Perturbation Theory Remixed:

Improved modeling of **matter power spectrum** and **bispectrum**

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*n*EPT
☑ better convergence behavior
☑ extended validity range

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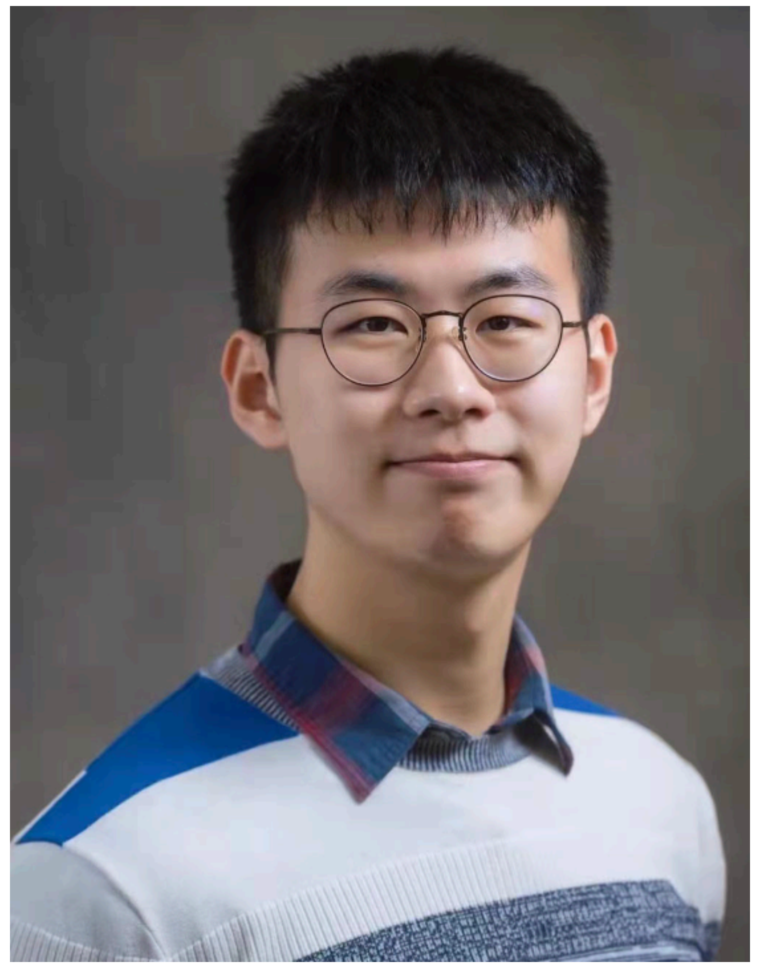
"New Strategies for Extracting Cosmology from Galaxy Surveys - 2nd edition"
Sesto, Italy (July 1st, 2024)

Collaborators



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1.1 Standard (Eulerian) Cosmological Perturbation Theory (SPT)

Mass conservation Law $\dot{\delta} + \nabla \cdot [(1+\delta)\boldsymbol{v}] = 0$

Euler's equation $\dot{\boldsymbol{v}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} + \frac{\dot{a}}{a}\boldsymbol{v} = -\nabla \phi$

Poisson's equation $\nabla^2 \phi = 4\pi G \bar{\rho}_{\mathrm{m}} a^2 \delta$

Solve $\{\delta,\ \nabla \cdot v\}$ **perturbatively**
in Fourier space

$$\delta(\boldsymbol{k}, z) = \sum_n \delta^{(n)}(k) D^n(z) = \delta^{(1)}(\boldsymbol{k}) D(z) + \delta^{(2)}(\boldsymbol{k}) D^2(z) + \delta^{(3)}(\boldsymbol{k}) D^3(z) + \cdots$$

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coupling kernel

$$\delta^{(n)}(k) = \int_{\boldsymbol{k}_1} \cdots \int_{\boldsymbol{k}_n} (2\pi)^3 \delta^{\mathrm{D}}(\boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n - \boldsymbol{k})\, F_n(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n)\, \delta^{(1)}(\boldsymbol{k}_1)\delta^{(1)}(\boldsymbol{k}_2)\cdots\delta^{(1)}(\boldsymbol{k}_n)$$

coupling between n linear modes

$$\theta^{(n)}(k) \equiv -\frac{\nabla \cdot \boldsymbol{v}}{aHf} = \int_{\boldsymbol{k}_1} \cdots \int_{\boldsymbol{k}_n} (2\pi)^3 \delta^{\mathrm{D}}(\boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n - \boldsymbol{k})\, G_n(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n)\, \delta^{(1)}(\boldsymbol{k}_1)\delta^{(1)}(\boldsymbol{k}_2)\cdots\delta^{(1)}(\boldsymbol{k}_n)$$

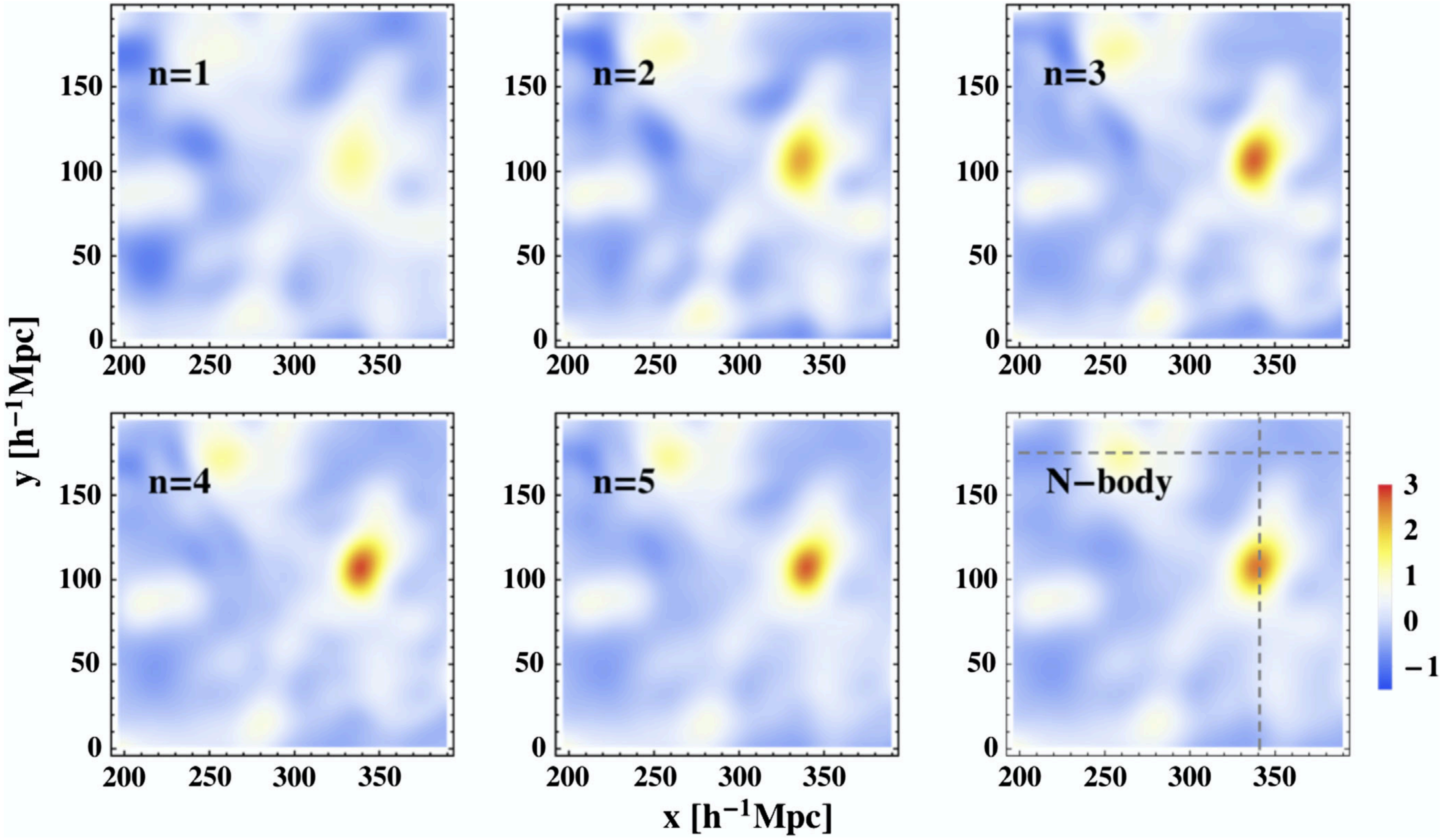
coupling kernel

Taruya, Nishimichi, Jeong (2018)

1.2 **SPT** at the field-level: **Grid**-based calculation (**GridSPT**)

- The **recursion relation** for the n-th order density perturbation and velocity field

$$\begin{pmatrix} \delta_n(\boldsymbol{x}) \\ \theta_n(\boldsymbol{x}) \end{pmatrix} = \frac{2}{(2n+3)(n-1)} \begin{pmatrix} n+\frac{1}{2} & 1 \\ \frac{3}{2} & n \end{pmatrix} \sum_{m=1}^{n-1} \begin{pmatrix} (\nabla\delta_m)\cdot\boldsymbol{u}_{n-m} + \delta_m\theta_{n-m} \\ [\partial_j(\boldsymbol{u}_m)_k][\partial_k(\boldsymbol{u}_{n-m})_j] + \boldsymbol{u}_m\cdot(\nabla\theta_{n-m}) \end{pmatrix}$$



linear Gaussian density perturbation δ_1

GridSPT → $\{\delta_1, \delta_2, \cdots, \delta_n\}$ → Measure P, B

N-body → $\delta_{N\text{-body}}$

2.1 Compute power spectrum from density fields

$$P_{ij}(k) = \left\langle \delta^{(i)}(\boldsymbol{k})\delta^{(j)}\left(-\boldsymbol{k}\right)\right\rangle'$$

The Standard Perturbation Theory (SPT)

(order-by-order calculation of power spectrum):

$$P_{\mathrm{Linear}} = P_{11}$$

$$P_{1-\mathrm{loop}} = (P_{22} + 2P_{13})$$

$$P_{2-\mathrm{loop}} = (2P_{15} + 2P_{24} + P_{33})$$

$$P_{\mathrm{SPT}} = P_{\mathrm{Linear}} + P_{1-\mathrm{loop}} + P_{2-\mathrm{loop}} + \cdots \ldots$$

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$$P_{\rm SPT} = P_{\rm Linear} + P_{\rm 1-loop} + P_{\rm 2-loop} + \cdots \ldots$$

"Perturbation Theory Remixed"
ZW *et al. (2023)*

The new way: ***n*EPT**

(order-by-order calculation at **field** level)

First add the non-linear density perturbation to order n

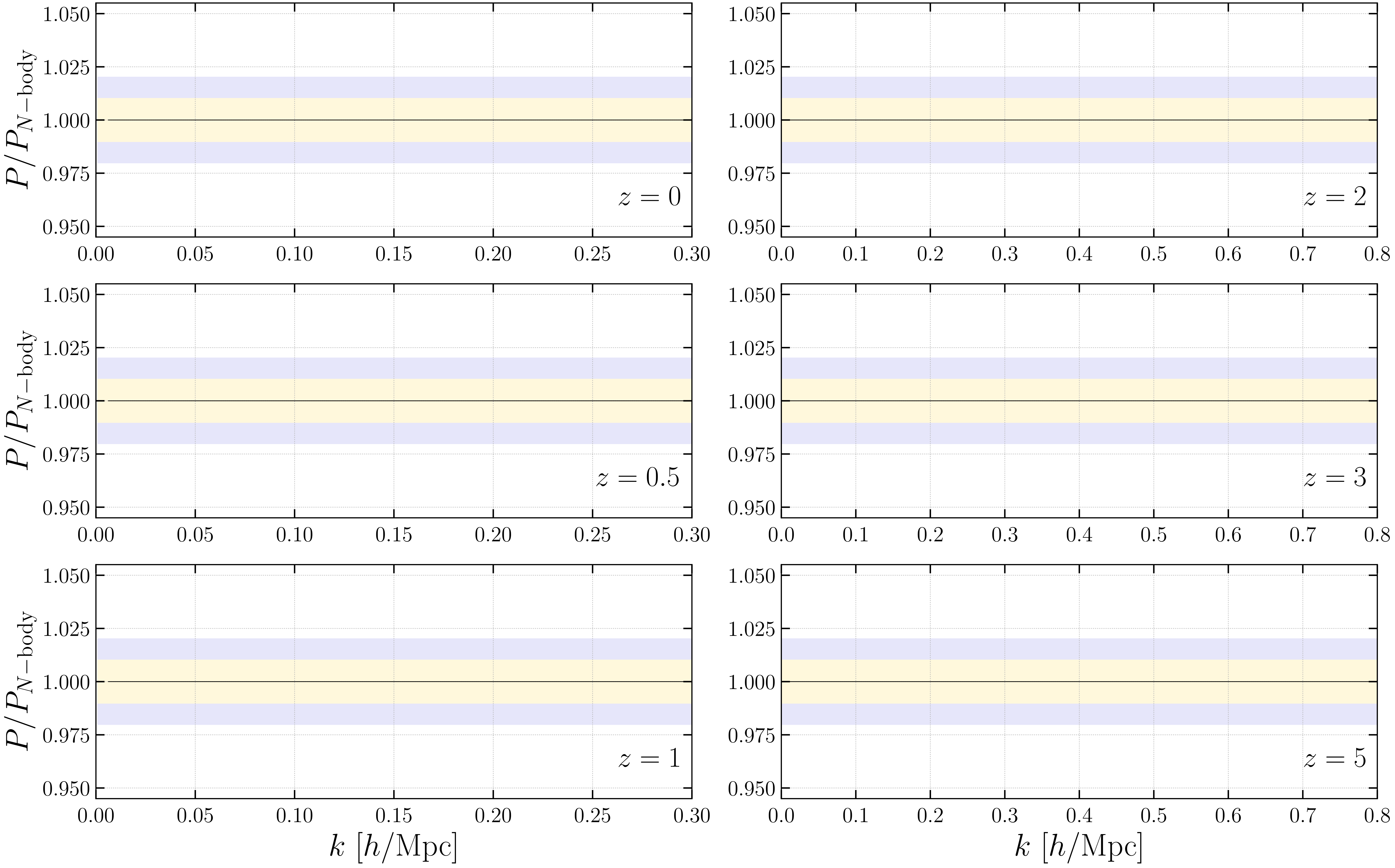
$$\delta_{n\rm EPT} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \cdots + \delta_n$$

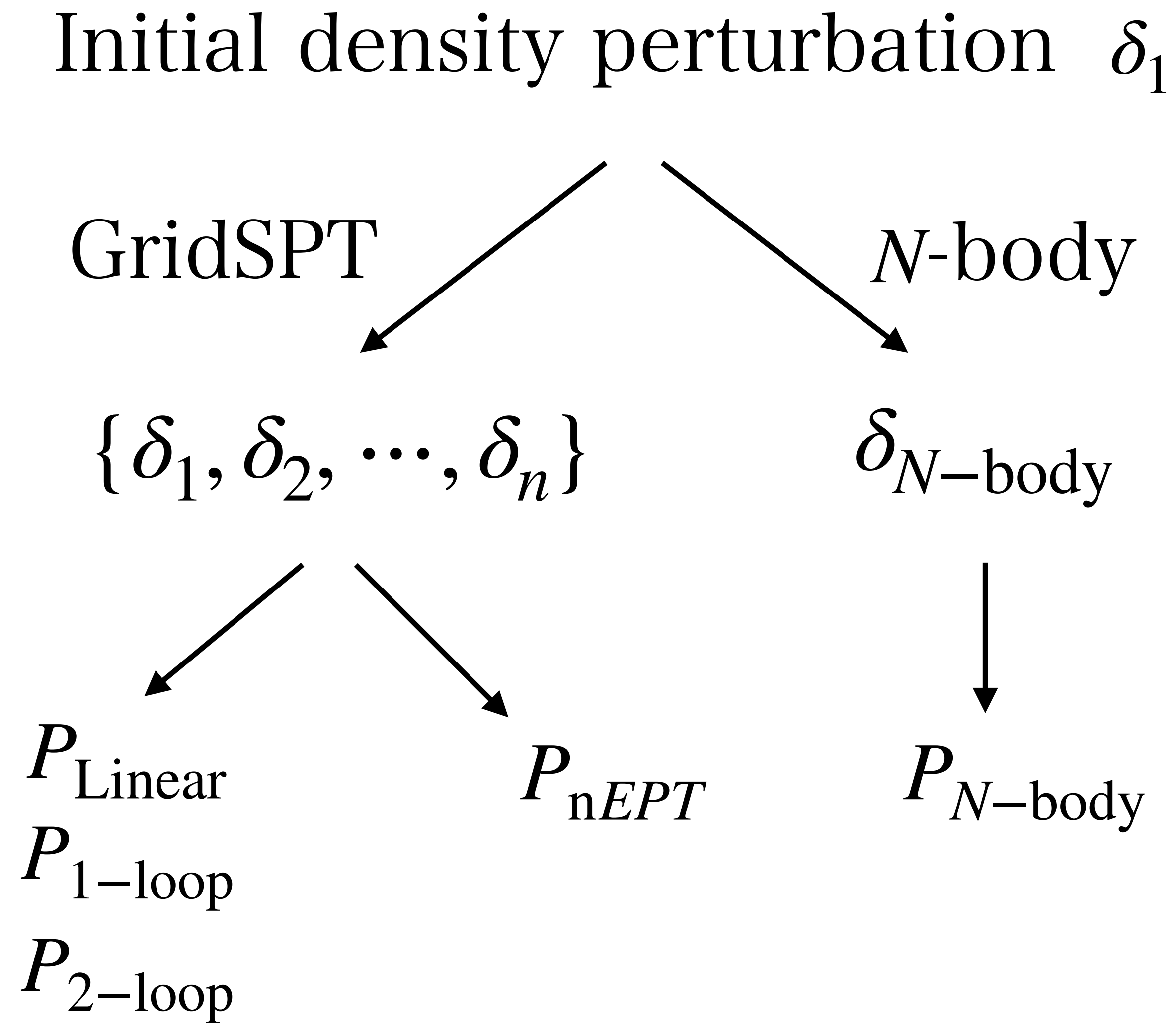
Then measure its power spectrum

$$P_{n\rm EPT} = \langle \delta_{n\rm EPT}(\boldsymbol{k})\delta_{n\rm EPT}(-\boldsymbol{k})\rangle'$$

$$\begin{aligned} P_{\rm 5EPT} = & P_{11} \\ & +2P_{12}+P_{22} \\ & +2P_{13}+2P_{23}+P_{33} \\ & +2P_{14}+2P_{24}+2P_{34}+P_{44} \\ & +2P_{15}+2P_{25}+2P_{35}+2P_{45}+P_{55} \end{aligned}$$

Result I: Matter Power Spectrum (WMAP cosmology)





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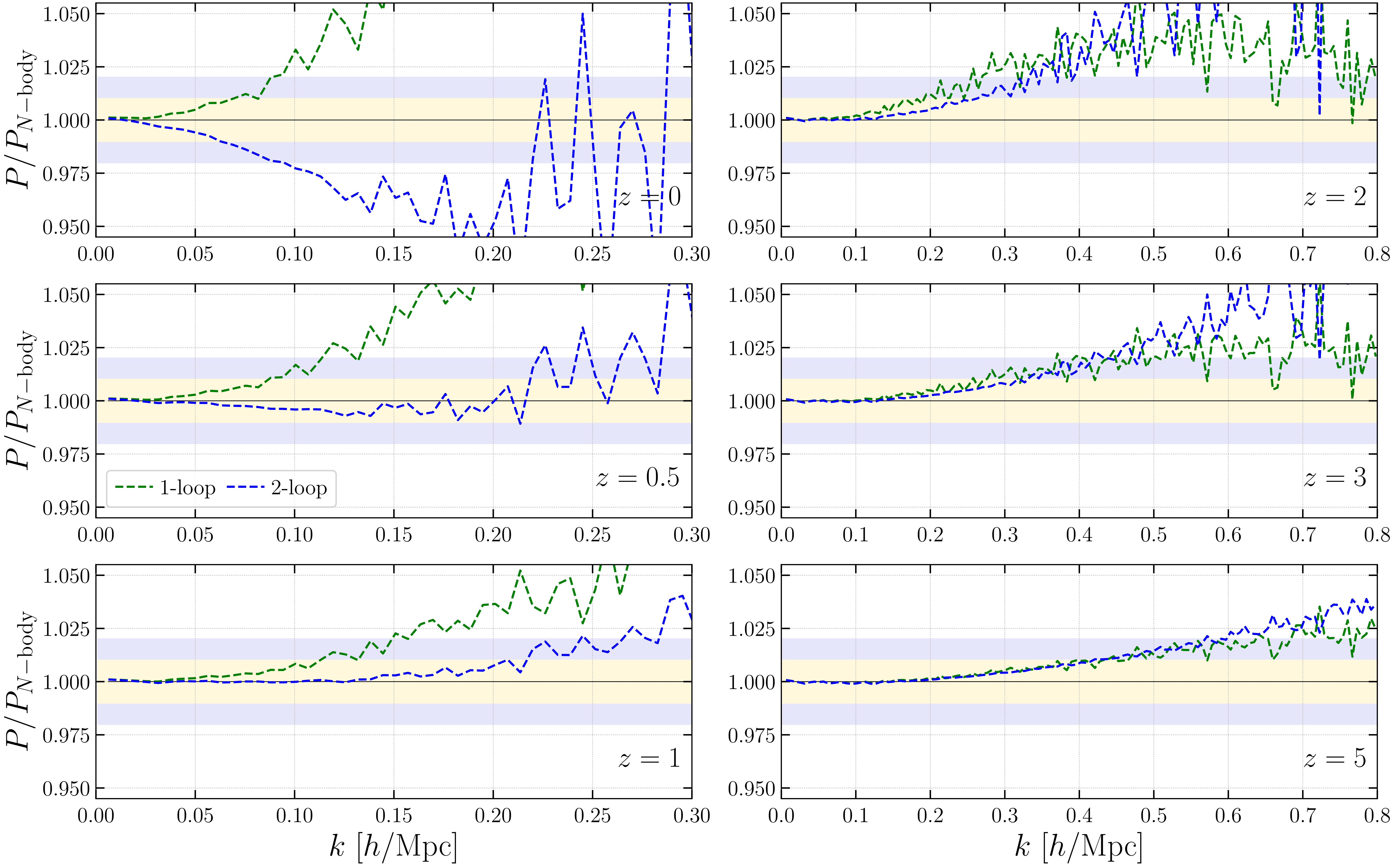

We run GridSPT and N-body simulation from the same random initial condition.

The Standard way: **SPT**

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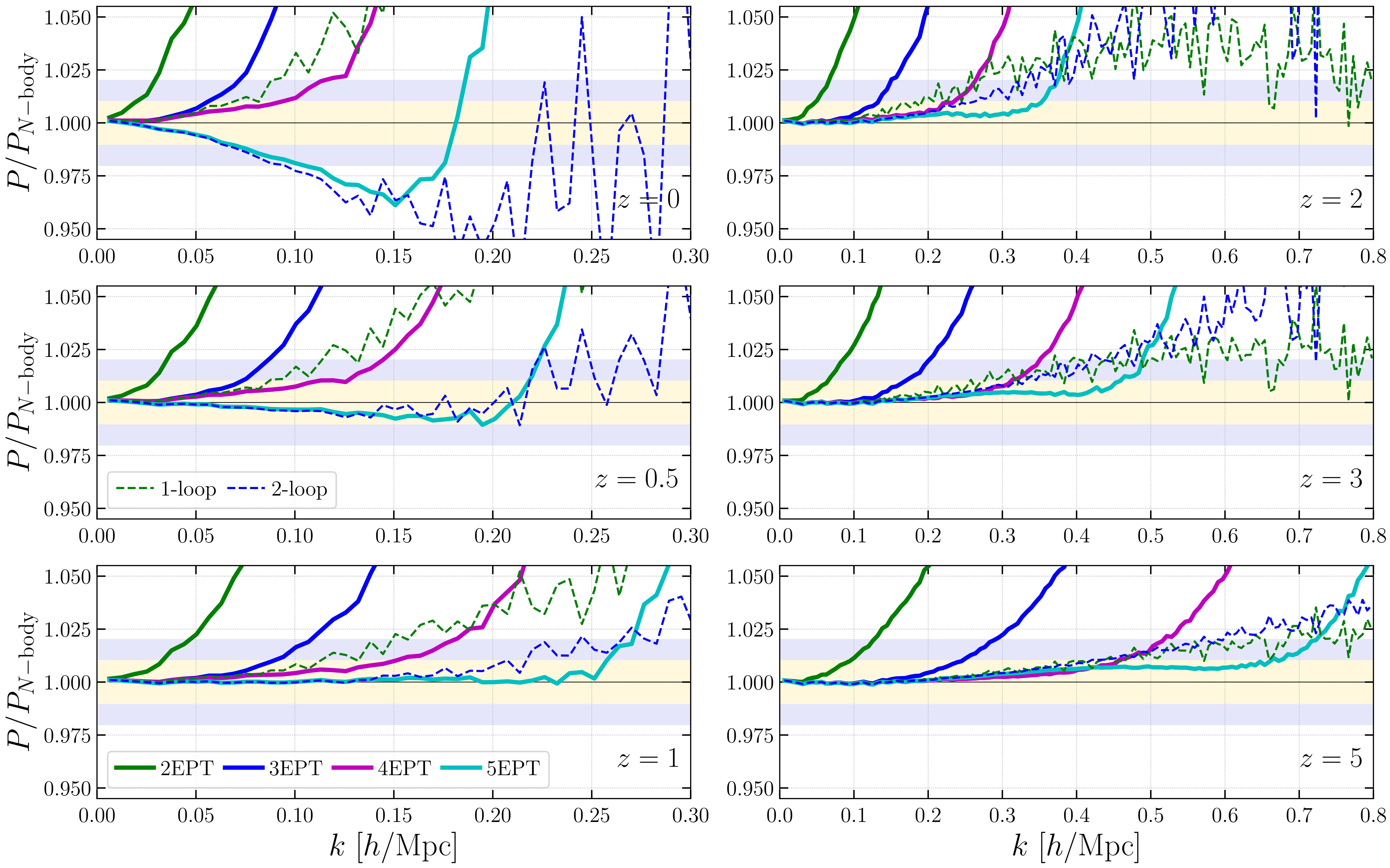

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$\delta_{n\text{EPT}} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \cdots + \delta_n$

Then measure its power spectrum

$P_{n\text{EPT}} = \langle \delta_{n\text{EPT}}(\boldsymbol{k})\delta_{n\text{EPT}}(-\boldsymbol{k})\rangle'$

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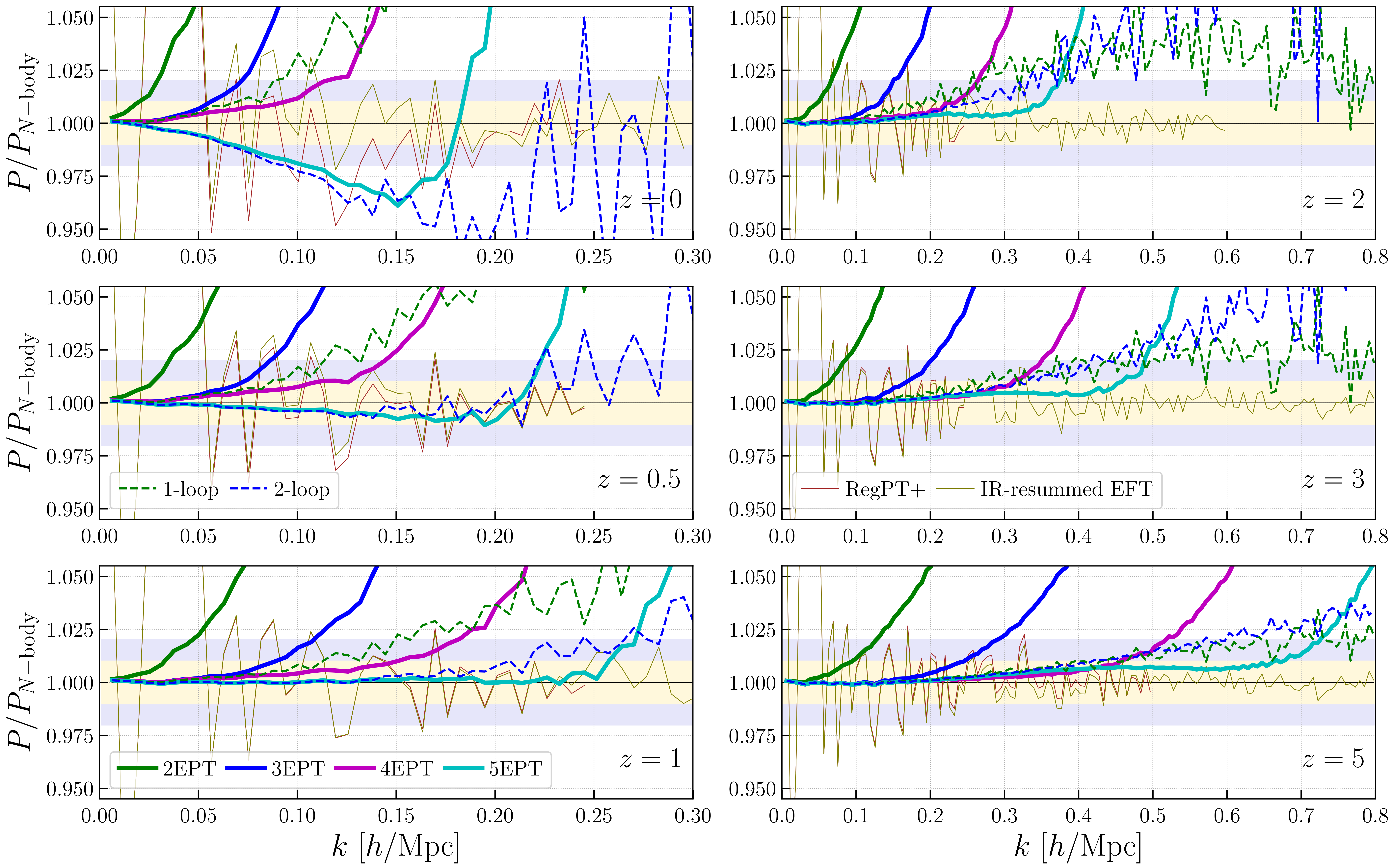

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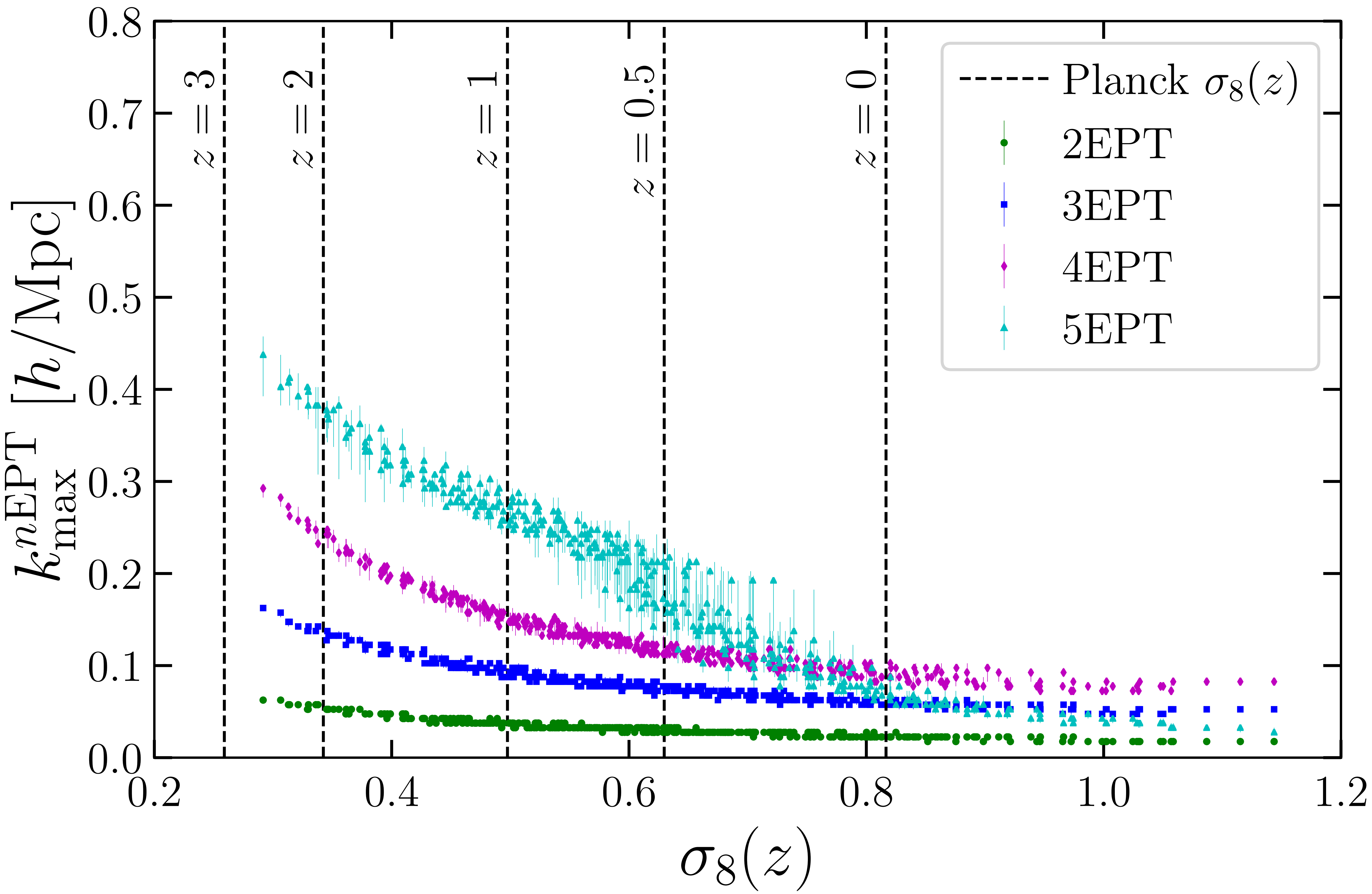
Then measure its power spectrum

$P_{n\mathrm{EPT}} = \langle \delta_{n\mathrm{EPT}}(\boldsymbol{k}) \delta_{n\mathrm{EPT}}(-\boldsymbol{k}) \rangle'$

*n*EPT needs **NO** free parameters!

Result II: Matter Power Spectrum (In wCDM cosmology)

- nEPT outperforms SPT in general wCDM cosmologies!
  (Test among 20 cosmologies, 21 redshifts between $z = 0$ and $z = 1.5$)



2.2 Compute bispectrum from density fields

$$B_{ijk}(k_1, k_2, k_3) = \left\langle \delta^{(i)}(\boldsymbol{k}_1)\delta^{(j)}(\boldsymbol{k}_2)\delta^{(k)}(\boldsymbol{k}_3) \right\rangle$$

The Standard Perturbation Theory (SPT)
(order-by-order calculation of bispectrum):

$$B_{\rm tree} = B_{211}$$

$$B_{1-\rm loop} = (B_{411} + B_{321} + B_{222})$$

$$B_{2-\rm loop} = (B_{611} + B_{521} + B_{431} + B_{422} + B_{332})$$

$$B_{\rm SPT} = B_{\rm tree} + B_{1-\rm loop} + B_{2-\rm loop}$$

The new way: *n*EPT

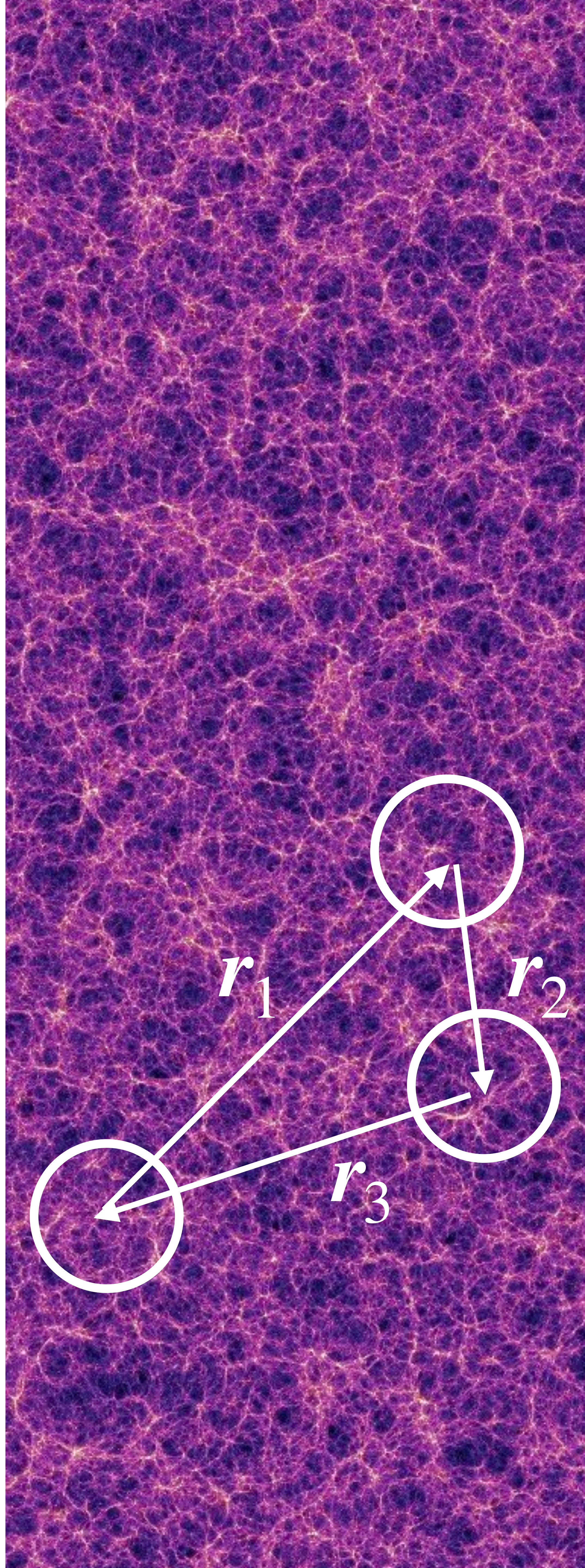
"Perturbation Theory Remixed II"
ZW *et al. (2024) in prep.*

First add the non-linear density perturbation to order n

$$\delta_{n\rm EPT} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \cdots + \delta_n$$

Then measure its power spectrum

$$B_{n\rm EPT} = \langle \delta_{n\rm EPT}(\boldsymbol{k}_1)\delta_{n\rm EPT}(\boldsymbol{k}_2)\delta_{n\rm EPT}(\boldsymbol{k}_3)\rangle'$$

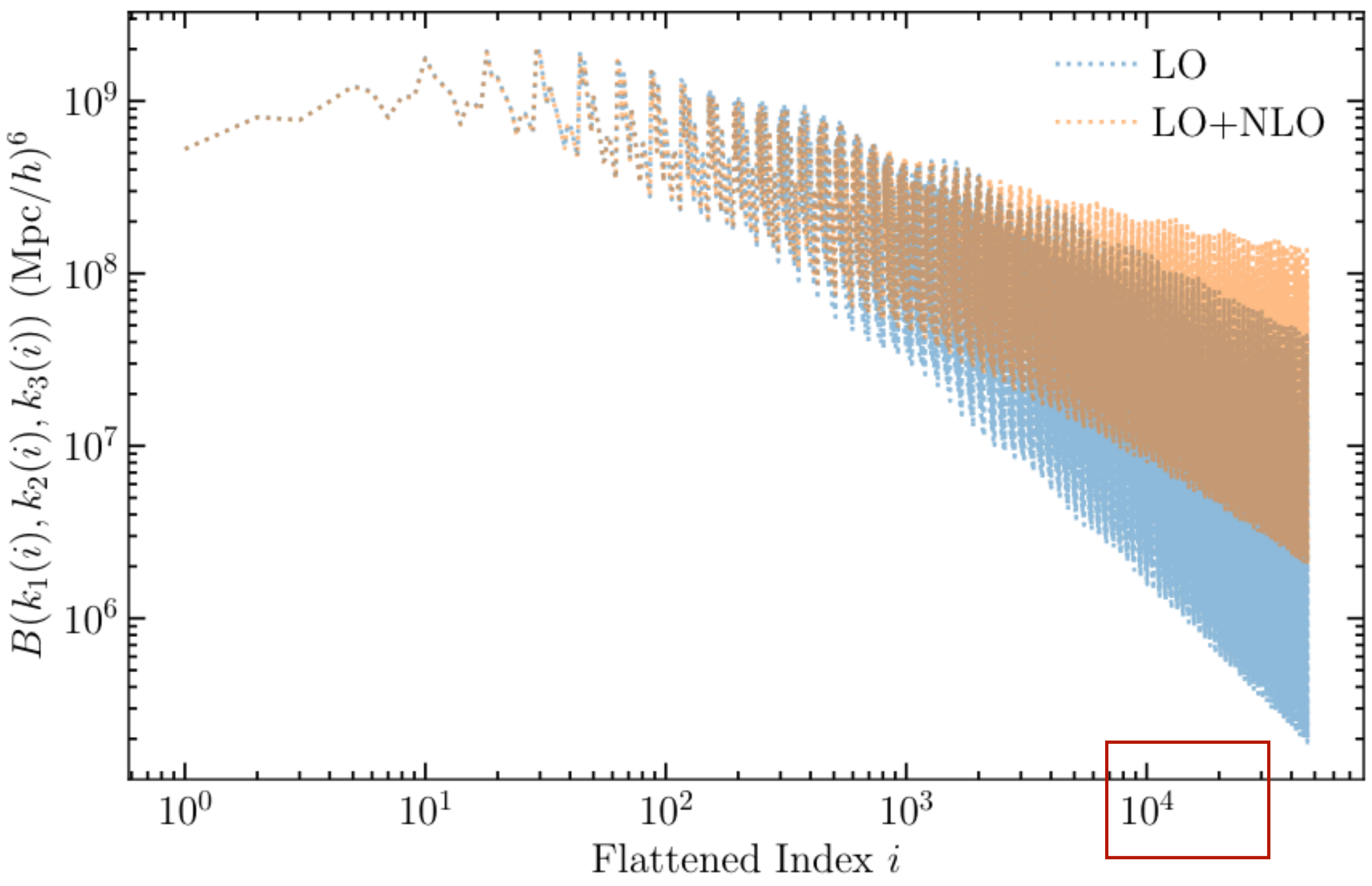


A novel way to visualize Bispectrum

Tomlinson & Jeong (2023)

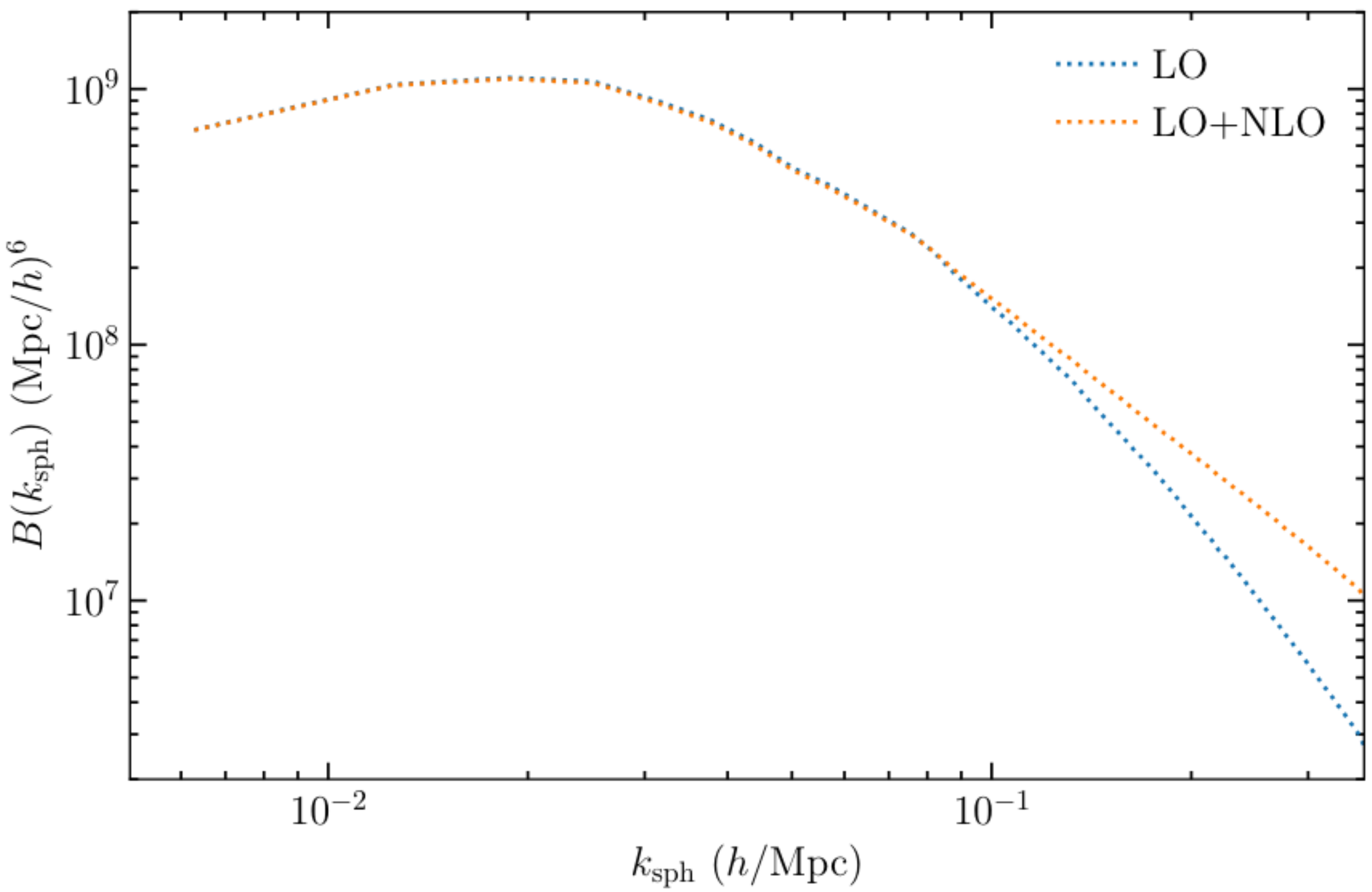
Standard way: $B(k_1, k_2, k_3)$

Bin bispectrum according to (k_1, k_2, k_3)

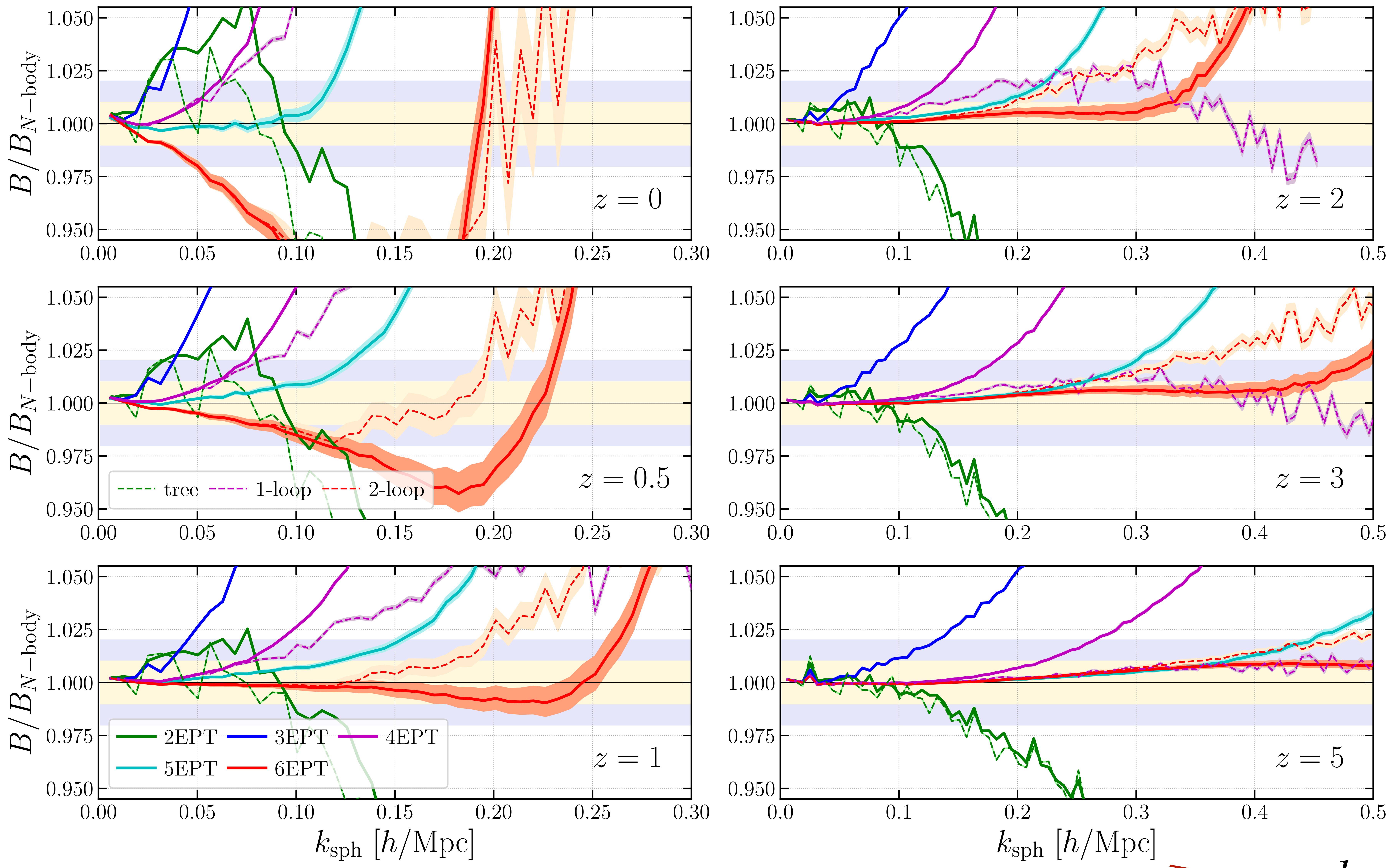


Spherical bispectrum: $B(k_{\rm sph})$

Bin bispectrum according to $k_{\rm sph} = \sqrt{(k_1^2 + k_2^2 + k_3^2)/3}$



Result III: Matter Spherical Bispectrum (WMAP)



The Standard way (order-by-order):

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$B_{1-\mathrm{loop}} = (B_{411} + B_{321} + B_{222})$

$B_{2-\mathrm{loop}} = (B_{611} + B_{521} + B_{431} + B_{422} + B_{332})$

The new way: nEPT

First add the non-linear density to order n

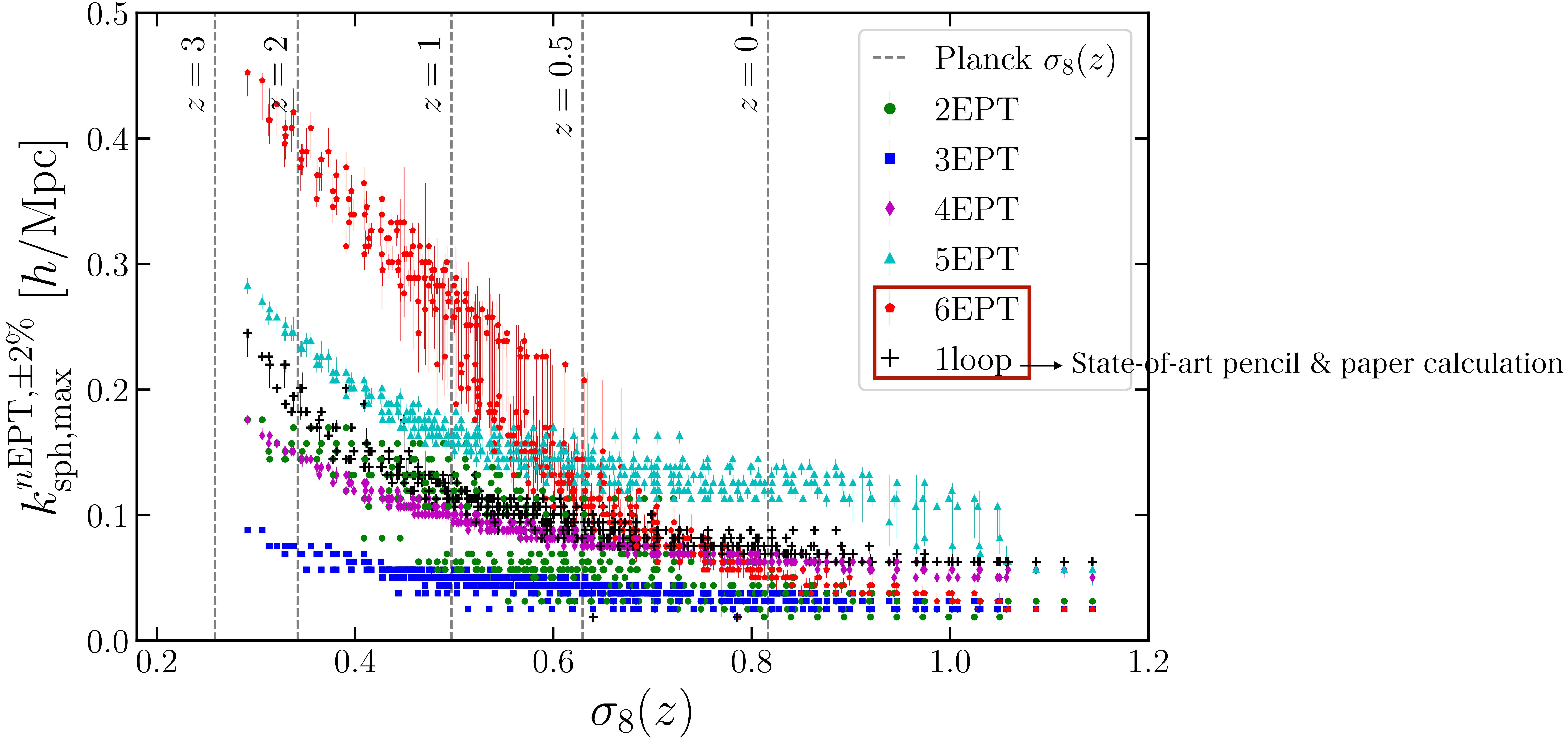
$\delta_{n\mathrm{EPT}} = \delta_1 + \delta_2 + \delta_3 + \cdots + \delta_n$

Then measure its **bispectrum**

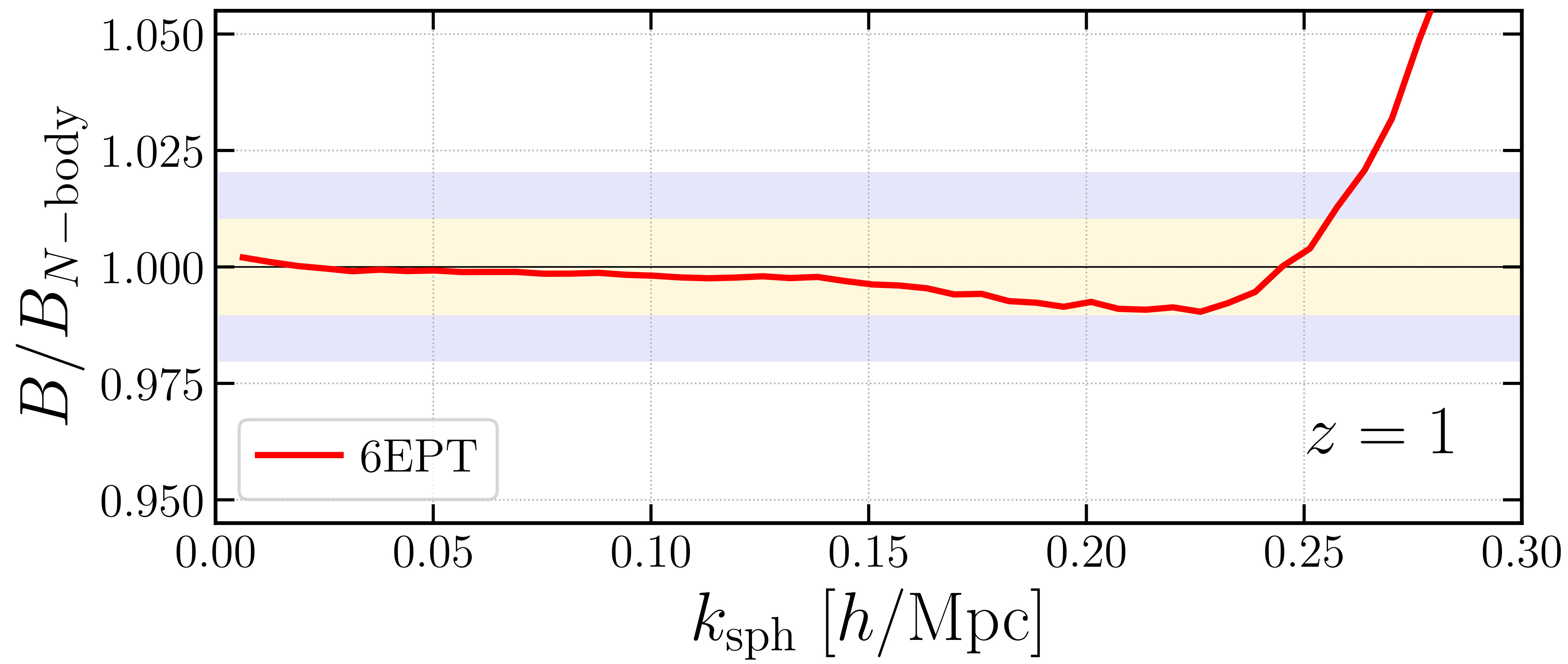
$B_{n\mathrm{EPT}} = \langle \delta_{n\mathrm{EPT}}(\boldsymbol{k}_1)\delta_{n\mathrm{EPT}}(\boldsymbol{k}_2)\delta_{n\mathrm{EPT}}(\boldsymbol{k}_3)\rangle'$

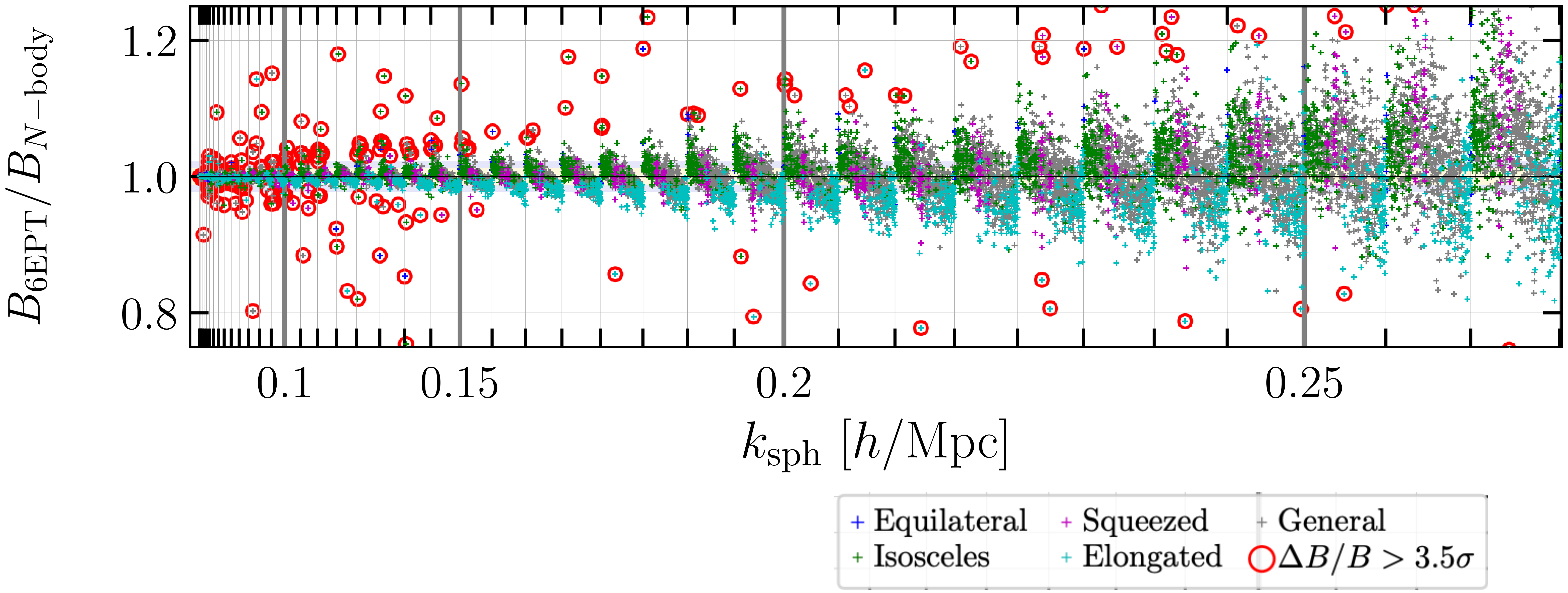
$$k_{\mathrm{sph}} \equiv \sqrt{(k_1^2 + k_2^2 + k_2^3)/3}$$

Result IV: Validity Range of nEPT spherical bispectrum



Result V: Validity Range of nEPT bispectrum (individual Δ)

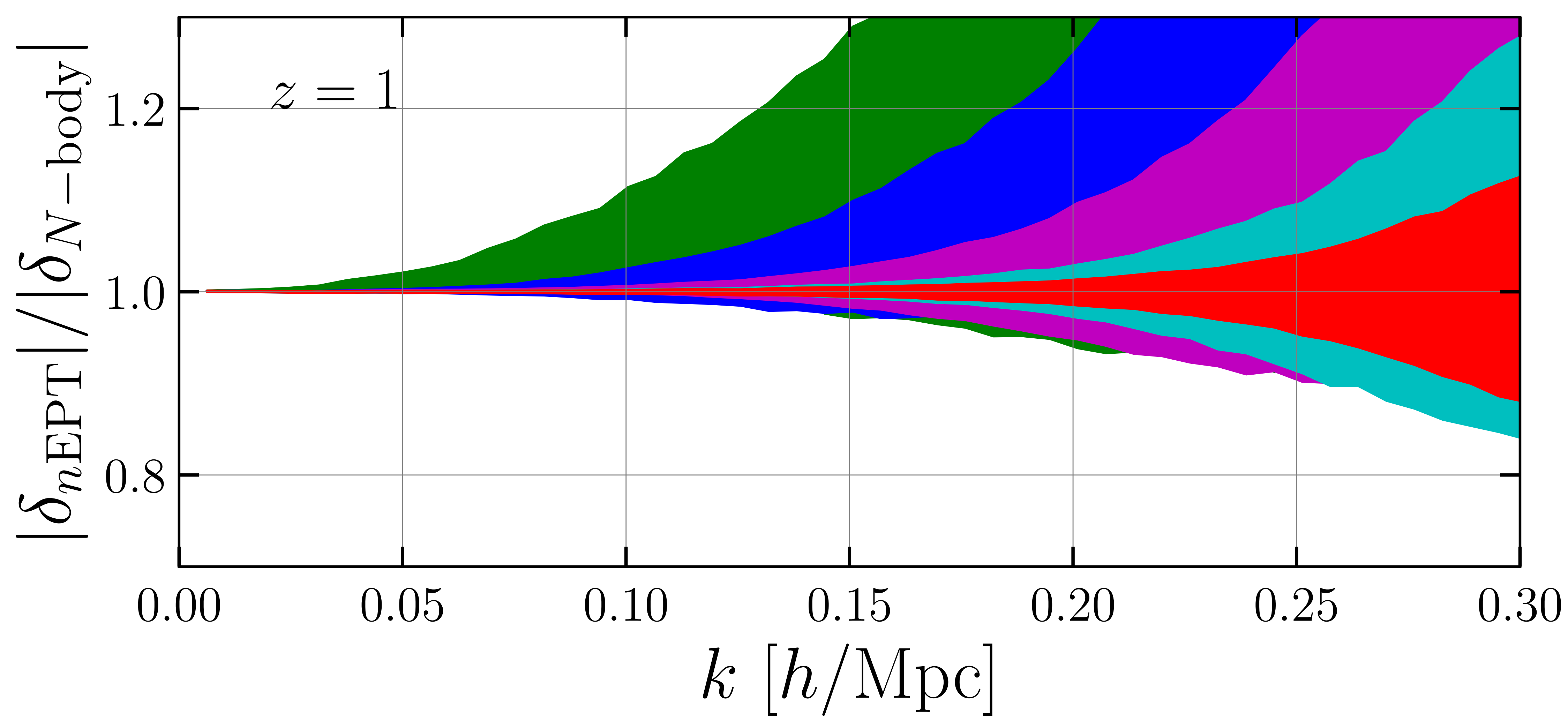




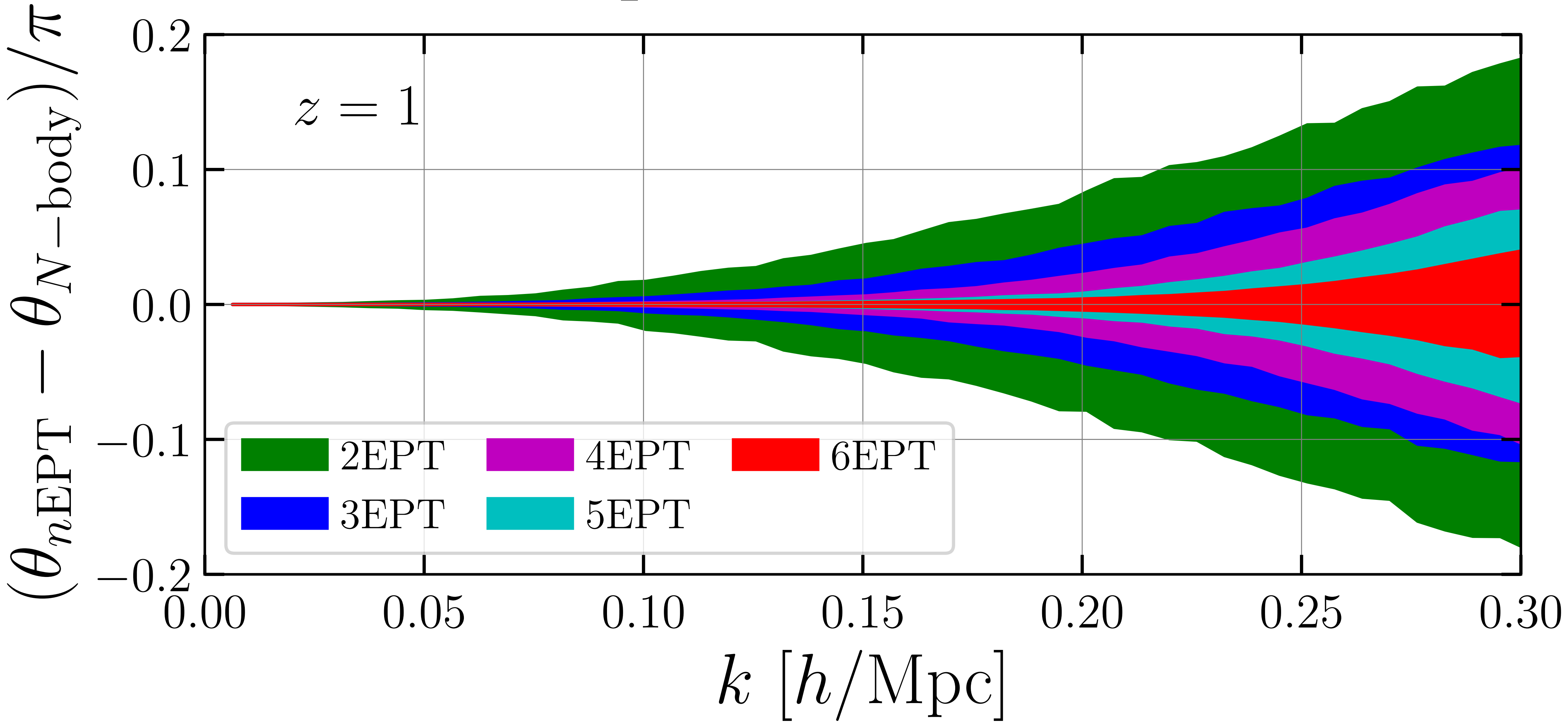
2.3 Field-level residual

$$\delta(\boldsymbol{k}) = \color{red}{\delta(\boldsymbol{k})} \; e^{i\theta}$$

modulus ratio



phase difference



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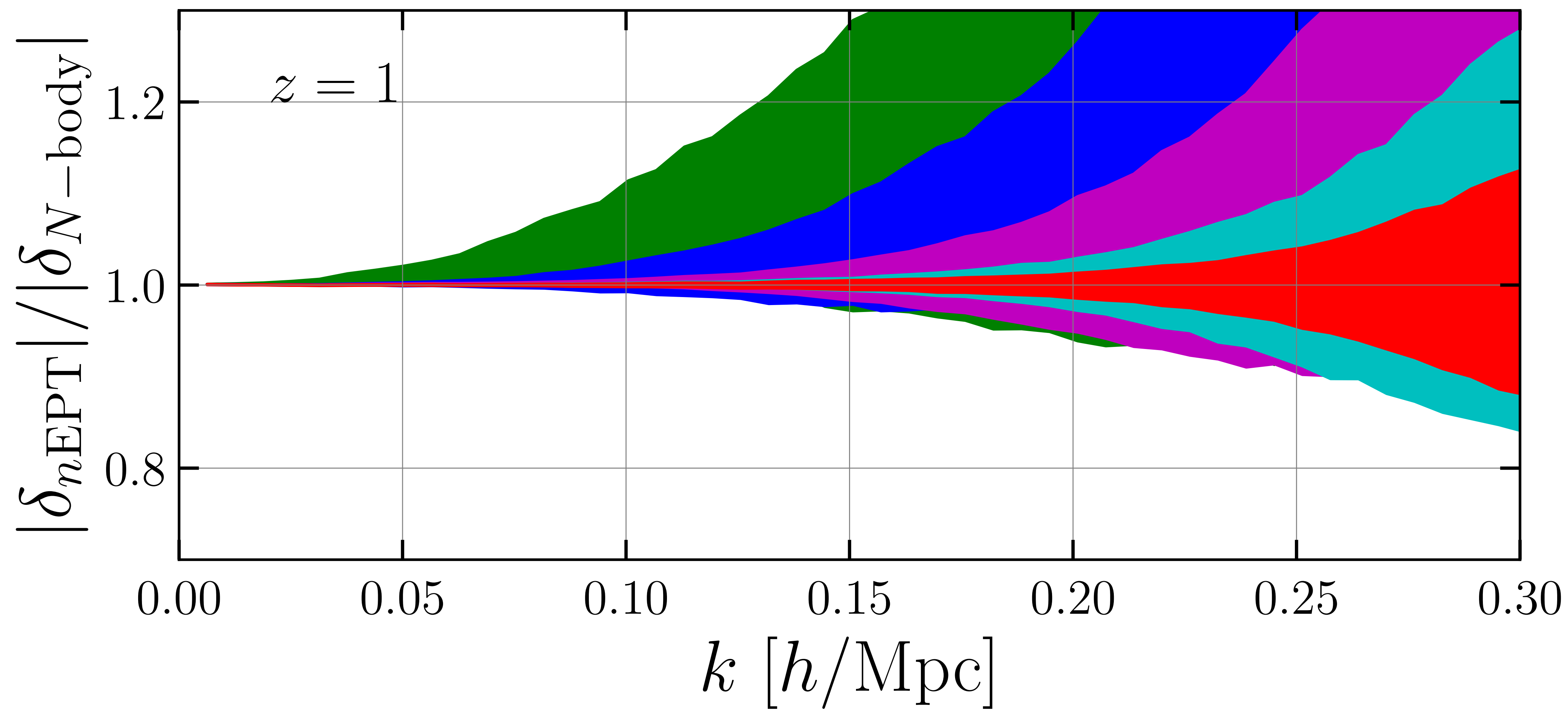

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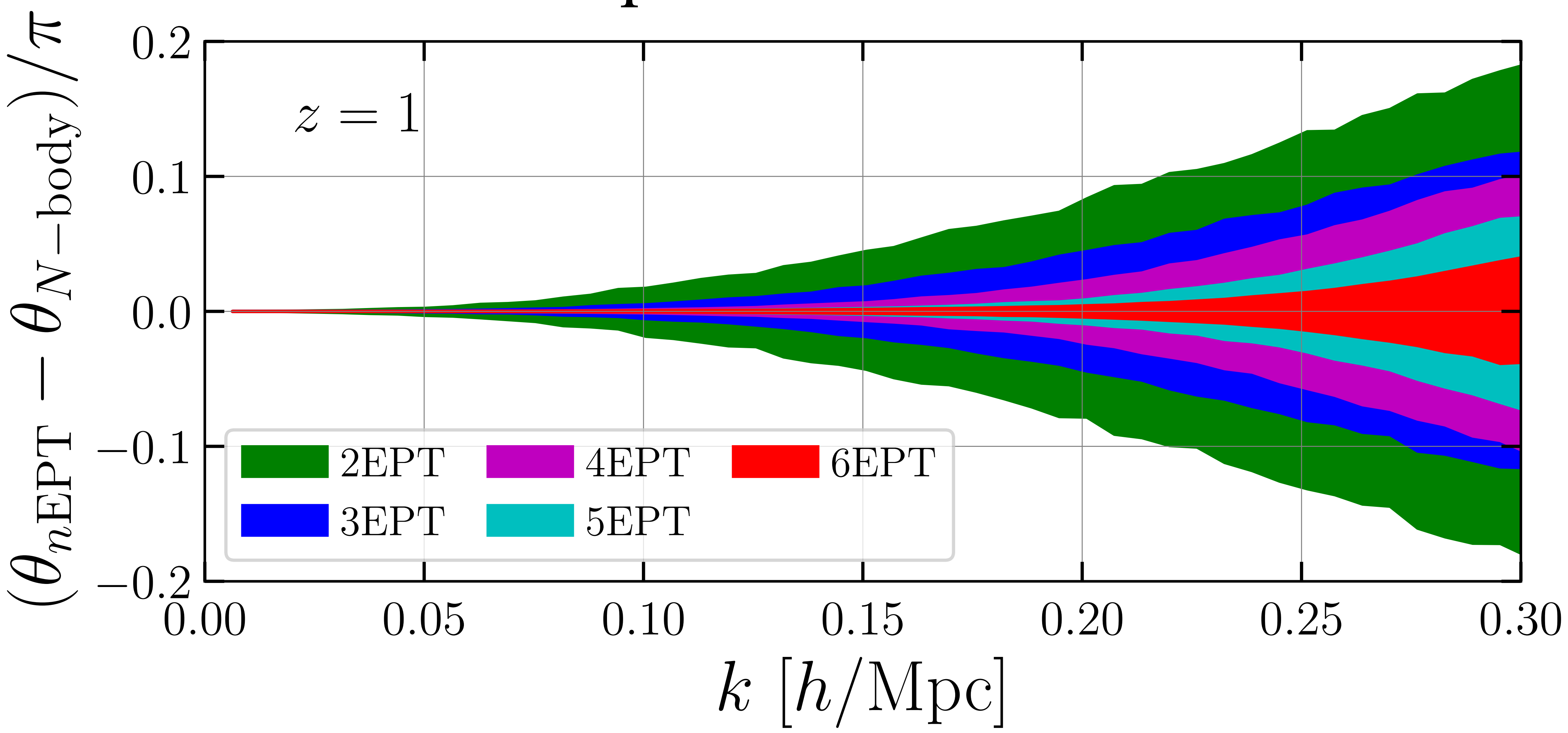
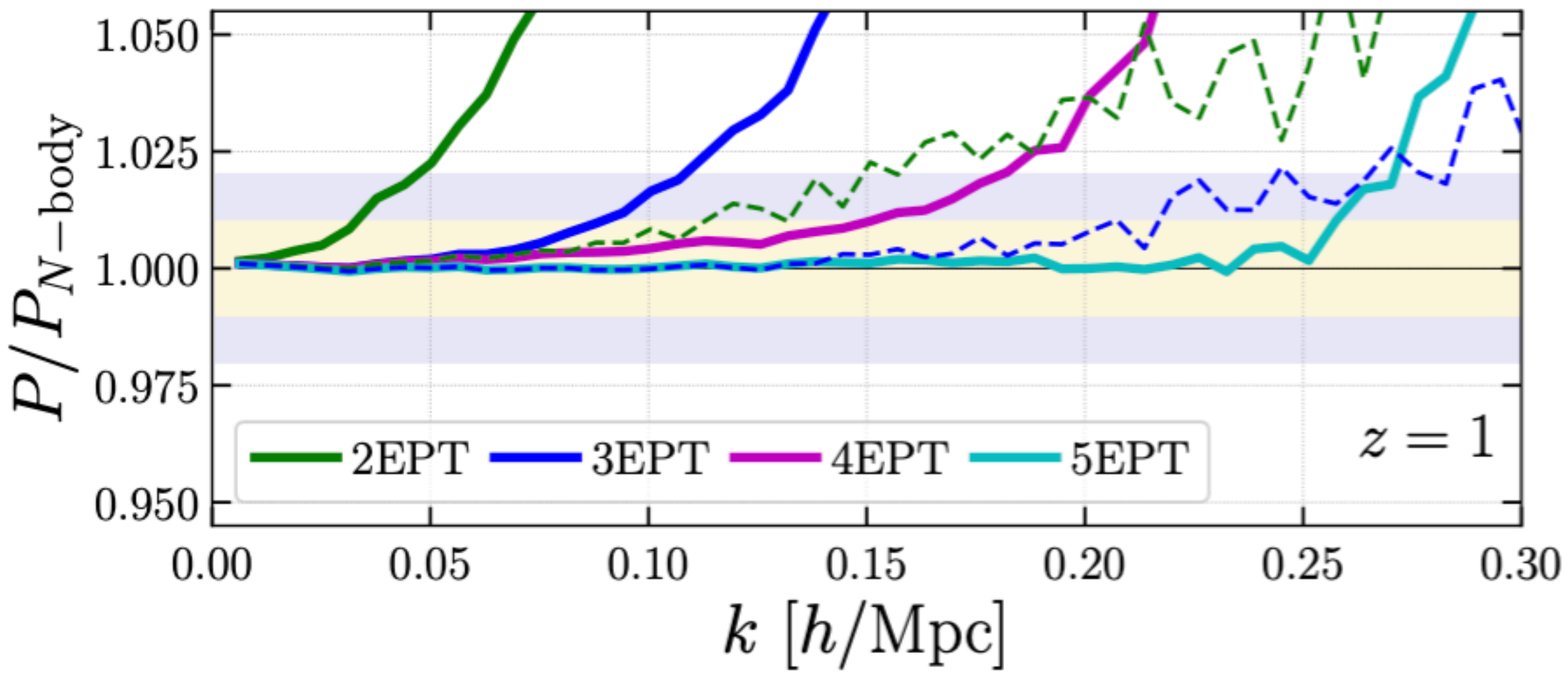
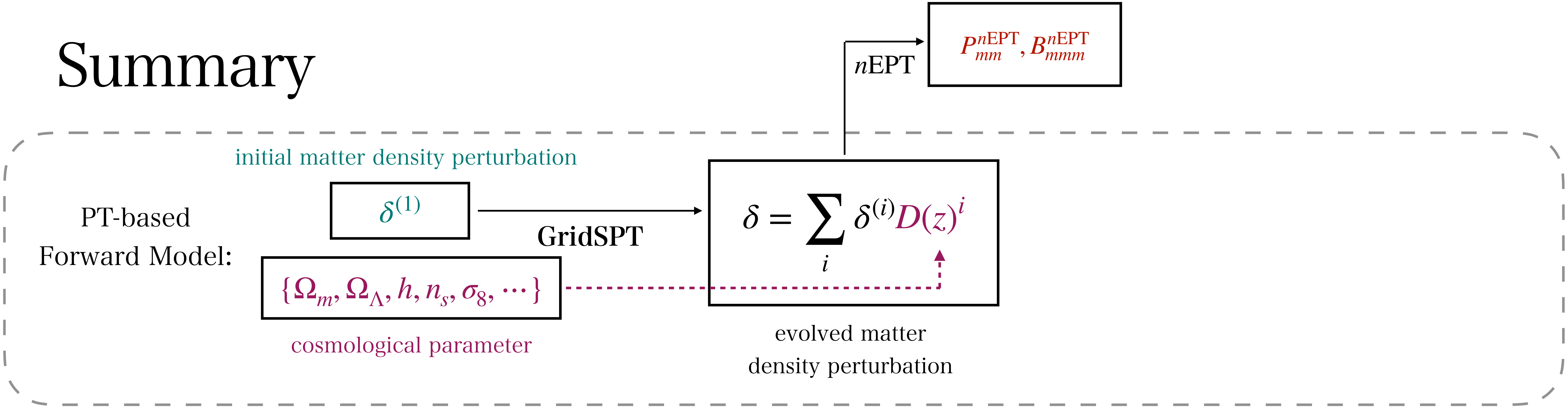


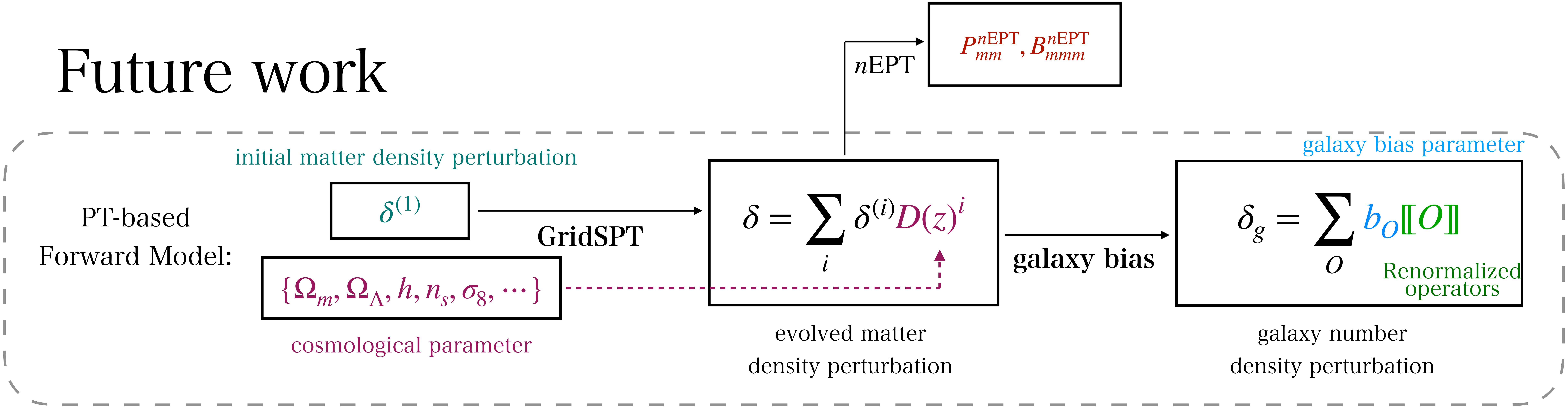


Summary



- nEPT $P(k)$ & $B(\{k_i\})$ has better convergence than SPT at $z > 1$
- 5EPT/6EPT extend the validity range k_{max} of matter $P(k)$ & $B(\{k_i\})$ modeling without any free parameters
- The k_{max} for PT-based field-level inference could be smaller than that of summary statistics

Future work



- Renormalization of galaxy bias

- Can nEPT extends the $k_{\max}$ of galaxy power spectrum and bispectrum?

- **Field-level Inference** with PT-based forward model