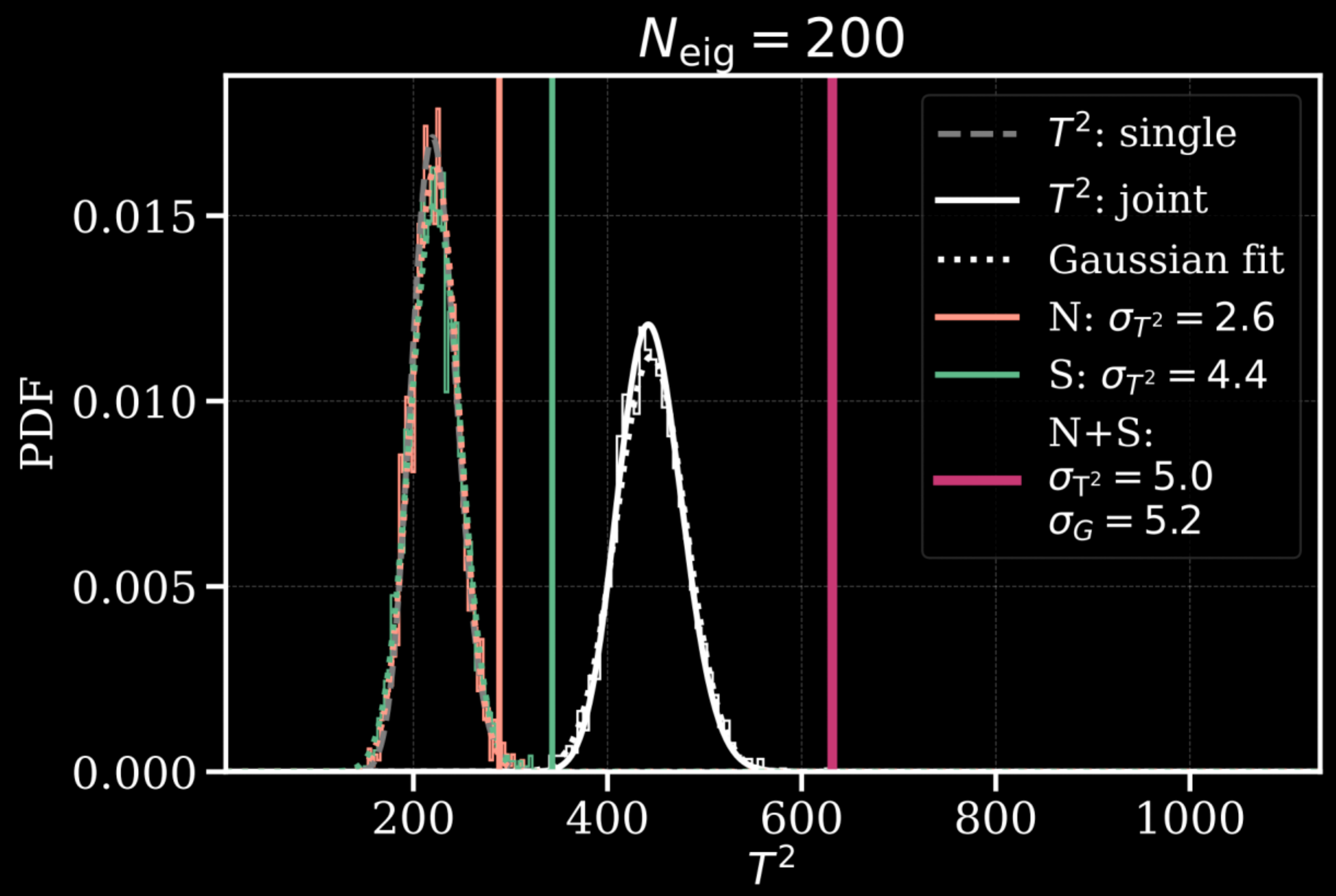
$$T = T_+ + iT_-$$

Observational Evidence

Detected in BOSS data with significance:

3.1σ–7.1σ Hou, Slepian, Cahn 2023

2.9σ Philcox 2022



Hou, Slepian, Cahn 2023

Observational Challenges:

- Covariance
  - High-dimensional data vector
  - Sensitivity to mocks?
  - Bias significance
- Observational systematics
  - Survey window/mask
  - Bias signal

CMB consistent with parity invariance

Philcox 2023

Philcox & Shiraishi 2024

Compressed statistics

- Build composite fields

$$\delta_\mathrm{m}(\mathbf{x})^2,\ \delta_\mathrm{m}(\mathbf{x})^3,\ \ldots$$

- Compute low-order correlation functions

$$\langle\delta_\mathrm{m}(\mathbf{x})\delta_\mathrm{m}(\mathbf{x}')^2\rangle,\ \langle\delta_\mathrm{m}(\mathbf{x})^2\delta_\mathrm{m}(\mathbf{x}')^2\rangle,\ \langle\delta_\mathrm{m}(\mathbf{x})\delta_\mathrm{m}(\mathbf{x})^3\rangle,\ \ldots$$

Skew spectrum

Schmittfull, Baldauf, Seljak 2015
Dizgah, Lee, Schmittfull, Dvorkin 2020
Hou et al. 2024

Kurt spectra

Munshi, Lee, Dvorkin, McEwena 2022

These are integrated N-point statistics

Compressed statistics

How to include parity sensitivity?

$$\nabla(?) \cdot \Big(\nabla(?) \times \nabla(?)\Big)$$

- Build composite fields

$$\delta_{\rm m}(\mathbf{x})^2,\ \delta_{\rm m}(\mathbf{x})^3,\ \ldots$$

- Compute low-order correlation functions

$$\langle\delta_{\rm m}(\mathbf{x})\delta_{\rm m}(\mathbf{x}')^2\rangle,\ \underbrace{\langle\delta_{\rm m}(\mathbf{x})^2\delta_{\rm m}(\mathbf{x}')^2\rangle,\ \langle\delta_{\rm m}(\mathbf{x})\delta_{\rm m}(\mathbf{x})^3\rangle}_{\text{Kurt spectra}},\ \ldots$$

Skew spectrum

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Hou et al. 2024

Kurt spectra

Munshi, Lee, Dvorkin, McEwena 2022

These are integrated N-point statistics

Parity-odd Power Spectra (POP Spectra)

Ingredients:

- Triple vector product: $\mathbf{k}_1 \cdot \left(\mathbf{k}_2 \times \mathbf{k}_3\right)$
- Composite fields: $\mathcal{O}(\Phi(\mathbf{x})^2)$ and $\mathcal{O}(\Phi(\mathbf{x})^3)$
- Smoothing kernels: $\Phi_a(\mathbf{k}) \equiv f_a(k)\Phi(\mathbf{k})$

Two possibilities:

Jamieson et al. arXiv:2406.15683

Vector POP Spectrum $\left\langle \Phi_a(\mathbf{x})\nabla\Phi_b(\mathbf{x}) \cdot \nabla\Phi_c(\mathbf{x}') \times \nabla_d\Phi_b(\mathbf{x}')\right\rangle$

Scalar POP Spectrum $\left\langle \Phi(\mathbf{x})\nabla\Phi_a(\mathbf{x}') \cdot \left(\nabla\Phi_b(\mathbf{x}') \times \nabla\Phi_c(\mathbf{x}')\right)_d\right\rangle$

Vector POP Spectrum

Composite fields:

$$\mathbf{V}_{ab}(\mathbf{x}) = \Phi_a(\mathbf{x})\nabla\Phi_b(\mathbf{x})$$

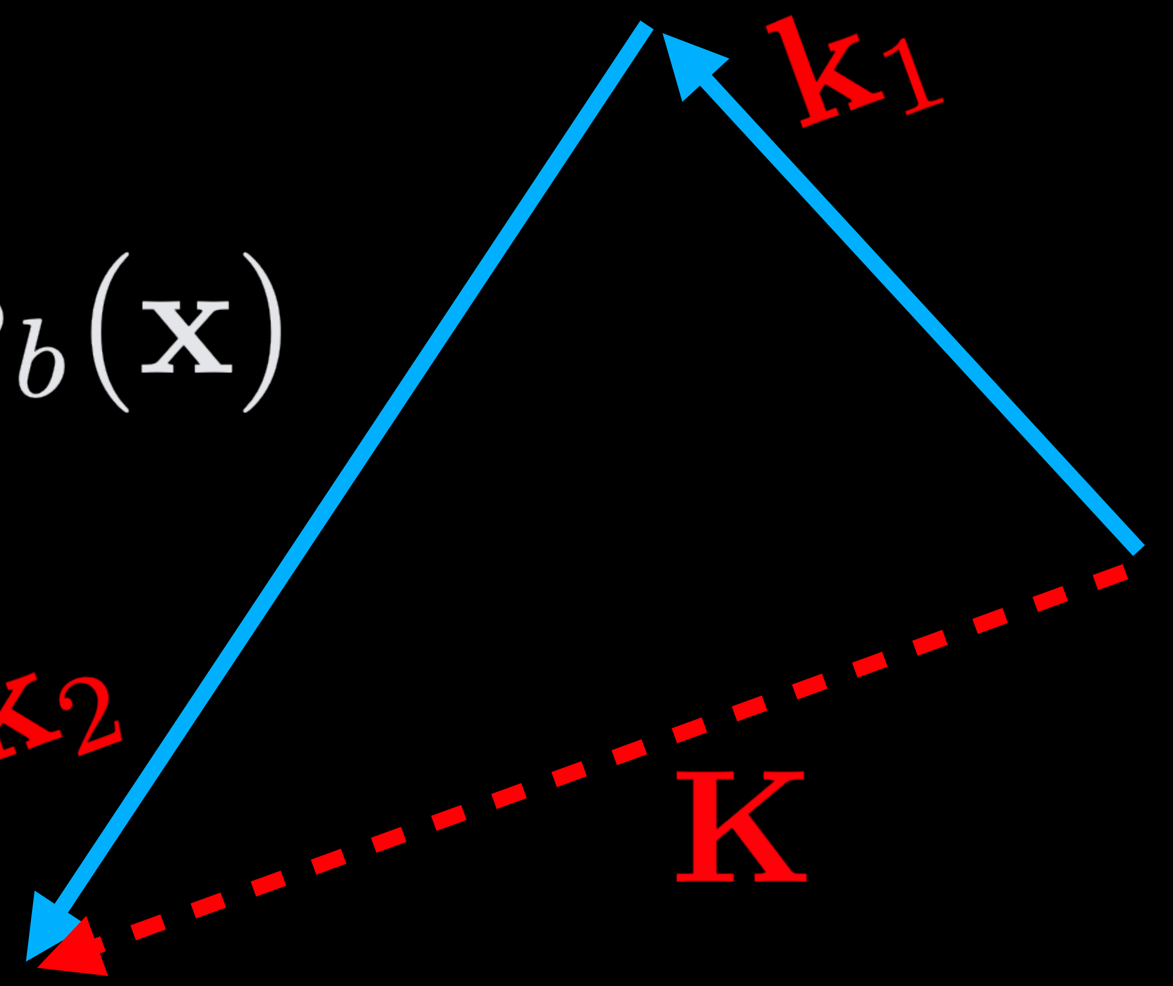
$$\mathbf{A}_{cd}(\mathbf{x}) = \nabla\Phi_c(\mathbf{x}) \times \nabla\Phi_d(\mathbf{x})$$

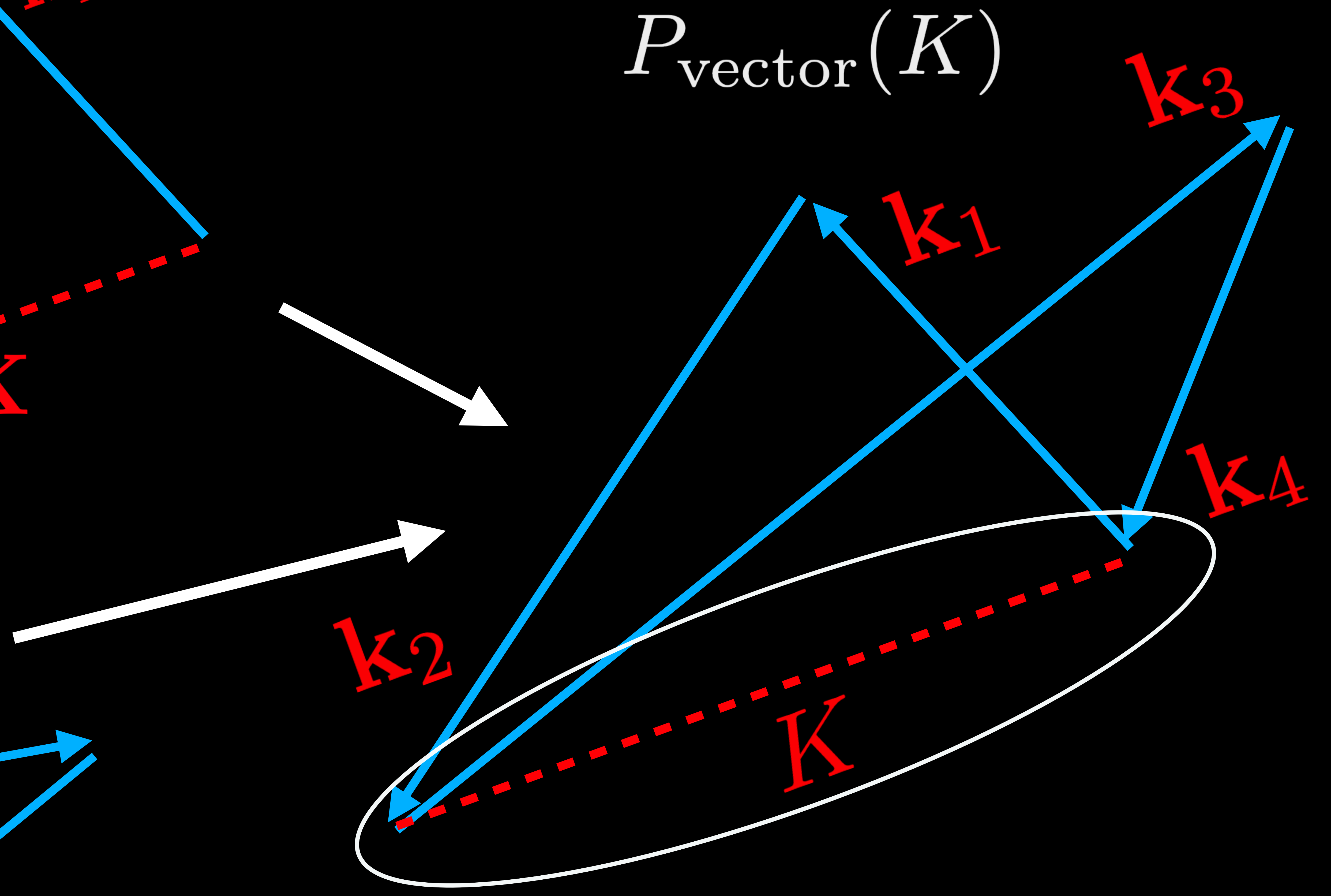
POP spectrum:

$$\langle \mathbf{V}_{ab}(\mathbf{k}) \cdot \mathbf{A}_{cd}(\mathbf{k}') \rangle = (2\pi)^3 \delta_{\mathrm{D}}^{(3)}(\mathbf{k} + k') P_{\mathrm{vector}}(k)$$

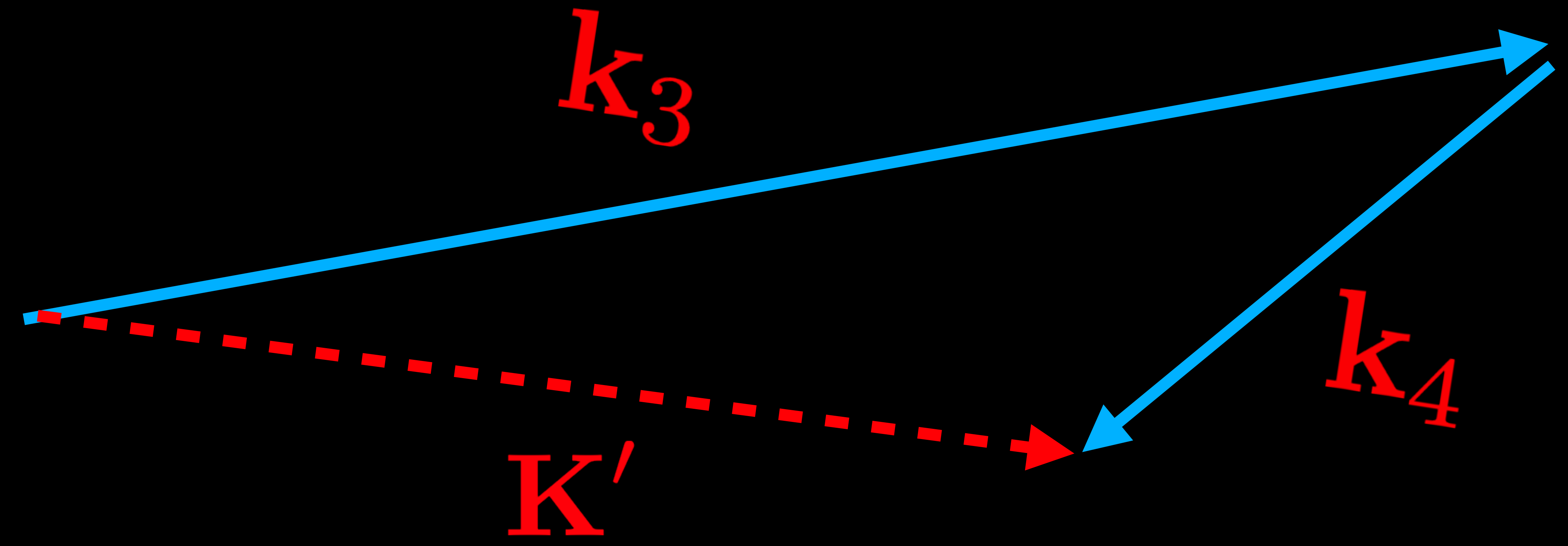
Vector POP Spectrum

$$\mathbf{V}_{ab}(\mathbf{x}) = \Phi_a(\mathbf{x})\nabla\Phi_b(\mathbf{x})$$





$$\mathbf{A}_{cd}(\mathbf{x}) = \nabla\Phi_c(\mathbf{x}) \times \nabla\Phi_d(\mathbf{x})$$



An Example Template

Non-Gaussian primordial potential with a parity-violating trispectrum

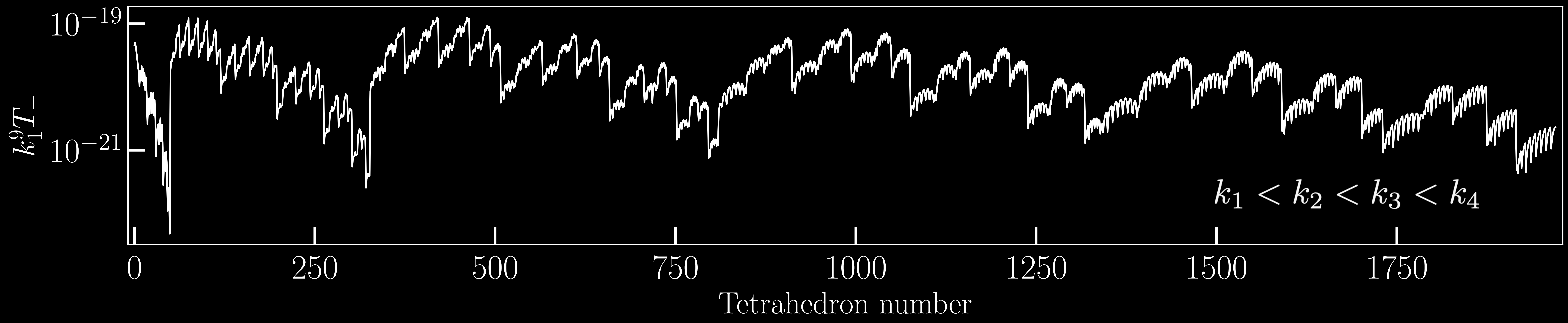
$$\Phi(\mathbf{x}) = \Phi_{\mathrm{G}}(\mathbf{x}) + g_- \nabla\Phi_{\mathrm{G}}(\mathbf{x}) \cdot \left(\nabla\Phi_{\mathrm{G}}^{[-1]}(\mathbf{x}) \times \nabla\Phi_{\mathrm{G}}^{[-2]}(\mathbf{x})\right)$$

Coulton, Philcox, Vallaescusa-Navarro 2023

$$\Phi_{\mathrm{G}}^{[a]}(\mathbf{k}) = k^a \Phi_{\mathrm{G}}(\mathbf{k})$$

Parity-odd trispectrum:

$$T\big(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\big) = i g_- \mathbf{k_1} \cdot \big(\mathbf{k}_2 \times \mathbf{k}_3\big)(2\pi^2 A_s)^3 \Big(k_1^{-5} k_2^{-4} k_3^{-3} k_4^{0} + 23 \text{ perms}\Big)$$



An example estimator

Choice of smoothing kernels:

$$f_a(k) \propto k^2$$

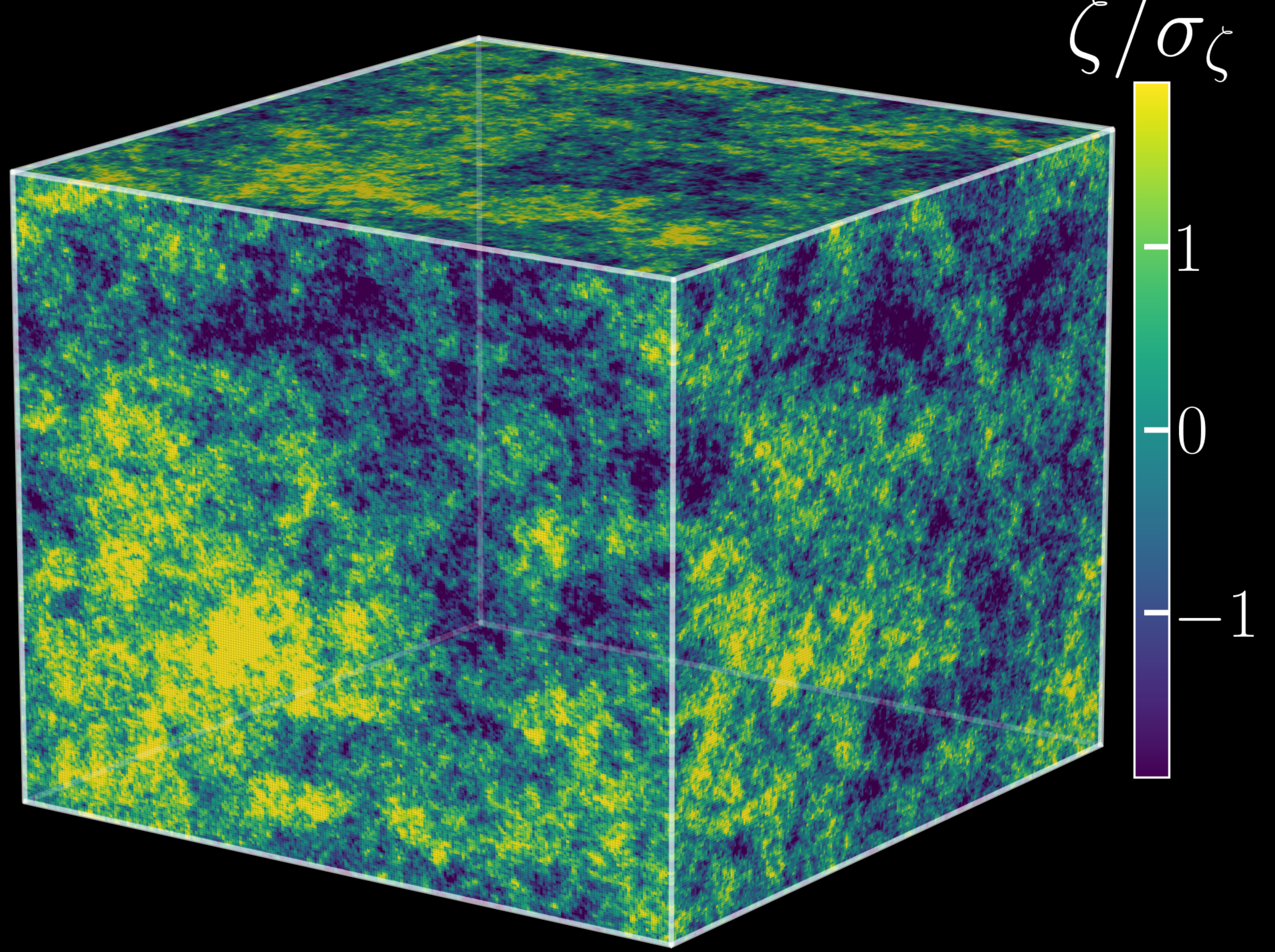
$$f_c(k) \propto k^{-2}$$

$$f_b(k) = f_d(k) \propto 1$$

Scale cuts:

$$k_{\rm min} = 5 \times 10^{-3}\ {\rm Mpc}^{-1}\ h$$

$$k_{\rm max} = 2 \times 10^{-1}\ {\rm Mpc}^{-1}\ h$$



Vector POP Spectrum

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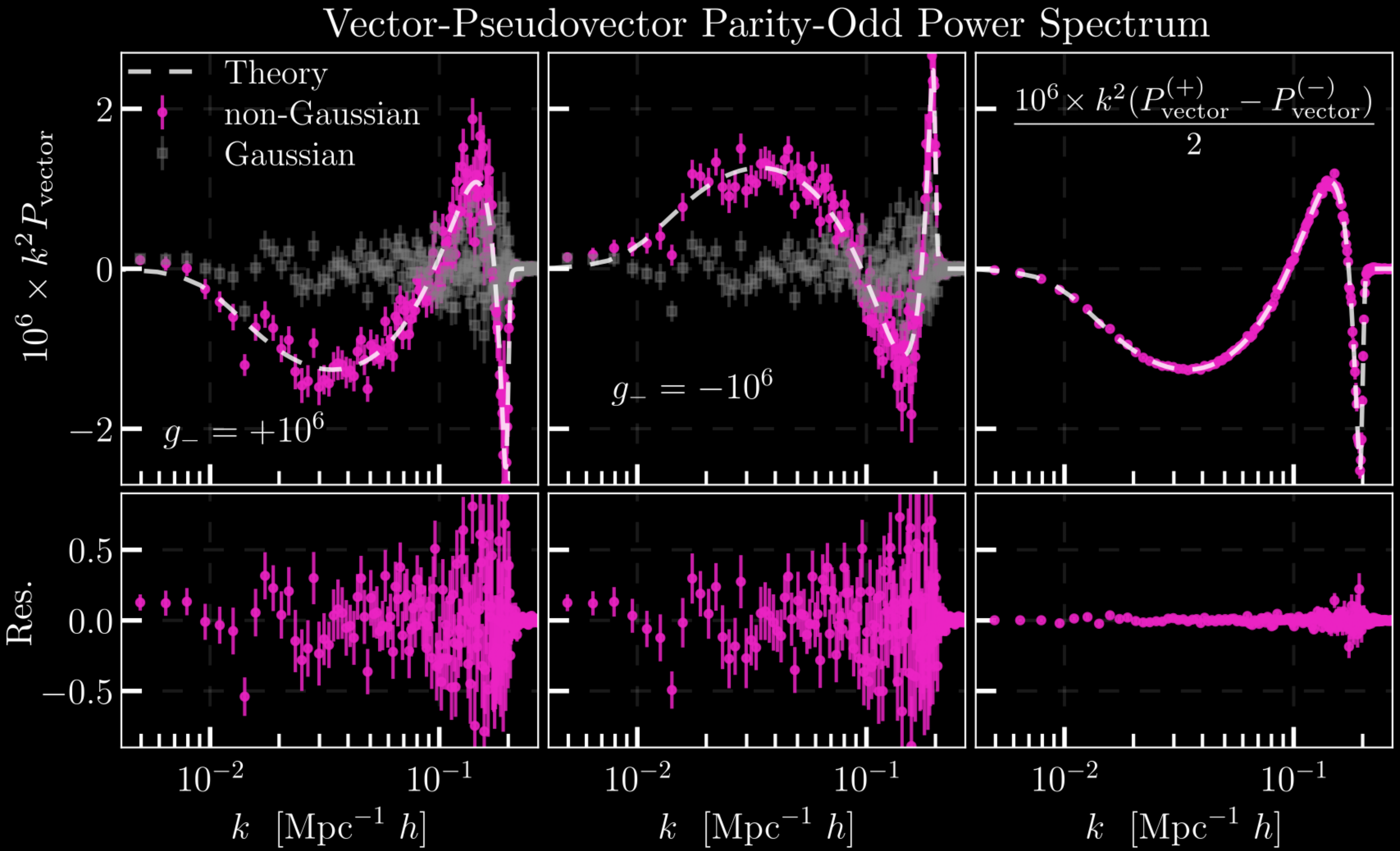


Scalar POP Spectrum

Composite field:

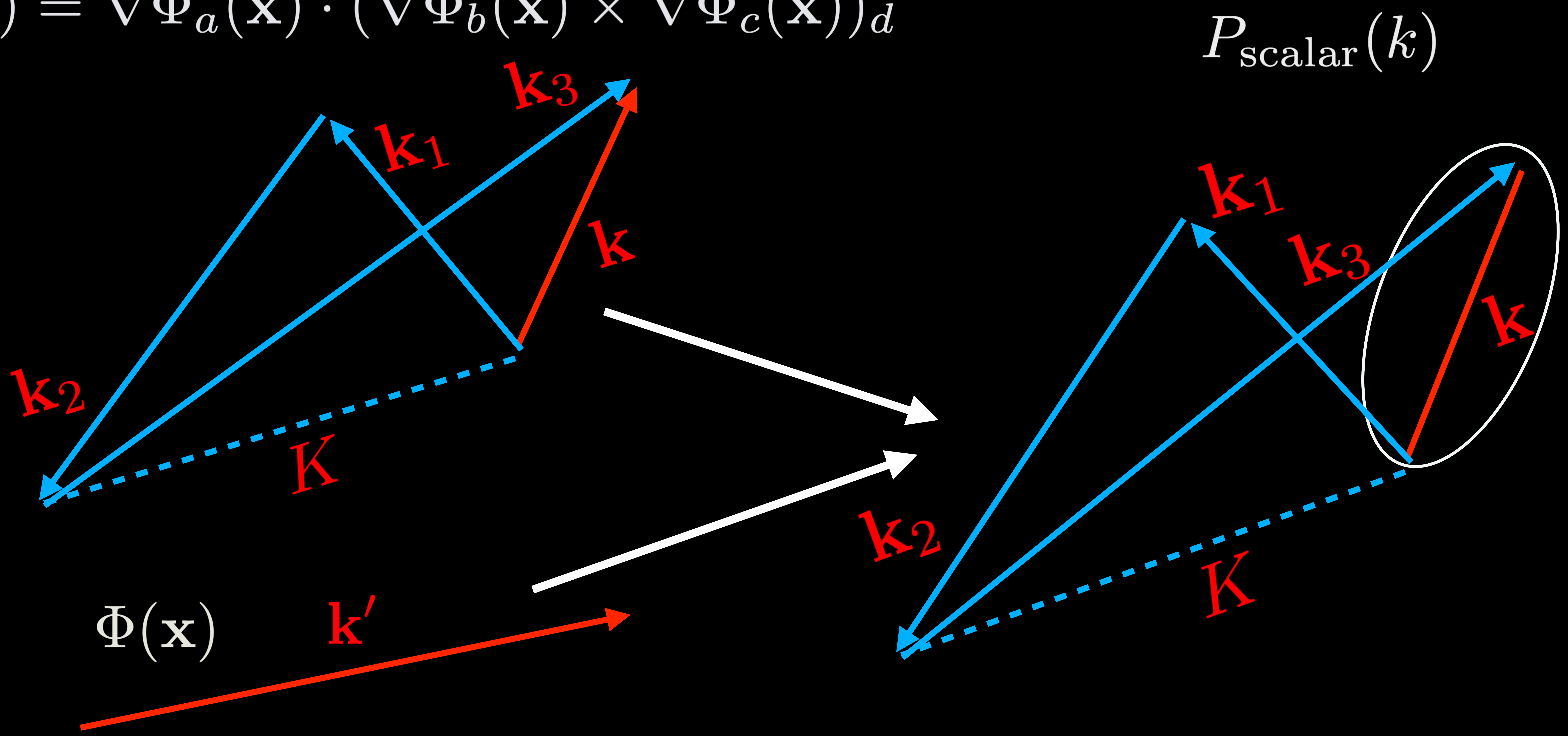
$$\Psi_{abcd}(\mathbf{x}) = \nabla\Phi_a(\mathbf{x}) \cdot (\nabla\Phi_b(\mathbf{x}) \times \nabla\Phi_c(\mathbf{x}))_d$$

POP spectrum:

$$\langle\Psi_{abcd}(\mathbf{k})\Phi(\mathbf{k}')\rangle = (2\pi)^3\delta_{\mathrm{D}}^{(3)}(\mathbf{k} + k')P_{\mathrm{scalar}}(k)$$

Scalar POP Spectrum

$$\Psi_{abcd}(\mathbf{x}) = \nabla\Phi_a(\mathbf{x}) \cdot (\nabla\Phi_b(\mathbf{x}) \times \nabla\Phi_c(\mathbf{x}))_d$$



Scalar POP Spectrum

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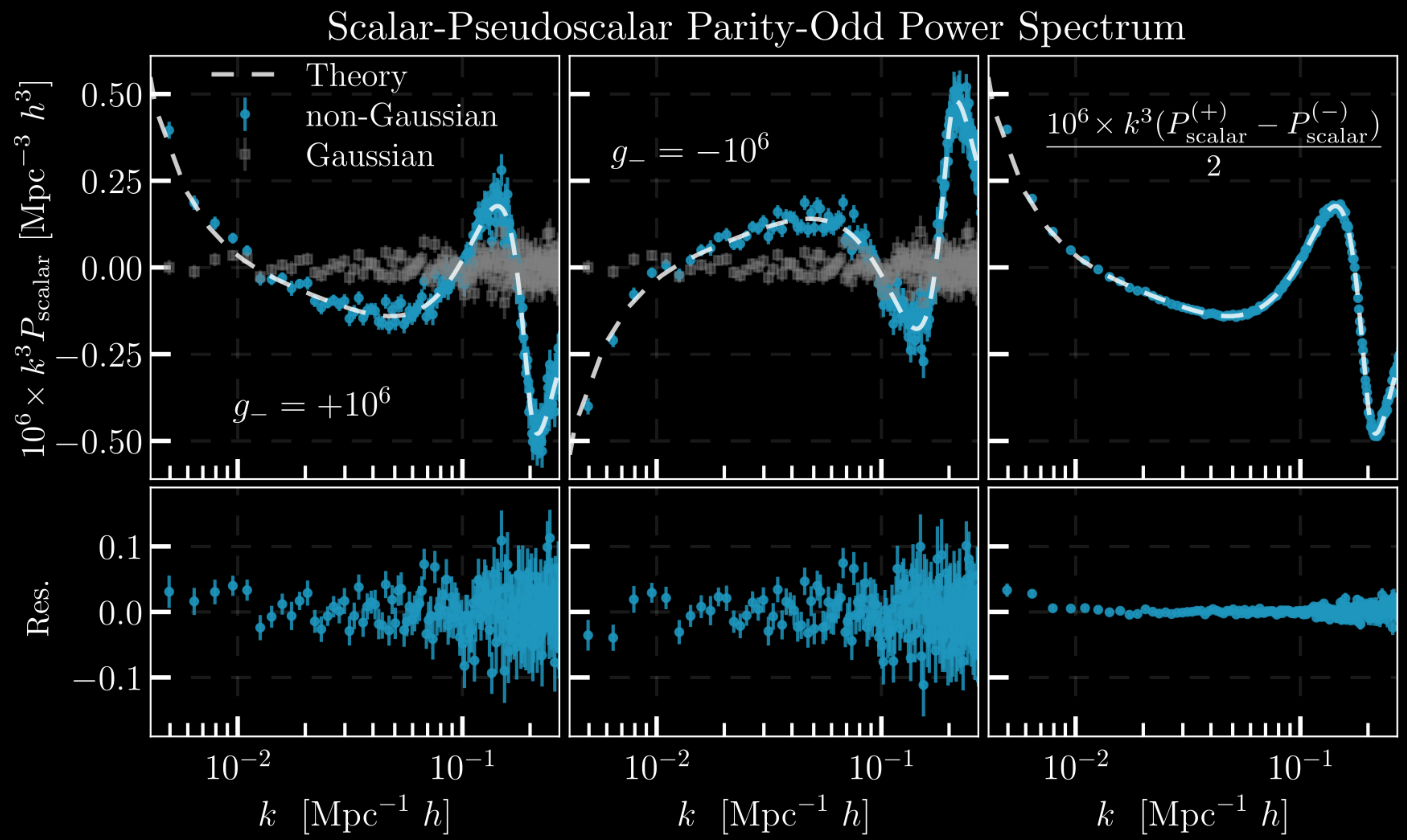

Sensitivity

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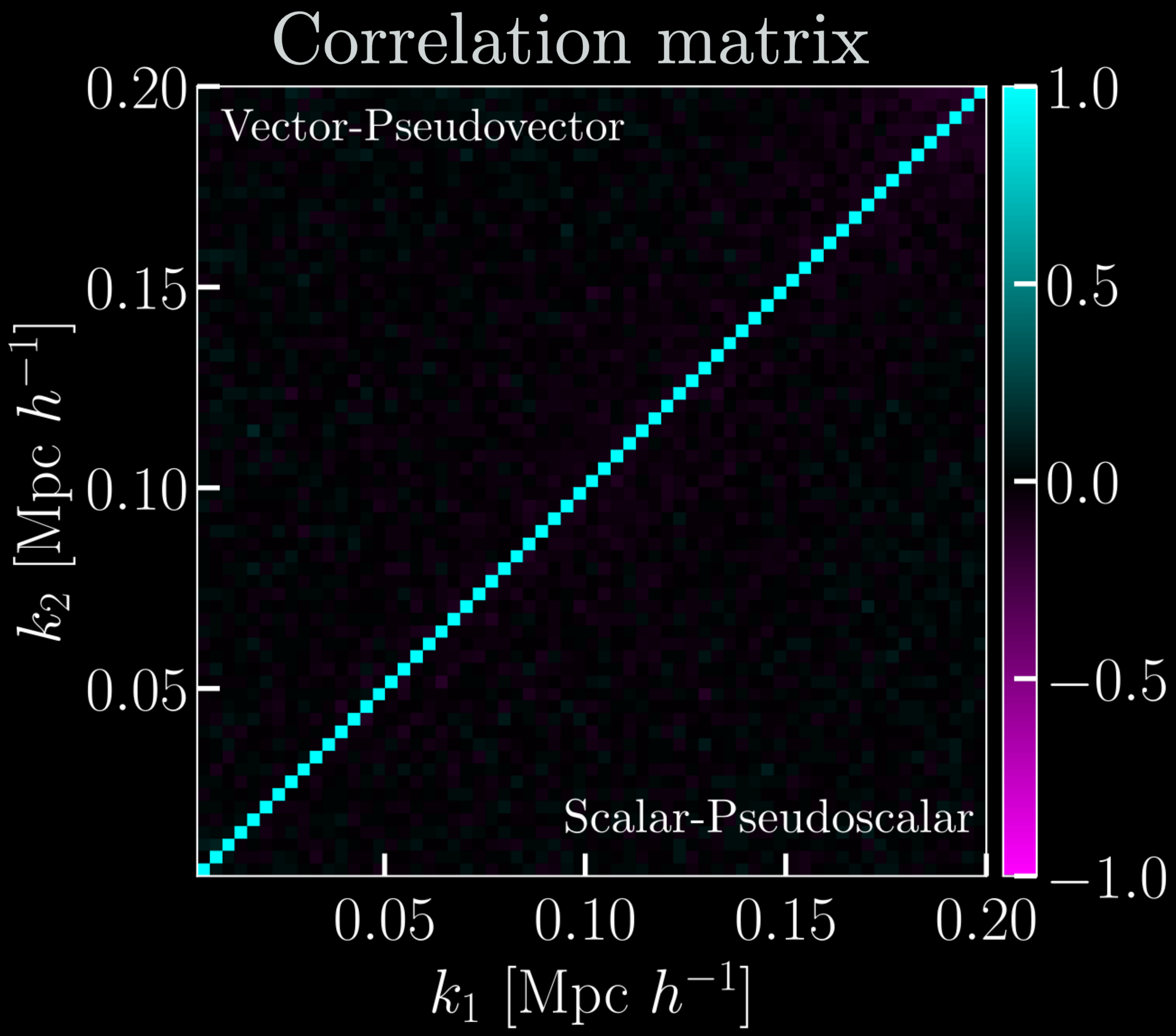

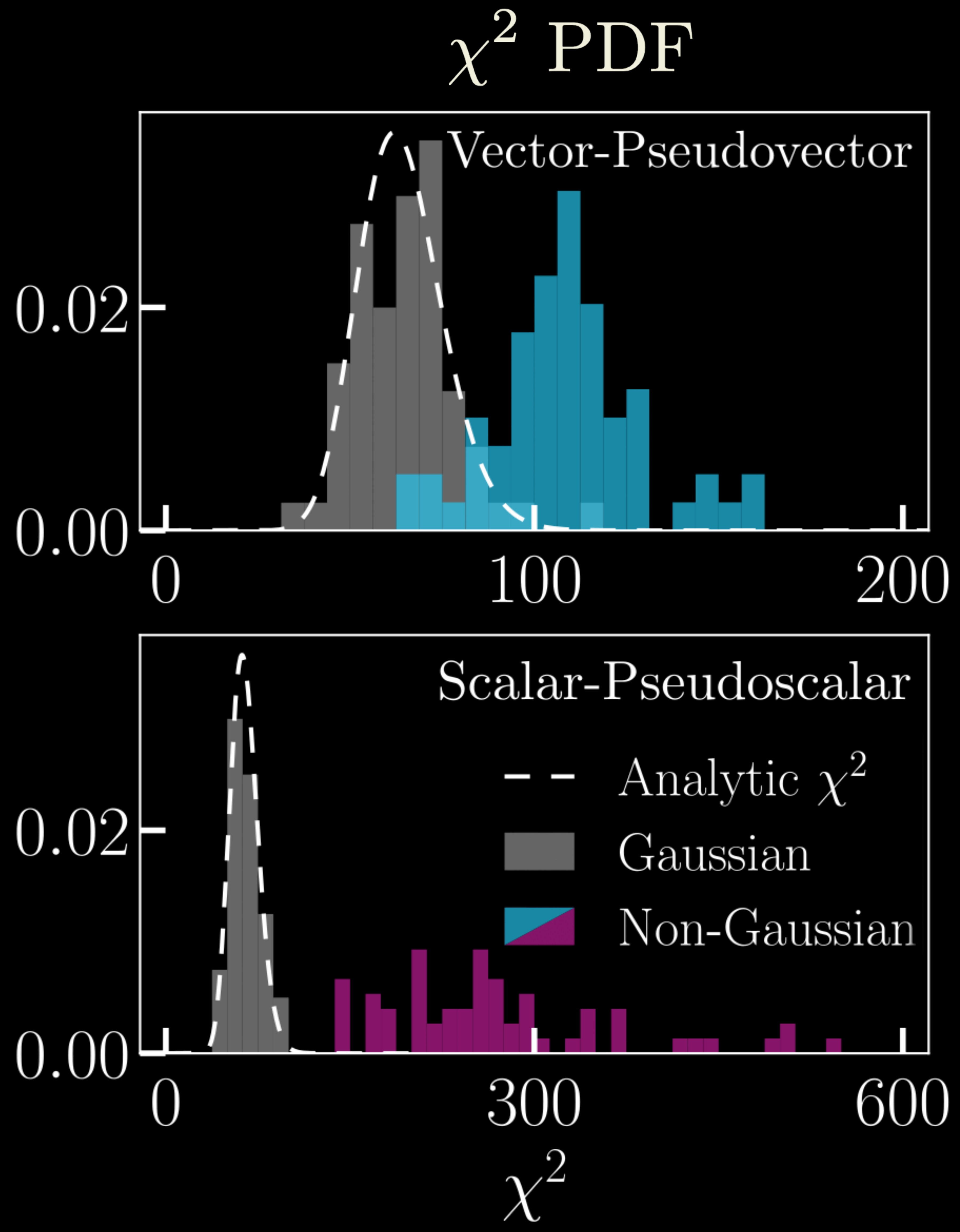

Outlook

- Real space counterpart
- Harmonic space/redshift space
- Effects of nonlinear evolution for LSS, bias, etc.
- Adapt to realistic survey data, window, etc.

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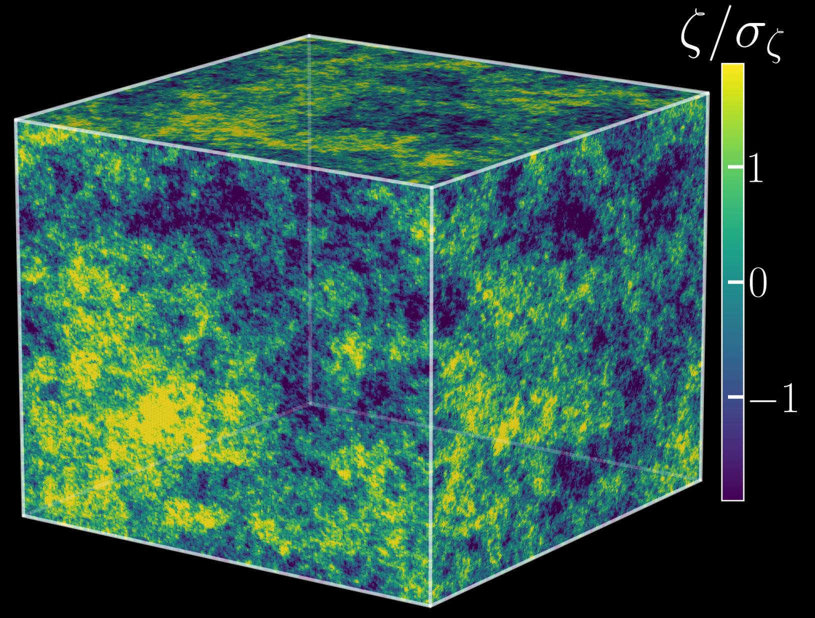


Thank you!

Relation to Helicity

Construct two vector fields:

$$\mathbf{V}_{ab}(\mathbf{x}) = \Phi_a(\mathbf{x})\nabla\Phi_b(\mathbf{x})$$

$$\mathbf{V}_{cd}(\mathbf{x}) = \Phi_c(\mathbf{x})\nabla\Phi_d(\mathbf{x})$$

The power spectrum matrix

$$\langle V^i_{ab}(\mathbf{k})V^j_{cd}(\mathbf{k})'\rangle = (2\pi)^3\delta^{(3)}_{\mathrm{D}}(\mathbf{k}+\mathbf{k}')P^{ij}(k)$$

Decompose into trace, traceless symmetric, and antisymmetric

$$P^{ij}(k) = \frac{1}{3}\delta^{ij}P_{\|}(k) + \left(\frac{k^ik^j}{k^2} - \frac{1}{3}\delta^{ij}\right)P_{\perp}(k) + i\epsilon^{ijl}\frac{k^l}{k}P_{-}(k)$$

Choose helicity polarization vectors:

$$i\mathbf{k}\times\mathbf{e}_{\mathrm{R/L}}(\mathbf{k}) = \pm k\mathbf{e}_{\mathrm{R/L}}(\mathbf{k})$$

Project onto this basis: $P_{\mathrm{RR}}(k),\ P_{\mathrm{LL}}(k)$

$$P_{-}(k) = \frac{1}{2}\Big(P_{\mathrm{RR}}(k) - P_{\mathrm{LL}}(k)\Big)$$

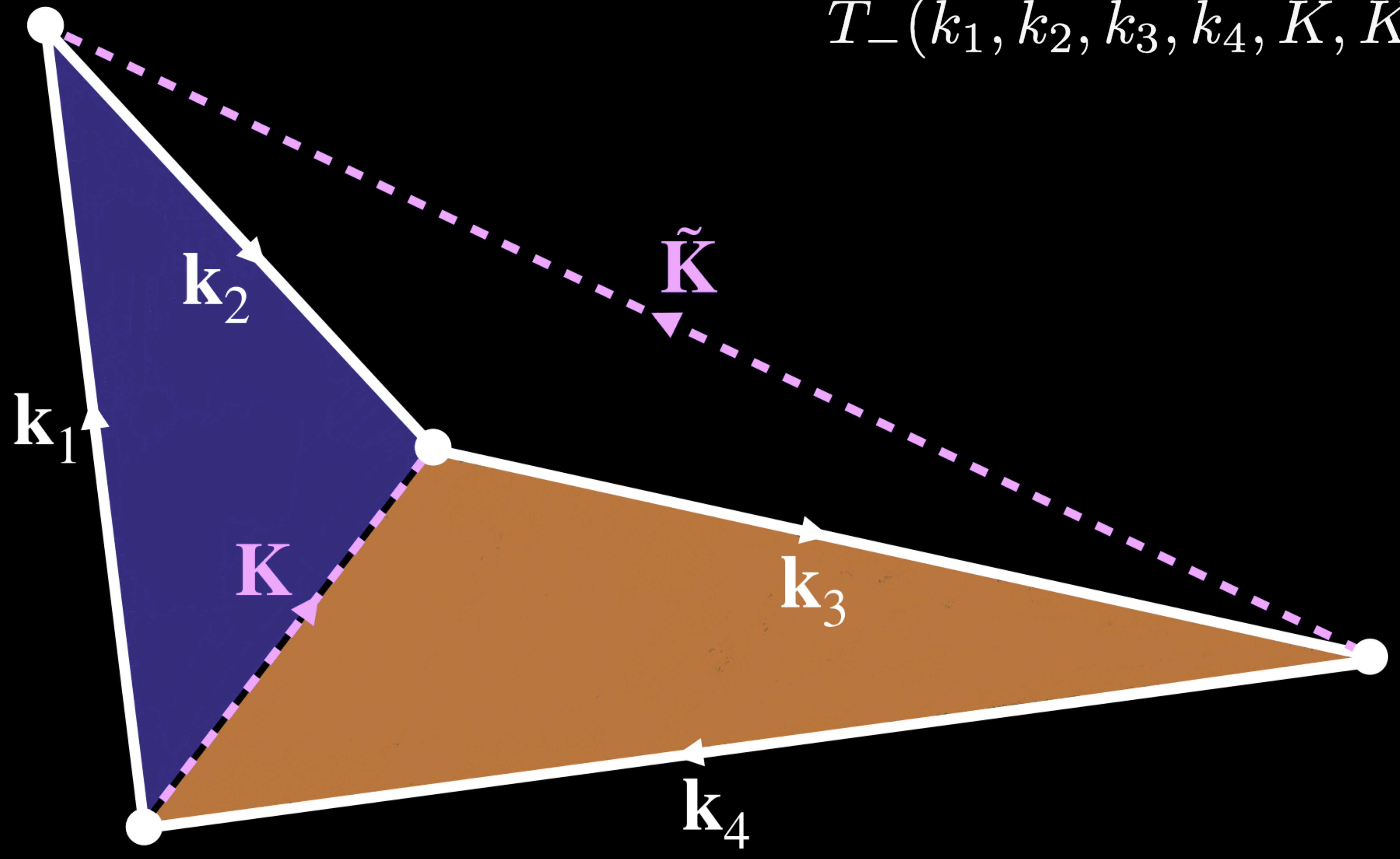
$$= -\frac{k}{2}P_{\mathrm{vector}}(k)$$

Template Peak

Ordering: $k_1 < k_2 < k_3 < k_4$

Dominant trispectrum behaviour:

$$T_-(k_1, k_2, k_3, k_4, K, \tilde{K}) \simeq \mathbf{k}_1 \cdot (\mathbf{k}_2 \times \mathbf{k}_3) \frac{\left(2\pi^2 A_s\right)^3}{k_1^5 k_2^4} \left(\frac{1}{k_3^3} - \frac{1}{k_4^3} \right)$$



- Peaks in soft limit of k_1
- Tradeoff:
  - Colinear wave vectors
  - Multually perpendicular wave vectors