Image: D. Schlegel

Probing primordial non-Gaussianity by reconstructing the initial conditions with machine learning

Xinyi Chen
Yale University

w/ Nikhil Padmanabhan,
Daniel Eisenstein

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Status of constraining local PNG with LSS

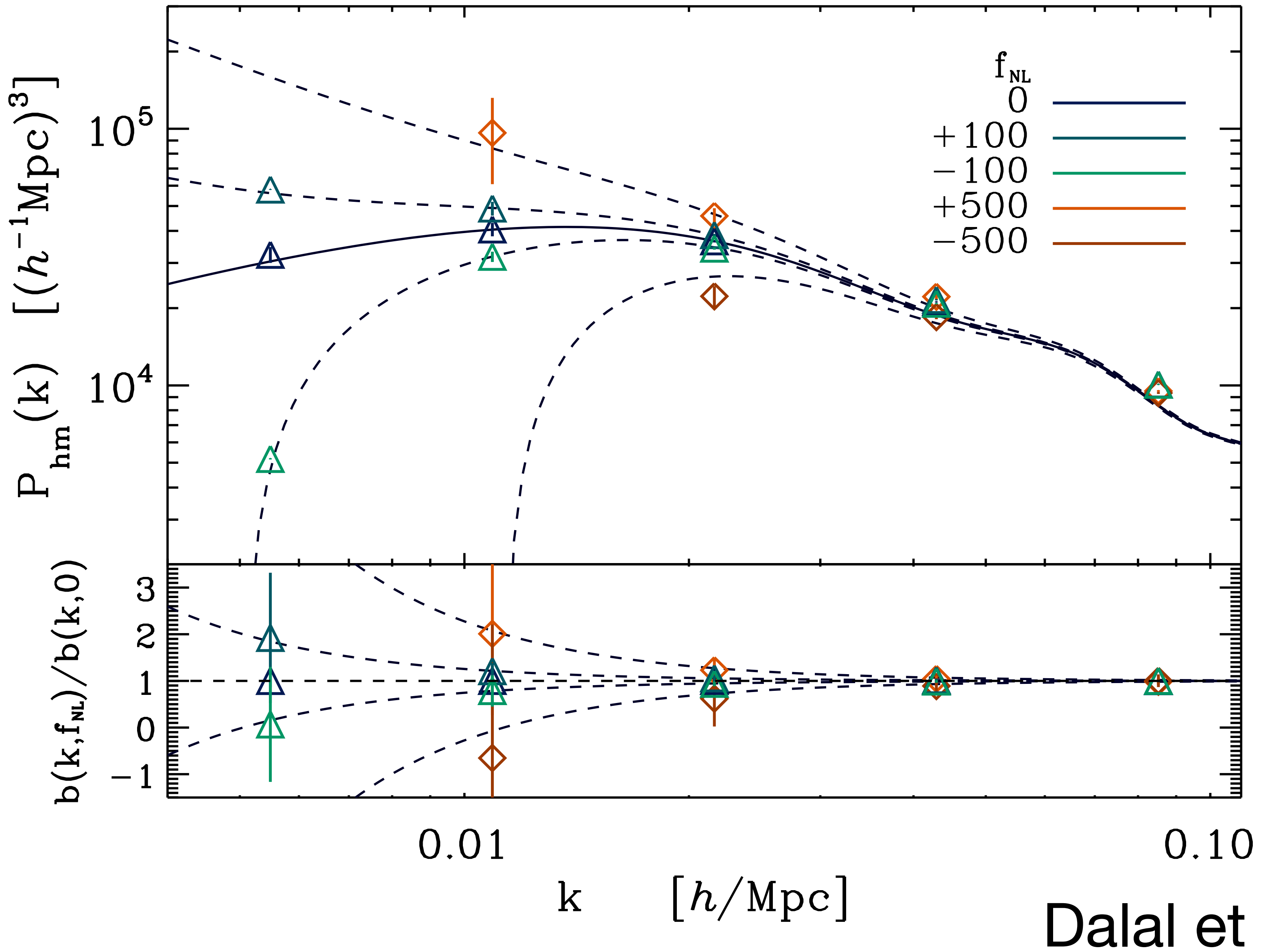
Primordial potential with local type $f_{\rm NL}$:

$$\Phi(\boldsymbol{x}) = \phi_G(\boldsymbol{x}) + f_{\rm NL}\{\phi_G^2(\boldsymbol{x}) - \langle\phi_G^2(\boldsymbol{x})\rangle\} + \dots$$

Gaussian potential

Sensitivity goal: $\sigma(f_{\rm NL}) < 1$

- Current best using Pk: -12±21 (eBOSS DR16 QSO, Mueller et al. 2022)
- Usual technique: scale-dependent bias on galaxy power spectrum
  - Systematics
  - Cosmic variance on large scales
  - Forecast DESI $\sigma(f_{\rm NL}) \sim 10$ (Sailer et al. 2021)
- **Adding Bispectrum -> tighter constraints**
  - e.g. a factor of ~2-4 Pk -> Pk+Bk (*SPHEREx*, Dore et al. 2014, Heinrich, Dore & Krause 2023)
  - **Large bispectrum from gravity**
  - **Large data vectors**



Dalal et al. 2008

$$\Delta b \propto \frac{f_{\rm NL}}{k^2 T(k)}$$

Transfer function

Reconstruction

Near-optimal 2-pt bispectrum estimator

New approach to constraining PNG

- Reconstructing the density field
- Computing and fitting a near-optimal 2-pt bispectrum estimator
- Information content at the field level

New approach to constraining PNG

- **Reconstructing the density field**
- Computing and fitting a near-optimal 2-pt bispectrum estimator
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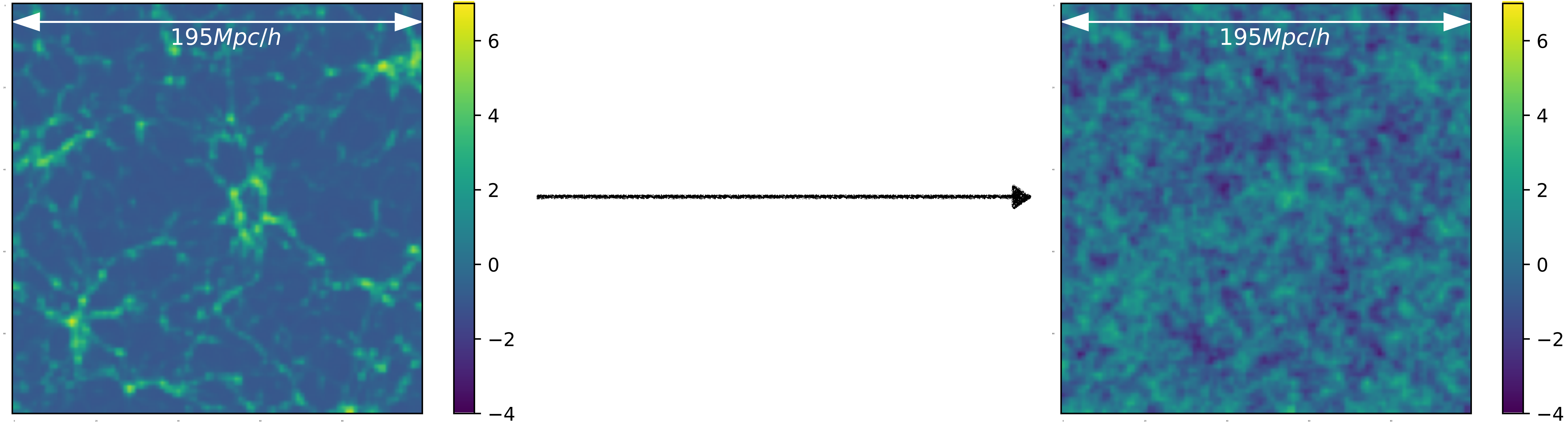
Reconstruction of the initial conditions: reverse a late-time density field back to initial density field

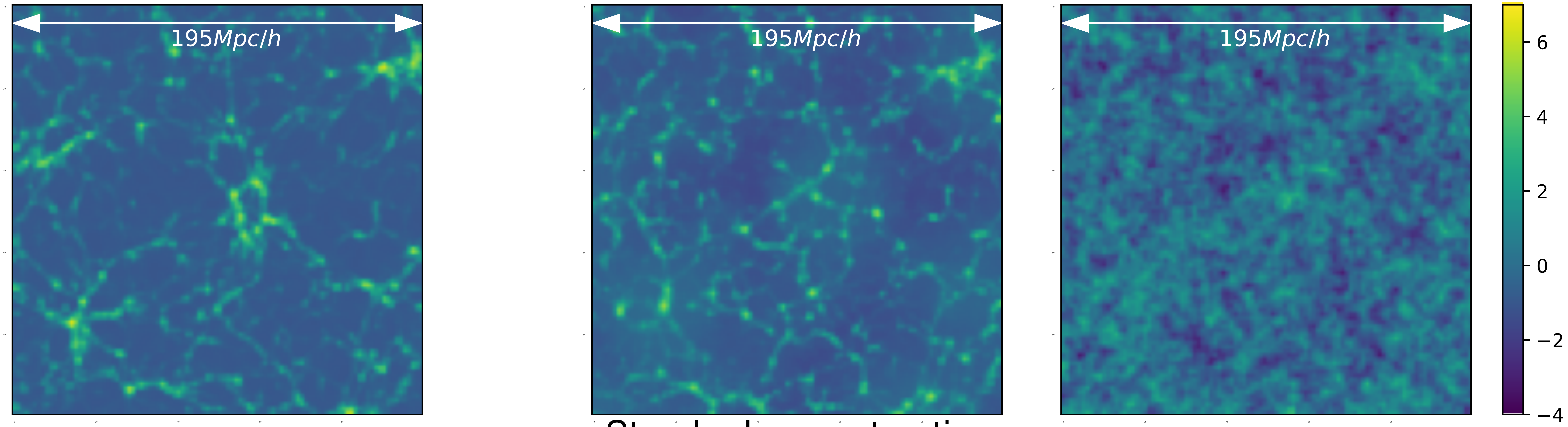


Late-time

Initial

Matter density fields at high resolution (1024^3 particles in 1 Gpc/h box) at z=0, on a 512^3 grid, using Quijote simulations (Villaescusa-Navarro et al. 2020)

Density field reconstructed by the standard reconstruction algorithm still nonlinear



Late-time

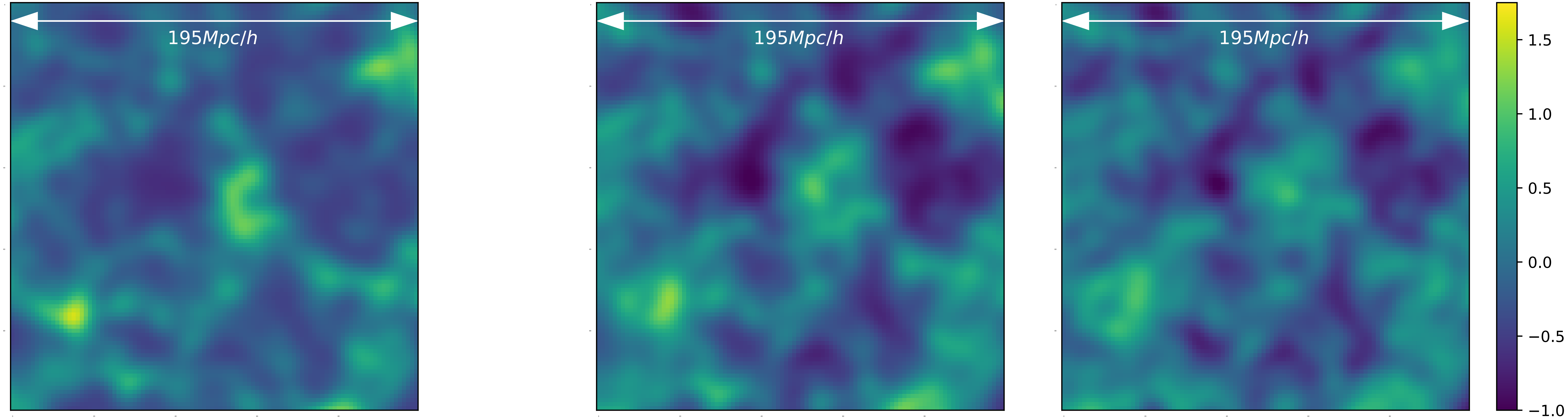
Standard reconstruction (Eisenstein et al. 2007)

Initial

Matter density fields at high resolution (1024^3 particles in 1 Gpc/h box) at z=0, on a 512^3 grid, using Quijote simulations (Villaescusa-Navarro et al. 2020)

Density field reconstructed by the standard reconstruction algorithm still nonlinear

Smoothed at 5 Mpc/h



Late-time

Standard reconstruction

Initial

Matter density fields at high resolution (1024^3 particles in 1 Gpc/h box) at z=0, on a 512^3 grid, using Quijote simulations (Villaescusa-Navarro et al. 2020)

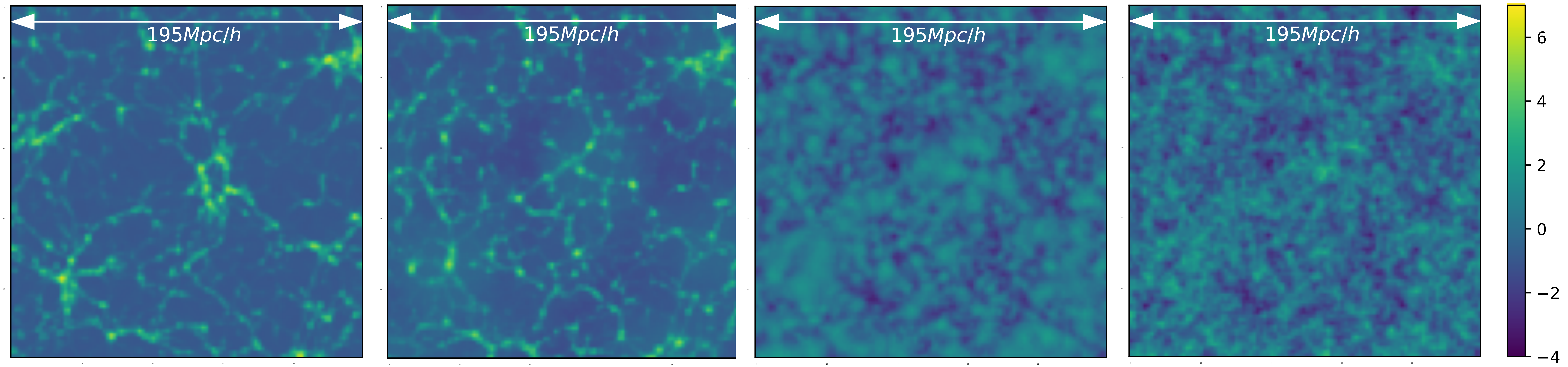
A new reconstruction method

A hybrid method that combines convolutional neural network (CNN) with a traditional algorithm based on perturbation theory (**Chen** et al. 2023, Shallue & Eisenstein 2023)

Matter density fields at high resolution (1024^3 particles in 1 Gpc/h box) at z=0, on a 512^3 grid, using Quijote simulations (Villaescusa-Navarro et al. 2020)

Large-scales use perturbation theory, small-scales use CNN

- First step: traditional algorithm
- Second step: train CNN with reconstructed density fields
- CNN is relatively local, but perturbation theory provides good approximation on large scales. **So traditional algorithm for large scales, CNN for smaller scales.**



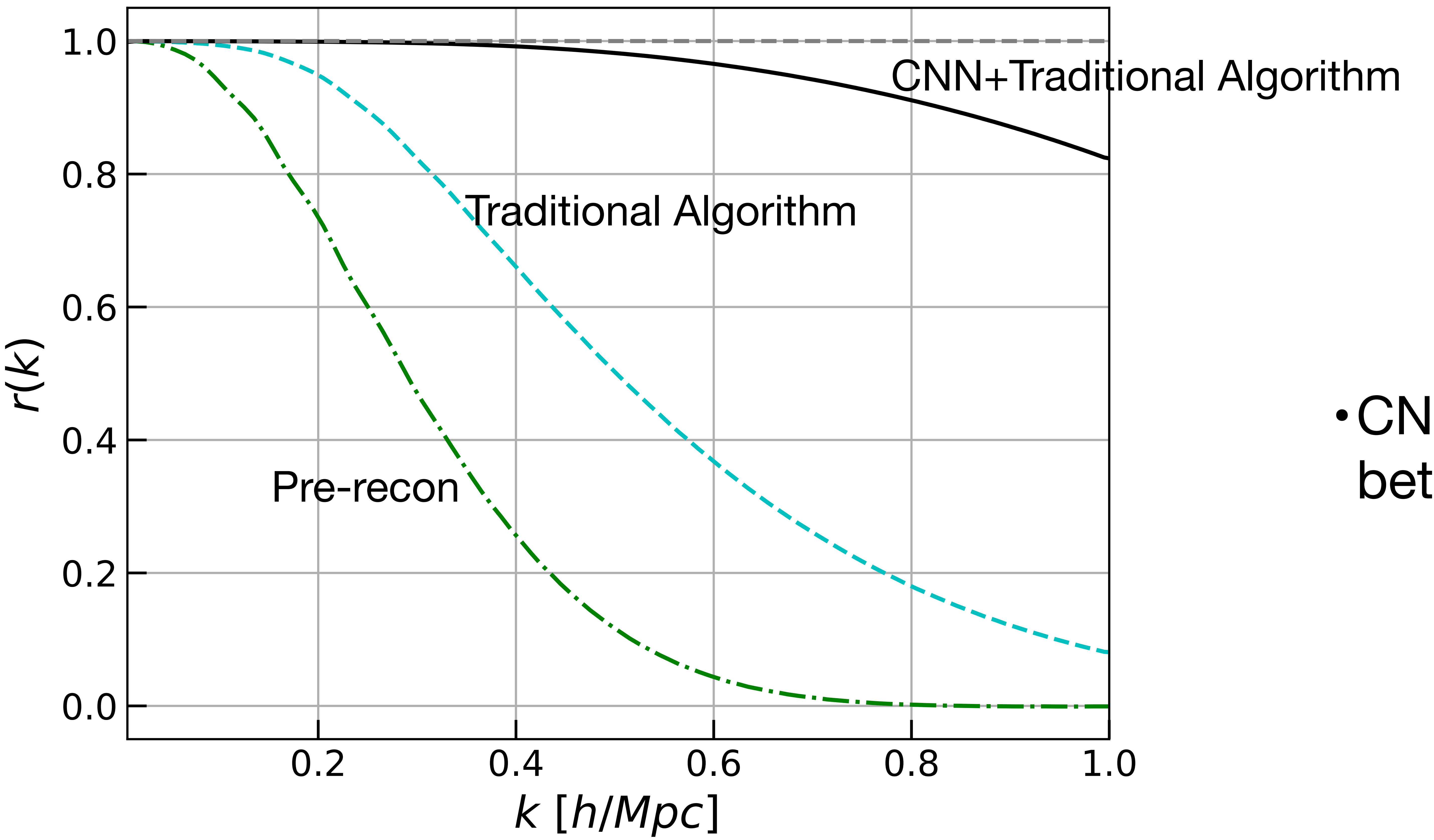
Late-time

Standard recon

CNN trained w/ standard recon field

Initial

CNN improves cross-correlation in matter field



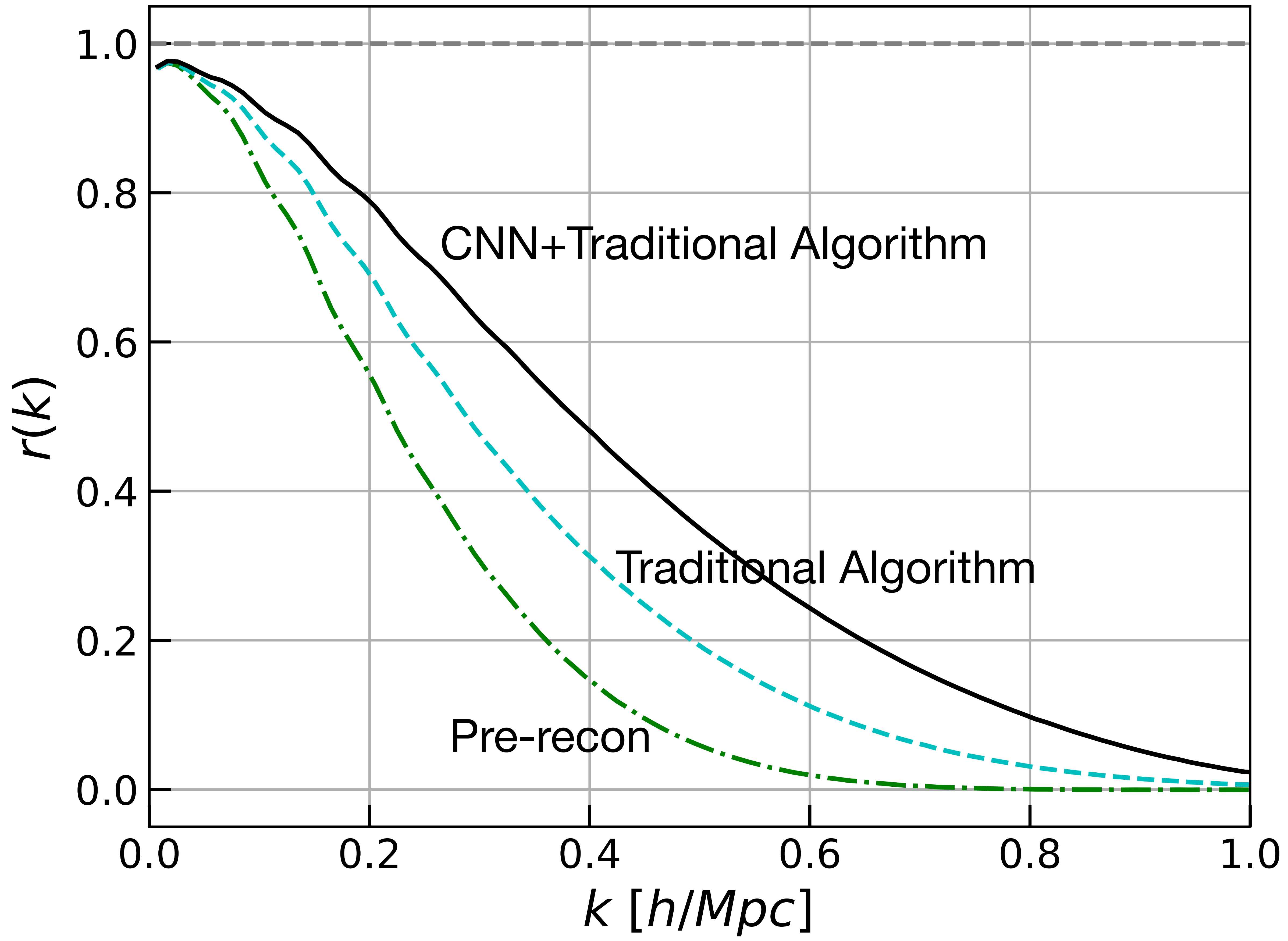
$$r(k) = \frac{\langle \delta^*(\boldsymbol{k})\delta_{\rm ini}(\boldsymbol{k})\rangle}{\sqrt{\langle \delta^2(\boldsymbol{k})\rangle\langle \delta_{\rm ini}^2(\boldsymbol{k})\rangle}}$$

- CNN+Algorithm performs significantly better

Real space matter field z=1, using Quijote simulations (Villaescusa-Navarro et al. 2020)

Reconstruction algorithm used: Hada & Eisenstein 2018 (HE18)

Hybrid recon boosts traditional algorithms in halo fields too



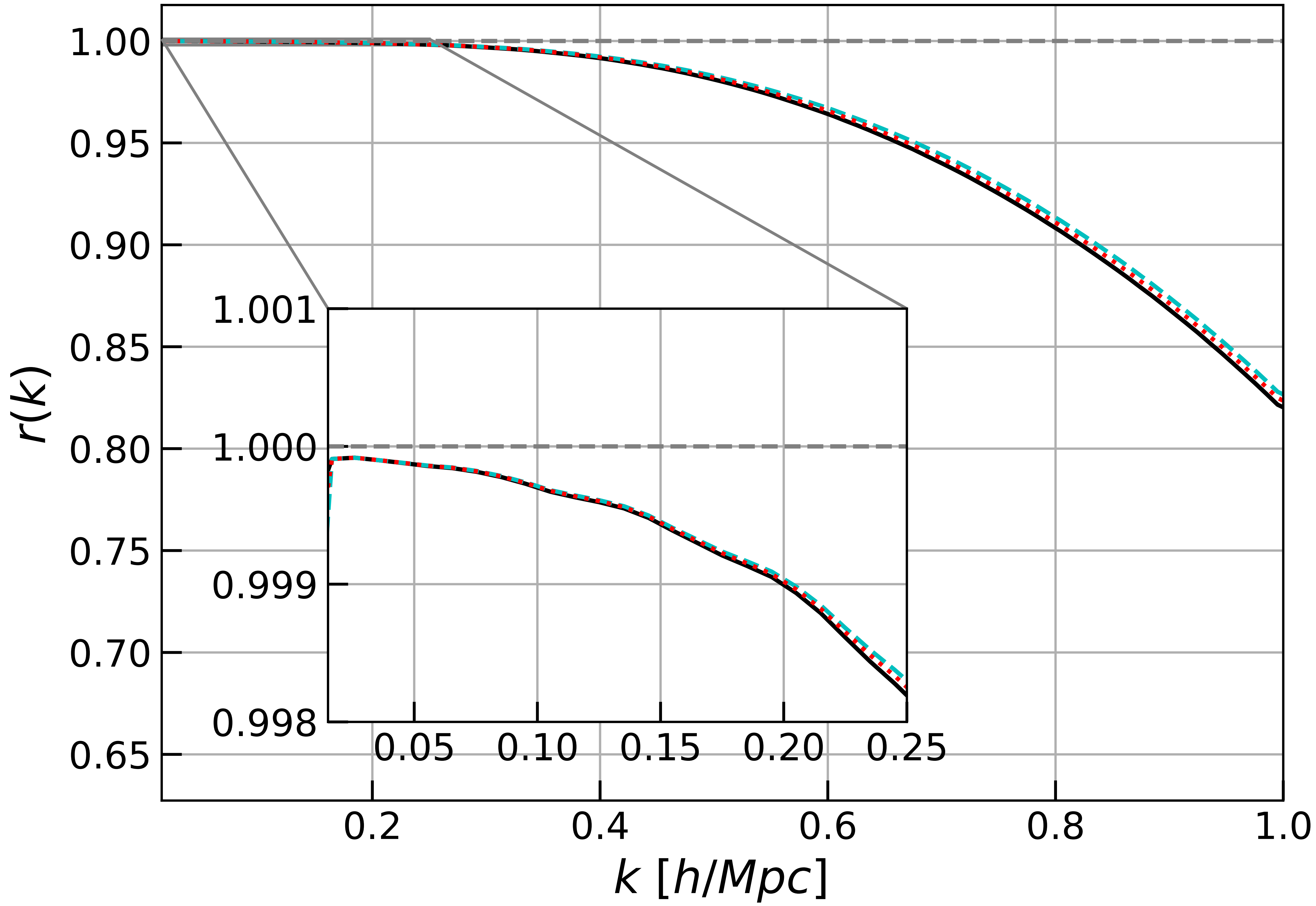
z=1

$\bar{n} = 2.0 \times 10^{-4} h^3 \mathrm{Mpc}^{-3}$
$b = 2.9$
$b^2\bar{n} = 1.7 \times 10^{-3} h^3 \mathrm{Mpc}^{-3}$

Similar to DESI Y1 LRG:
$b^2\bar{n} \sim 1.4 \times 10^{-3} h^3 \mathrm{Mpc}^{-3}$

Reconstruction algorithm used: Hada & Eisenstein 2018 (HE18)

Model trained with no PNG works for PNG



$$r(k) = \frac{\langle \delta^*(\boldsymbol{k}) \delta_{\mathrm{ini}}(\boldsymbol{k}) \rangle}{\sqrt{\langle \delta^2(\boldsymbol{k}) \rangle \langle \delta^2_{\mathrm{ini}}(\boldsymbol{k}) \rangle}}$$

CNN+Algorithm

– $f_{\mathrm{NL}} = +100$
··· $f_{\mathrm{NL}} = 0$
- - $f_{\mathrm{NL}} = -100$

Real space matter field z=1, using Quijote-PNG simulations (Coulton et al. 2022)

Reconstruction algorithm used: Hada & Eisenstein 2018

New approach to constraining PNG

- Reconstructing the density field
- **Computing and fitting a near-optimal 2-pt bispectrum estimator**
- Information content at the field level

Cross-power estimator

Primordial potential with local type f_{NL}:

$$\Phi(\boldsymbol{x}) = \phi_G(\boldsymbol{x}) + f_{\mathrm{NL}}\{\phi_G^2(\boldsymbol{x}) - \langle \phi_G^2(\boldsymbol{x})\rangle\} + \ldots$$

Gaussian potential

$$\langle \Phi^2 \delta \rangle$$

Reconstructed/Linear

Primordial potential

$$\Phi(\boldsymbol{k}) = \frac{\delta(\boldsymbol{k})}{M_\Phi(k)}$$

Transfer function

$$M_\Phi(k) = \frac{2}{3}\frac{k^2 T(k)}{\Omega_{\mathrm{m},0} H_0^2}$$

$$\Phi^2(\boldsymbol{k}) = \int \mathrm{d}\boldsymbol{x} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \Phi^2(\boldsymbol{x}) = \frac{1}{(2\pi)^3}\int \mathrm{d}\boldsymbol{k}_1 \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}-\boldsymbol{k}_1)$$

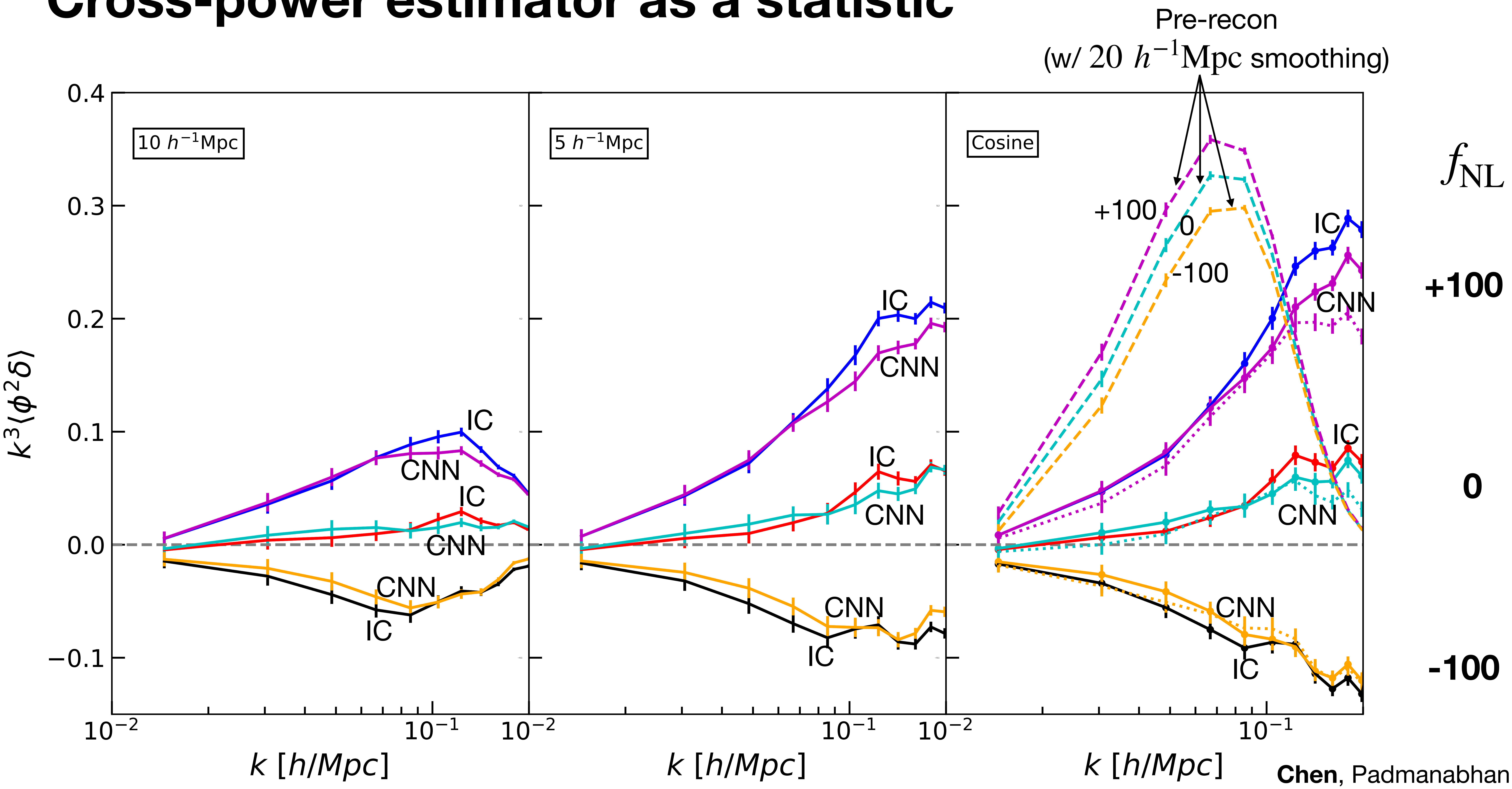
$$\langle \Phi^2(\boldsymbol{k})\delta(-\boldsymbol{k})\rangle = \frac{1}{(2\pi)^3}\int \mathrm{d}\boldsymbol{k}_1 M_\Phi(k) \boxed{\langle \Phi(\boldsymbol{k})\Phi(\boldsymbol{k}-\boldsymbol{k}_1)\Phi(-\boldsymbol{k})\rangle}$$

Primordial bispectrum

Integral of bispectrum. Need to set the integration limit same as bispectrum k_{max} to have same information as bispectrum

Near optimal by maximum likelihood estimation, first proposed by Schmittfull, Baldauf & Seljak 2015

Cross-power estimator as a statistic



Chen, Padmanabhan & Eisenstein
(in prep.)

Cosine filter between
k=0.2-0.25 h/Mpc

Real space matter field z=1

Fisher error $\sigma(f_{\rm NL})$ for cross-power with matter density field of 1 Gpc/h volume

$k_{\rm max}$	Smoothing	IC	CNN+HE18 $z=1$	NL $z=1$
$0.1\ h/{\rm Mpc}$	$10\ h^{-1}$ Mpc	52.7	57.2	76.2
	$5\ h^{-1}$ Mpc	48.0	52.4	
	Cosine	46.4	50.7	
$0.2\ h/{\rm Mpc}$	Cosine	15.8	17.4	54.5

3x

(Smoothed at $20\ h^{-1}$Mpc)

For eBOSS QSO survey volume (2.9 Gpc/h): $\sigma(f_{\rm NL}) \sim 4$

- Single parameter forecast: CNN+HE18 $\sigma(f_{\rm NL})$~50, pre-recon $\sigma(f_{\rm NL})$~76 ($k_{\rm max} = 0.1\ h/{\rm Mpc}$, z=1) — ~1.5x improvement
- Hybrid reconstruction allows higher $k_{\rm max}$
- Optimistic without including other bias terms (squared, shift, tidal) -> can compute similar cross-power estimators

New approach to constraining PNG

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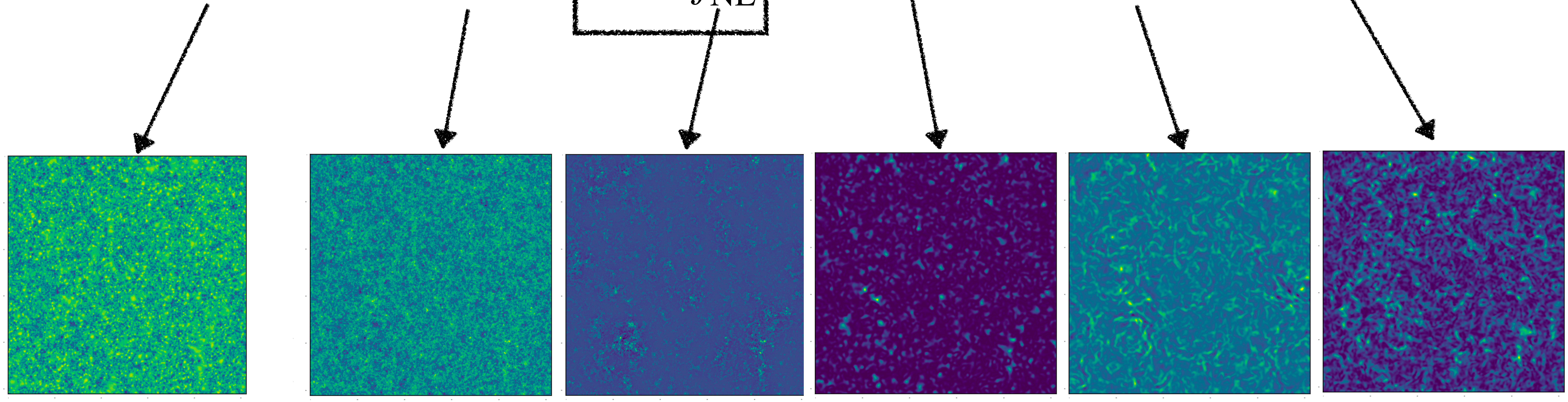
- Computing and fitting a near-optimal 2-pt bispectrum estimator
- **Information content at the field level**

Templates for fitting $f_{\rm NL}$

- δ_G=No PNG IC
- $\delta_{f_{\rm NL}} = \phi_G^2(k)M_\phi(k)$
- δ^2,δ_{∇^2},δ_{s^2} all computed using δ_G

$$\delta_{\rm CNN} = \overset{\text{Gaussian}}{b_G\delta_G} + \boxed{f_{\rm NL}\delta_{f_{\rm NL}}} + \overset{\text{Growth}}{b_2\delta^2} + \overset{\text{Shift}}{b_{\nabla^2}\delta_{\nabla^2}} + \overset{\text{Tidal}}{b_{s^2}\delta_{s^2}} + \ldots$$



Small error but fits are slightly biased

- Errors in 1 Gpc volume, std of 90 sims
- With 3 Mpc/h smoothing for the quadratic fields
- k cut at 0.1 h/Mpc

Fits for $\delta_{\rm CNN}$
z=1 real space

	b_G	$f_{\rm NL}$	b_2	b_{∇^2}	b_{s^2}
$f_{\rm NL} = 0$	0.9998±0.0006	0.6±3.1	0.004±0.001	-0.018±0.001	0.017±0.001
$f_{\rm NL} = + 100$	1.0015±0.0006	93.0±3.2	0.004±0.001	-0.018±0.001	0.018±0.001
$f_{\rm NL} = - 100$	0.9980±0.0006	-91.7±3.1	0.004±0.001	-0.018±0.001	0.016±0.001

Chen, Padmanabhan & Eisenstein in prep.

Accounting for the shift in the mean at $f_{\rm NL} = 0$:

$f_{\rm NL} = + 100$: ~+92
$f_{\rm NL} = - 100$: ~-92

Slightly biased

For >2 Gpc survey volume (e.g. DESI):
$\sigma(f_{\rm NL}) \sim 1$

Much lower error

Small error but fits are slightly biased

- Reconstruction **significantly reduces nonlinearities** at the second order, and still **preserves most PNG and gives tighter constrains**

Fits for $\delta_{\rm CNN}$

z=1 real space

~1.5x improvement

	b_G	$f_{\rm NL}$	b_2	b_{∇^2}	b_{s^2}
$f_{\rm NL}=0$	0.9998±0.0006	0.6±3.1	0.004±0.001	-0.018±0.001	0.017±0.001
$f_{\rm NL}=+100$	1.0015±0.0006	93.0±3.2	0.004±0.001	-0.018±0.001	0.018±0.001
$f_{\rm NL}=-100$	0.9980±0.0006	-91.7±3.1	0.004±0.001	-0.018±0.001	0.016±0.001

Chen, Padmanabhan & Eisenstein in prep.

	b_G	$f_{\rm NL}$	b_2	b_{∇^2}	b_{s^2}
$f_{\rm NL}=0$	0.9994±0.0002	-5.8±4.5	0.820±0.003	-1.023±0.004	0.202±0.002
$f_{\rm NL}=+100$	0.9994±0.0002	92.6±4.3	0.820±0.004	-1.023±0.005	0.202±0.002
$f_{\rm NL}=-100$	0.9994±0.0002	-104.4±4.7	0.820±0.003	-1.023±0.003	0.201±0.002

Fits for $\delta_{\rm NL}$

From F2 kernel: $b_2=\frac{17}{21}\sim 0.81 \quad b_{\nabla^2}=-1 \quad b_{s^2}=\frac{4}{21}\sim 0.19$

Summary

- Reconstruction with CNN+Algorithm removes most gravitational nonlinearity and strengthens the primordial signal
- Cross-power estimator easy to compute and promising to estimate f_{NL}
- Application of reconstruction on cross-power estimator gives low $\sigma(f_{\mathrm{NL}})$ although slightly biased mean

Ongoing work and outlook

- Including quadratic gravitational bias terms in the model (estimate each bias term —square, shift, tidal — with its own cross-power)
- High shot noise biased tracer
- Fitting templates in reality: fitting coefficients together with δ_G, forward model
- Applying to non-local types of PNG — extending cross-power estimator — can be more helpful since can't rely on scale-dependent bias