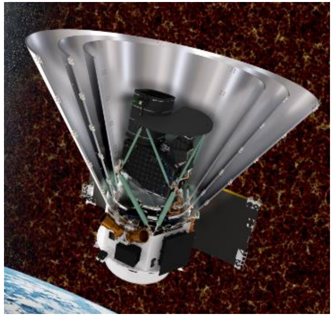


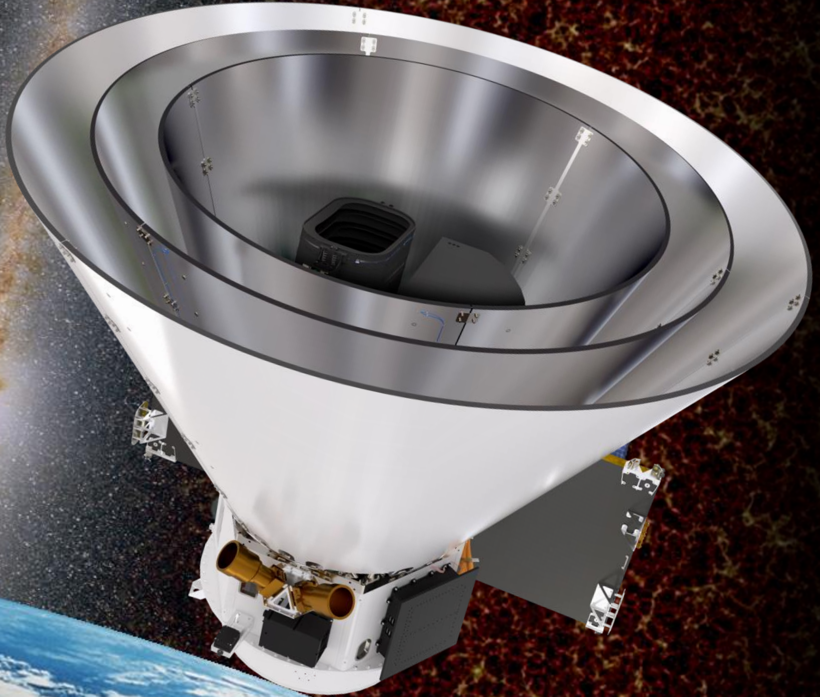
Large-Scale Modeling of Galaxy Power Spectrum for SPHERE^{x}

Robin Wen
California Institute of Technology

Outline

- SPHERE^x Overview
- Large-scale Modeling Challenges
- Power Spectrum Multipole (PSM) modeling through Spherical Fourier Bessel (SFB) basis

SPHERE^{X}: An All-Sky Infrared Spectral Survey Satellite



Designed to Explore

- Origin of the Universe
- Origin and History of Galaxies
- Origin of Water in Planetary Systems

First All-Sky Near-IR Spectral Survey

102 bands in 0.75-5 μm
Scan full-sky 4 times in 2 years

Elegantly Simple

- Single Observing Mode
- No Moving Parts in Instrument

PI: Jamie Bock -- Caltech/JPL
PS: Olivier Doré -- JPL

JPL Jet Propulsion Laboratory California Institute of Technology · Ball · ipac · CfA · UCI · THE OHIO STATE UNIVERSITY · ASU · R·I·T · JOHNS HOPKINS UNIVERSITY · ARIZONA · Argonne NATIONAL LABORATORY

SPHERE^{X} ADDRESSES 3 CENTRAL QUESTIONS

…as stated in the NASA 2014 Science Plan

Cosmic Inflation

How Did the Universe Begin?

"Probe the origin and destiny of our universe, including the nature of black holes, dark energy, dark matter and gravity"

Galaxy Formation

How Did Galaxies Begin?

"Explore the origin and evolution of the galaxies, stars and planets that make up our universe"

Interstellar Ices

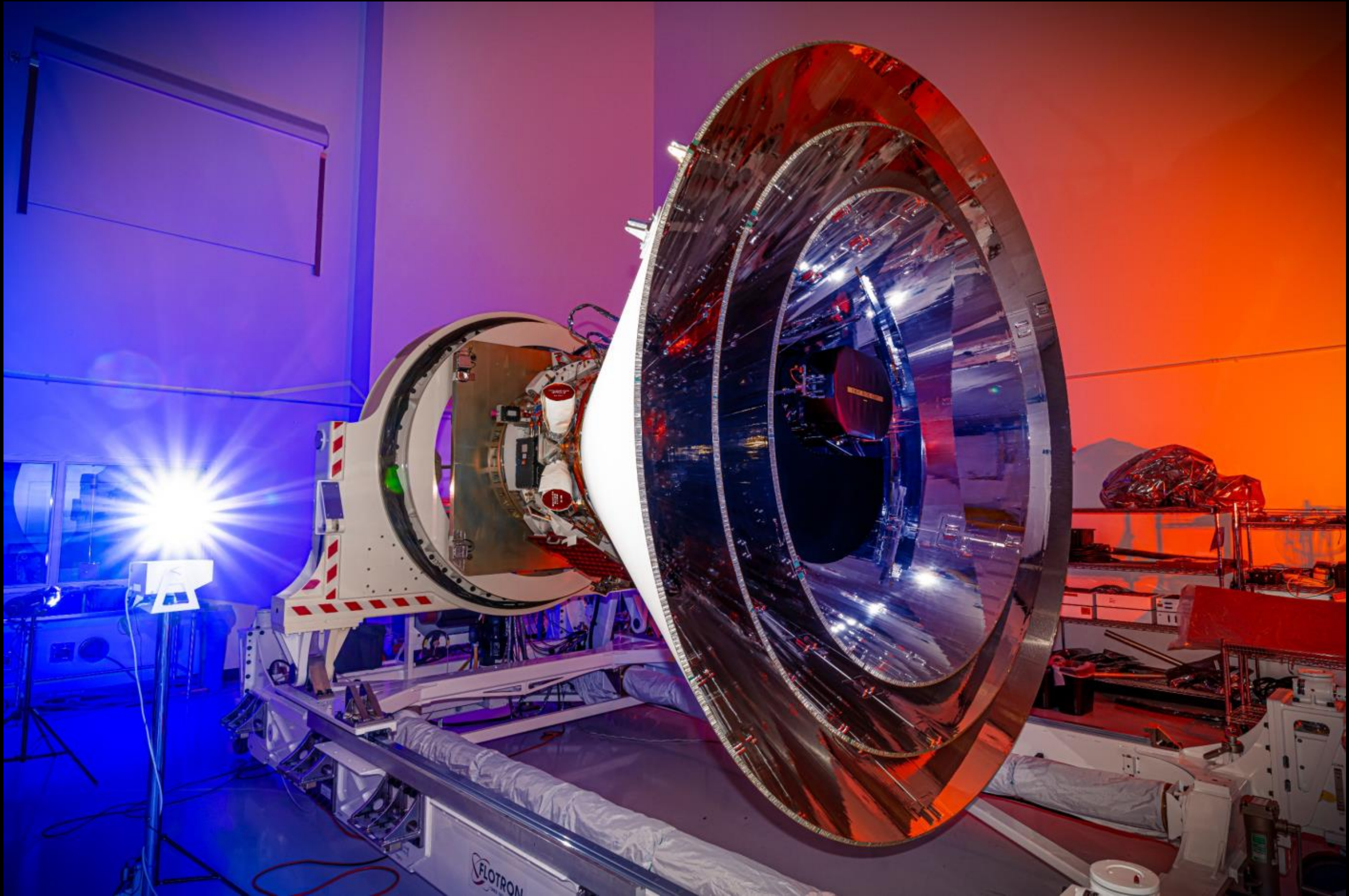
What are the Conditions for Life Outside the Solar System?

"Discover and study planets around other stars, and explore whether they could harbor life"

Legacy Survey

…While Creating a Unique All-Sky Spectral Survey

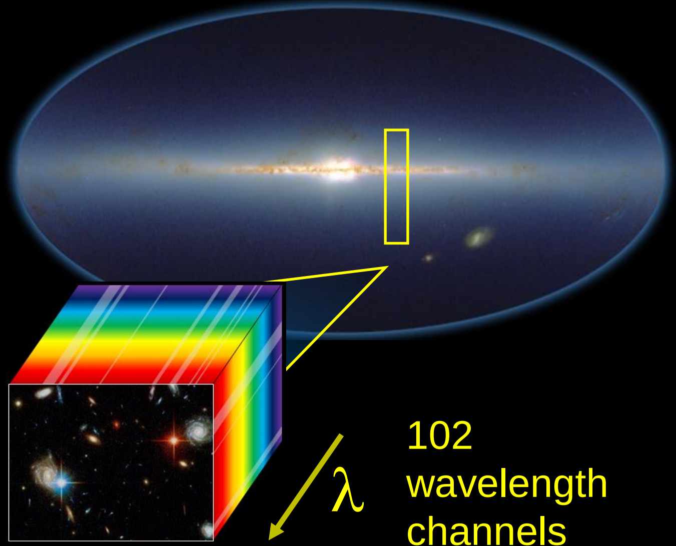
Launch in early 2025!



Legacy Survey

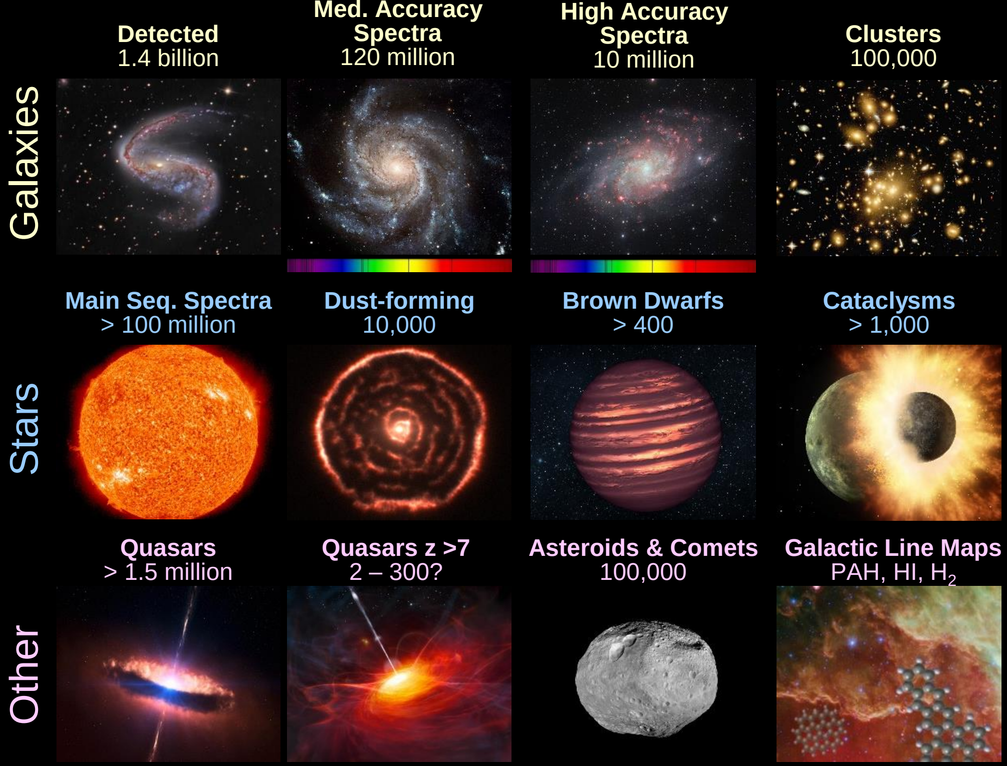
SPHERE^{X} PROVIDES A RICH ALL-SKY SPECTRAL CATALOG

All-Sky Survey



102 wavelength channels

Spectral Data Cube



Galaxies

Detected 1.4 billion

Med. Accuracy Spectra 120 million

High Accuracy Spectra 10 million

Clusters 100,000

Stars

Main Seq. Spectra > 100 million

Dust-forming 10,000

Brown Dwarfs > 400

Cataclysms > 1,000

Other

Quasars > 1.5 million

Quasars z >7 2 – 300?

Asteroids & Comets 100,000

Galactic Line Maps PAH, HI, H_2

SPHEREx provides a new and unique dataset

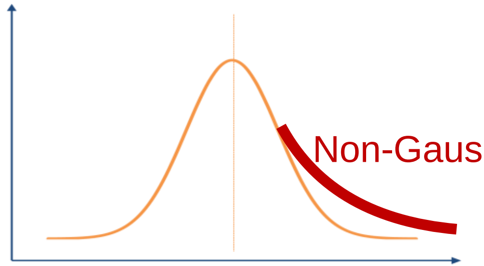
a complete near-infrared spectrum for every 6” pixel on the sky

All-Sky surveys demonstrate high scientific return with lasting data legacy used across astronomy (COBE, IRAS, GALEX, WMAP, Planck, WISE)

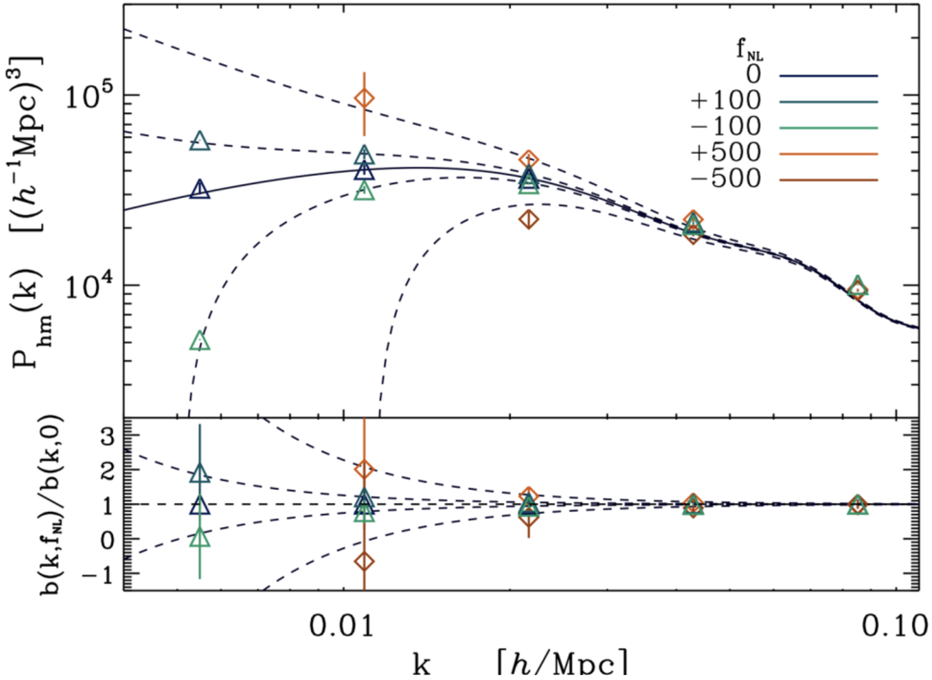
Many exciting discoveries will come from the community

SPHEREx Constrains Local Primordial Non-Gaussianity (PNG)

$$\Phi(\boldsymbol{x}) = \varphi(\boldsymbol{x}) + f_{\mathrm{NL}} \left(\varphi^2(\boldsymbol{x}) - \langle \varphi^2 \rangle\right)$$



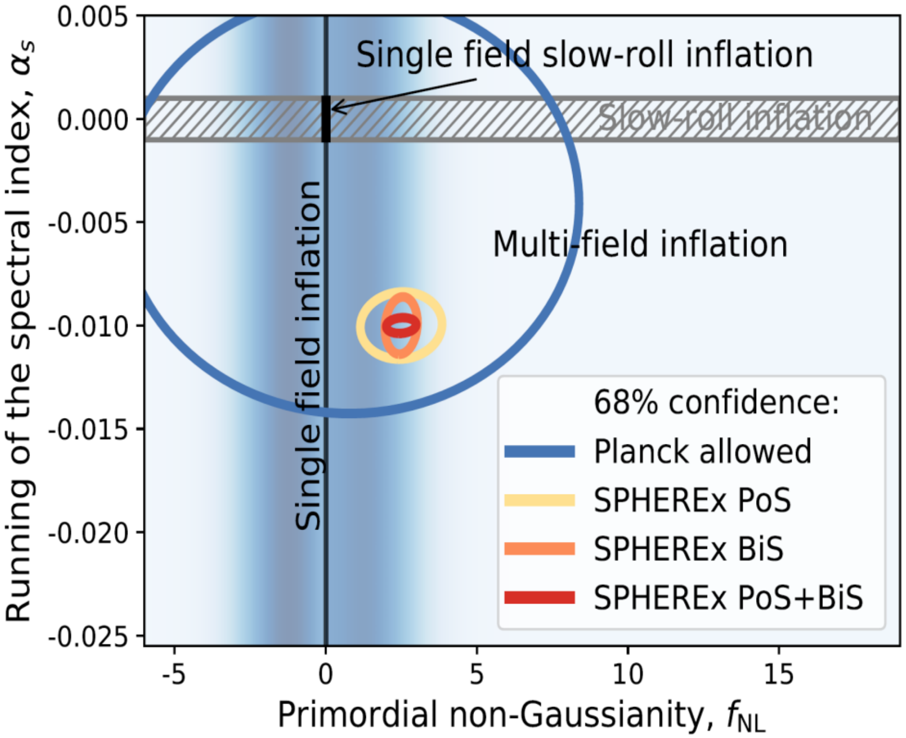
- Single-field inflation generically predicts $f_{\mathrm{NL}} < 0.01$
- Multi-field inflation generically predicts $f_{\mathrm{NL}} \sim \mathcal{O}(1)$
- Power Spectrum (PS): scale-dependent bias
- Bispectrum (BS): primordial non-Gaussian perturbation



Dalal, Doré et al. 2008

$$\delta_g(k) = \left[b_1 + b_\phi f_{NL}\, \alpha^{-1}(k)\right] \delta_m(k)$$

SPHERE^x Tests Inflation through local PNG



PS: $\sigma(f_{\rm NL}) \sim 1$

BS: $\sigma(f_{\rm NL}) \sim 0.7$

PS+BS: $\sigma(f_{\rm NL}) \sim 0.5$

Doré et al. 2014
Heinrich, Doré, Krause 2024

*Multi-tracer analysis exploiting LPNG bias (b_ϕ) may offer further improvement!

Challenges

- Systematics
  - SPHEREx is designed to minimize systematics (in space, stable gain)
  - However, galactic foregrounds (dust, star,…)
  - Crowding
  - Systematics in reference catalogues

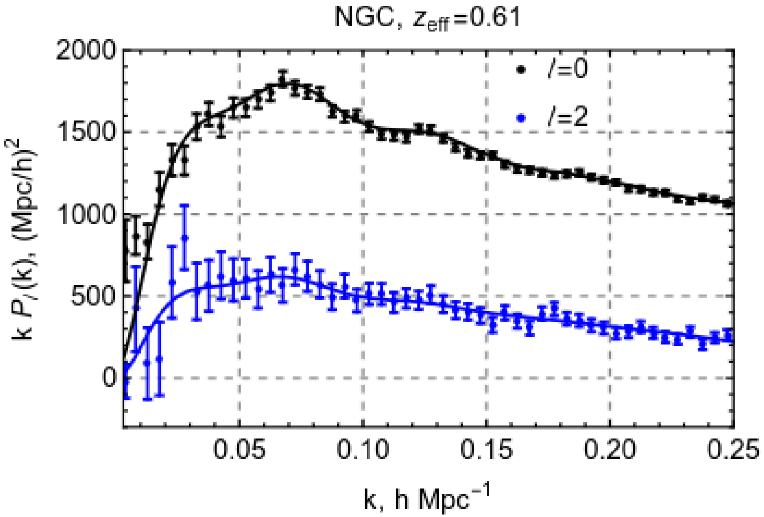
- Redshift-error modelling (spectro-photometric survey)
- Theoretical Modeling at Large-scale
  - Wide-angle (WA) Effects
  - General Relativistic (GR) Effects
  - Covariance

Yamamoto Estimator: Power Spectrum Multipole (PSM)

$$\langle \widehat{P}_L(k)\rangle = \frac{(2\ell+1)}{I_{22}} \int_{\hat{\mathbf{k}}} \int_{\mathbf{x}_1,\mathbf{x}_2} e^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)} \langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle W(\mathbf{x}_1)W(\mathbf{x}_2)\mathcal{L}_L(\hat{\mathbf{x}}_1\cdot\hat{\mathbf{k}})$$

- Fast Implementation through FFT
- Multipoles L=0,2,4 capture redshift-space distortion (RSD)
- Mature non-linear modeling at 1-loop order
- Covariance mostly diagonal
- Standard for local PNG measurement

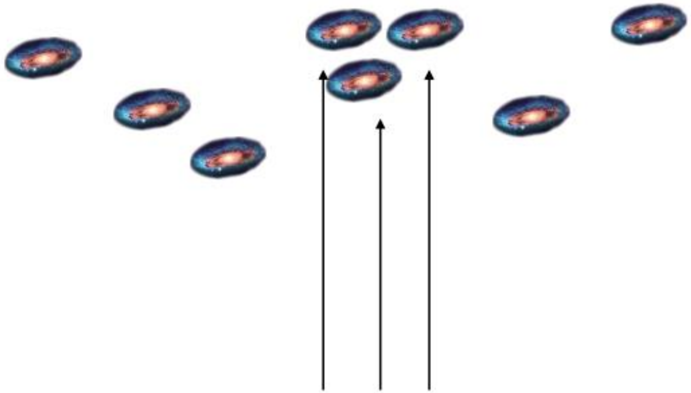
- Mixing angular and radial scales
- No* exact covariance at large scales



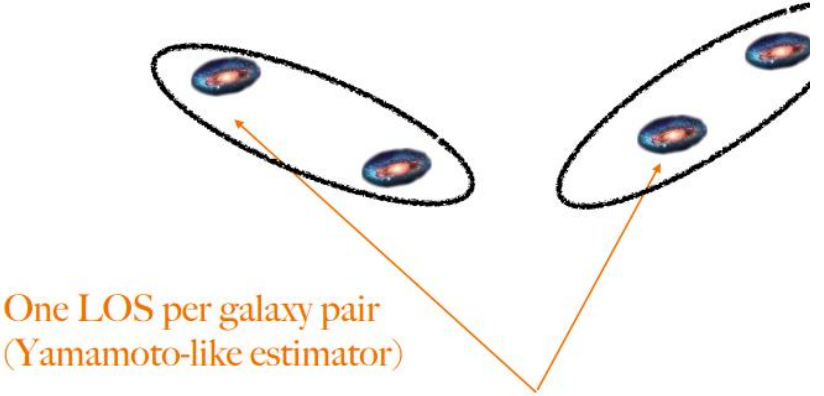
Ivanov et al. 2019

Wide-Angle Effects

- PSM are in Cartesian coordinate
- Do not obey the curved-sky geometry at large angular separation
- WA effects need to be modeled in theory



Plane-parallel approximation

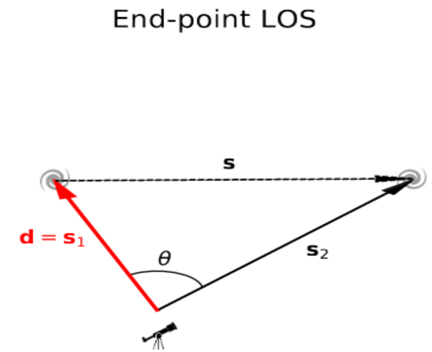


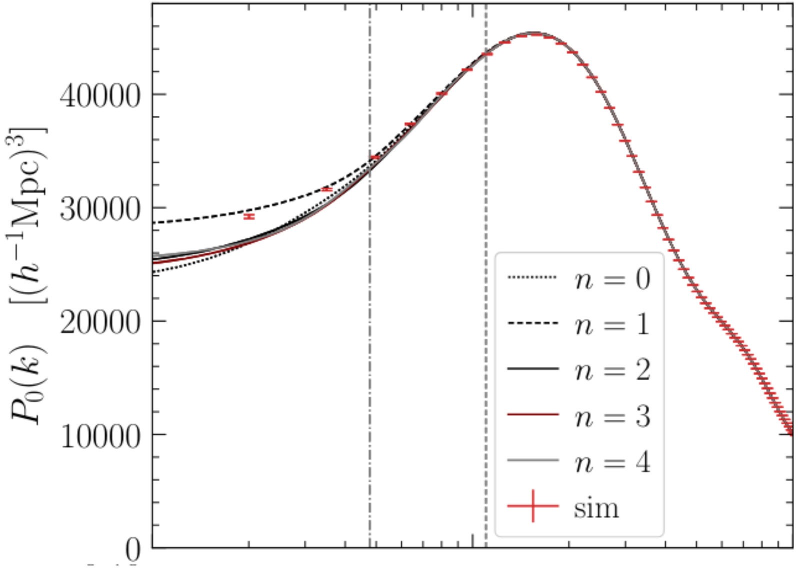
End-point LOS

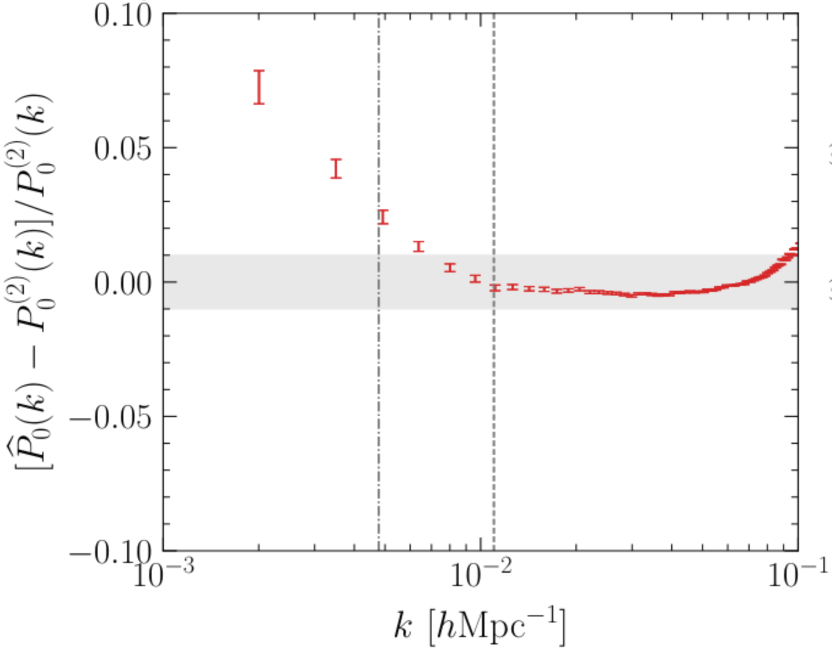
Perturbative WA modeling of PSM

$$P(\mathbf{k},\mathbf{d}) = \sum_{L,n} (\frac{1}{kd})^n P_L^{(n)}(k)\mathcal{L}_L(\hat{\mathbf{k}}\cdot\hat{\mathbf{d}})$$

Perturbative WA can bias fNL~5







Benabou, Sands, et al. 2024

PSM Modeling with Spherical Fourier Bessel (SFB) basis

Based on arxiv:2404.04812
Exact Modeling of Power Spectrum Multipole through Spherical Fourier-Bessel Basis

Key People

Henry Gebhardt

Chen Heinrich

Olivier Doré

SFB Transform

Laplacian Eigenfunctions: $\nabla^2 f = -k^2\, f$

Cartesian Coordinates:

$$f(\mathbf{k},\mathbf{r}) = e^{-i\mathbf{k}\cdot\mathbf{r}}$$

Fourier transform:

$$\tilde{\delta}(\mathbf{k}) = \int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, \delta(\mathbf{r})$$

Spherical Coordinates:

$$f_{\ell m}(k,\mathbf{r}) = j_\ell(kr)\, Y_{\ell m}(\hat{\mathbf{r}})$$

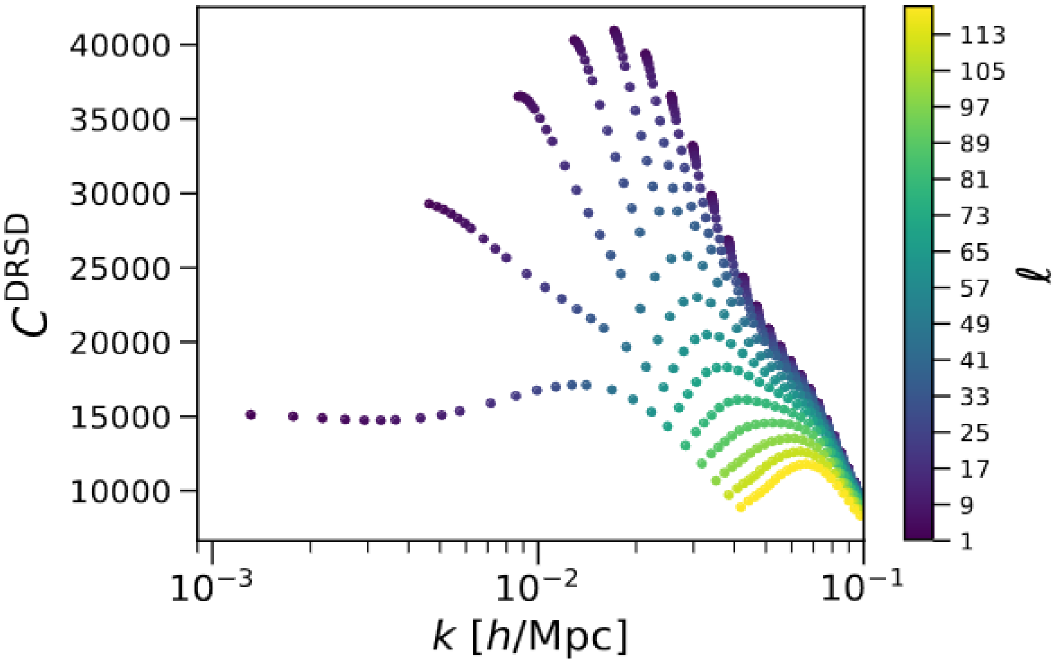
SFB transform:

$$\tilde{\delta}_{\ell m}(k) = \int d^3r\, j_\ell(kr)\, Y_{\ell m}(\hat{\mathbf{r}})\, \delta(\mathbf{r})$$

- Obey curved-sky geometry. Retain angular modes
- Naturally include WA effects
- Remain in Fourier space. Mode separation

SFB Power Spectrum

$$\langle \delta_{\ell_1 m_1}(k_1)\delta^*_{\ell_2 m_2}(k_2)\rangle = C_{\ell_1}(k_1,k_2)\delta^K_{\ell_1\ell_2}\delta^K_{m_1 m_2}$$



Drawbacks:

- Unfamiliar statistics
- Large Data Vector
- No FFT estimator
- Challenging non-linear modeling

SFB-to-PSM Mapping

$$P_L(k) = \frac{(4\pi)^2(2L+1)}{I_{22}} \sum_{a,b} i^{-a+b}(2a+1)(2b+1) \begin{pmatrix} a & L & b \\ 0 & 0 & 0 \end{pmatrix}^2 C_b^{ab,\mathrm{W}}(k,k)$$

Castorina & White 2017

Generalized SFB:

$$\delta^L_{\ell m}(k) = \int_{\mathbf{x}} j_{\enclose{circle}{L}}(kx) Y^*_{\enclose{circle}{\ell m}}(\hat{\mathbf{x}})\delta(\mathbf{x})$$

$$C_{\enclose{circle}{\ell}}^{\enclose{circle}{ab,}\mathrm{W}}(k_1,k_2) \equiv \frac{1}{2\ell+1}\sum_m \langle \delta^{a,\mathrm{W}}_{\ell m}(k_1)\delta^{b*,\mathrm{W}}_{\ell m}(k_2)\rangle$$

Reduces to the canonical SFB for a = b = ell

SFB-to-PSM Mapping

PS monopole:

- Only modes in a homogenous and isotropic Universe
- Same Fourier modes k on both sides

$$P_0(k) = \frac{(4\pi)^2}{I_{22}} \sum_b (2b+1) C_b^{\mathrm{W}}(k,k)$$

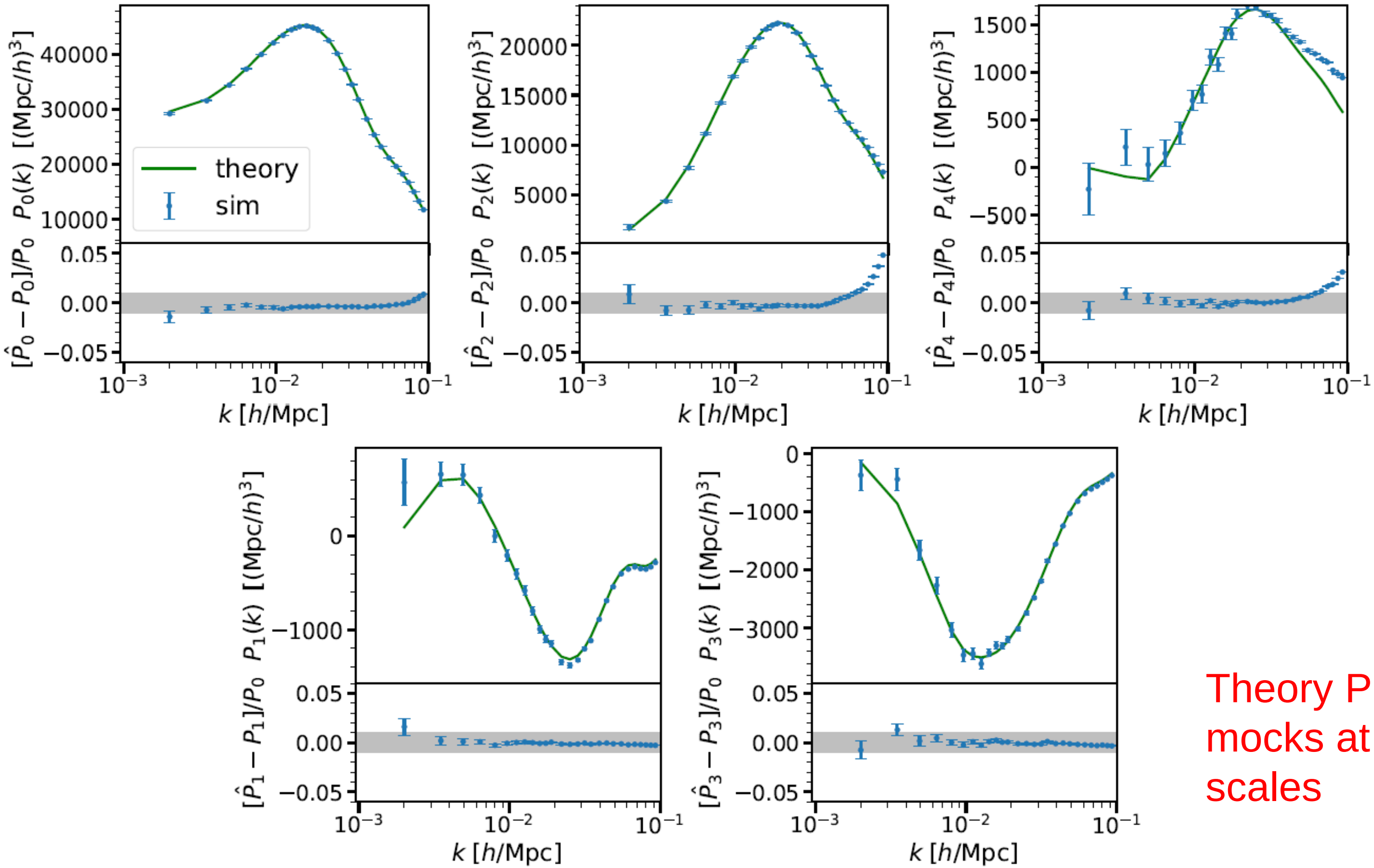
Higher multipoles:

- Off-diagonal components to be (partially) brought back in higher multipoles
- folded in through the upper indices of generalized SFB (a,b).

$$P_L(k) = \frac{(4\pi)^2(2L+1)}{I_{22}} \sum_{a,b} i^{-a+b}(2a+1)(2b+1) \begin{pmatrix} a & L & b \\ 0 & 0 & 0 \end{pmatrix}^2 C_b^{\boxed{ab,\mathrm{W}}}(k,k)$$

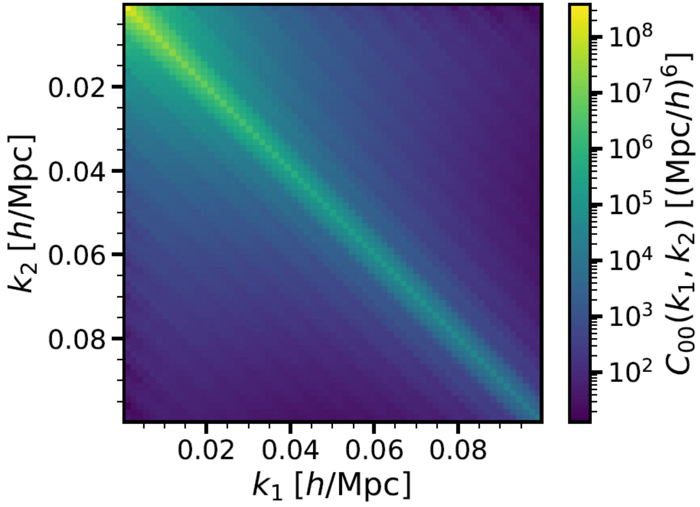
Exact calculation of WA Effects in PSM through SFB

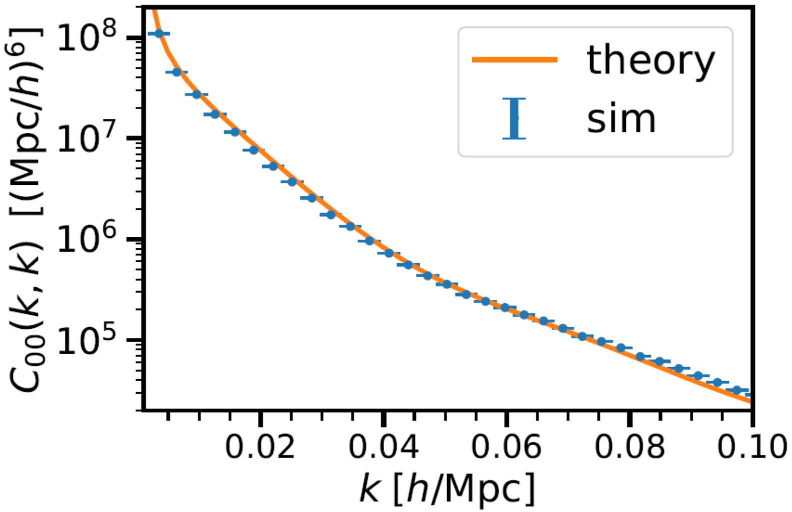
Mock Validation



Theory PSM match mocks at the largest scales

PSM Gaussian Covariance (Exact Window and WA)





full sky (radial window only, z = 0.2 - 0.5)

We now can calculate exact PSM Gaussian covariance without approximation

Mapping Benefits

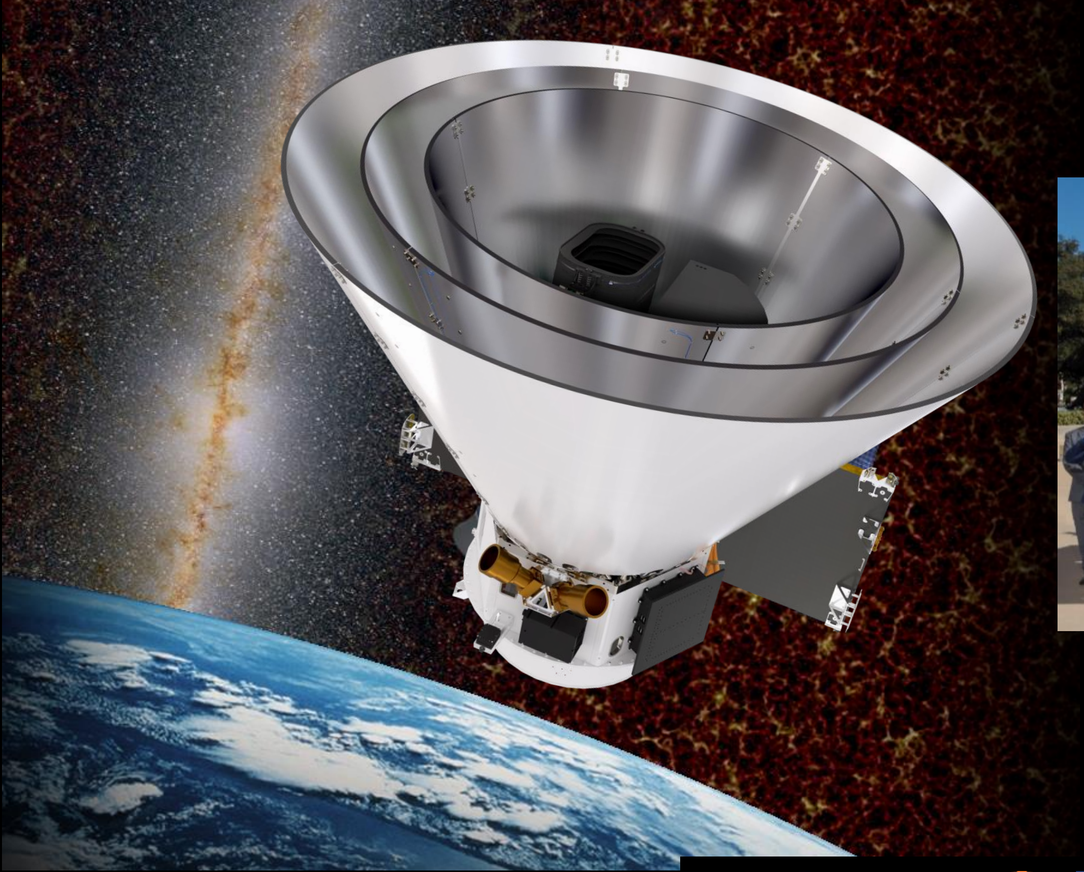
- PSM as natural compression of SFB
- Use the same estimator (Yamamoto) for all scales
- Model integral constraint
  - remove monopole in SFB
- Model GR effects
  - GR effects in SFB PS (will post on arxiv this week)
- Model observer's terms
  - potential in monopole, velocity in dipole
- Model redshift evolution
  - move beyond effective redshift approximation
- Control and remove systematics
  - remove certain angular modes

*A version for Discrete SFB basis exists and developed

Summary

- SFB basis offers angular and radial separation, ideal basis for large-scale analysis of spec-z surveys
- PSM signal can be exactly calculated through SFB basis at large-scale
- Can be extended for Bispectrum multipoles
  WA and GR effects important for surveys aiming at $\sigma(f_{\mathrm{NL}}) \sim 1$
- Ultra-large scale galaxy clustering offers new window for fundamental physics such as inflation, gravity, and dark energy

SPHERE^{x}: An All-Sky Infrared Spectral Survey Satellite



Looking forward to launch in early 2025!

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Backup Slides

PSM Gaussian Covariance

$$\mathbf{C}^{\mathrm{G}}_{L_1L_2}(k_1,k_2) = \frac{(2L_1+1)(2L_2+1)}{I_{22}^2}\left[\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2} \langle F_{L_1}(\mathbf{k}_1)F_0(-\mathbf{k}_1)F_{L_2}(\mathbf{k}_2)F_0(-\mathbf{k}_2)\rangle\right] - \langle \widehat{P}_{L_1}(k_1)\rangle \, \langle \widehat{P}_{L_2}(k_2)\rangle$$

$$\mathbf{C}^{\mathrm{G}}_{L_1L_2}(k_1,k_2) = (4\pi)^4\frac{(2L_1+1)(2L_2+1)}{I_{22}^2}\sum_{a,b,c,d,\ell_1,\ell_2} i^{-a-c+b+d}(2a+1)\begin{pmatrix} a & L_1 & b \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} c & L_2 & d \\ 0 & 0 & 0 \end{pmatrix}^2$$
$$\left[(2c+1)S_{b\ell_1 d\ell_2} + (-1)^{L_2}(2d+1)S_{b\ell_1 c\ell_2}\right] C^{ad,\mathrm{R}}_{\ell_1}(k_1,k_2)C^{bc,\mathrm{R}}_{\ell_2}(k_1,k_2)\,,$$

Radial window only (full sky):

$$\mathbf{C}^{\mathrm{G}}_{00}(k_1,k_2) = \frac{(4\pi)^4}{I_{22}^2}\sum_b 2(2b+1)\left[C^{\mathrm{R}}_b(k_1,k_2)\right]^2$$

Wen, Grasshorn Gebhardt, Heinrich, Doré 2024

Continuous versus Discrete SFB

Continuous

$$\delta_{\ell m}(k) = \int_{\mathbf{x}} j_\ell(kx) Y^*_{\ell m}(\hat{\mathbf{n}}) \delta(\mathbf{x}) \qquad \langle \delta_{\ell_1 m_1}(k_1) \delta^*_{\ell_2 m_2}(k_2) \rangle = C_{\ell_1}(k_1, k_2) \delta^K_{\ell_1 \ell_2} \delta^K_{m_1 m_2}$$

Discrete

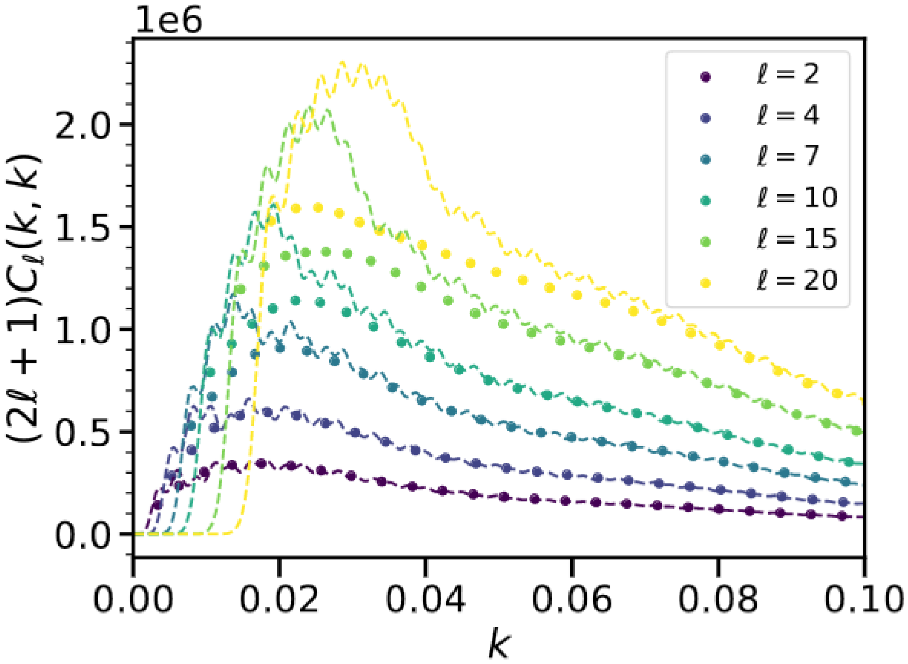
$$\delta_{n\ell m} = \int_{\mathbf{x}} g_{n\ell}(x)\, Y^*_{\ell m}(\hat{\mathbf{n}}) \delta(\mathbf{x}) \qquad \langle \delta_{n_1 \ell_1 m_1} \delta^*_{n_2 \ell_2 m_2} \rangle = C_{\ell_1 n_1 n_2} \delta^K_{\ell_1 \ell_2} \delta^K_{m_1 m_2}$$

$$g_{n\ell}(x) = c_{n\ell}\, j_\ell(k_{n\ell} x) + d_{n\ell}\, y_\ell(k_{n\ell} x)$$

$$\int_{x_{\min}}^{x_{\max}} dx\, x^2\, g_{n\ell}(x)\, g_{n'\ell}(x) = \delta^K_{nn'}$$

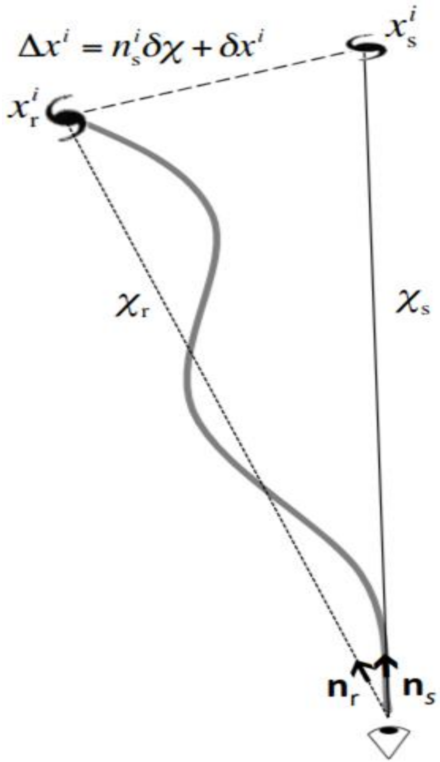
Benefit of Discrete SFB

- Numerical Stability
- Complete decomposition of the finite volume
- Efficient for large scale
- Explicit angular-fourier mode dependence
- Matching the estimator



- GR effects only in continuous basis (Yoo & Desjacques 2014, Semenzato et al. 2024)

Redshift Space Distortion



Observed Galaxy Number Count

$$\delta_g(\hat{\mathbf{n}}, z) = \frac{N_g(\hat{\mathbf{n}}, z) - \langle N_g(\hat{\mathbf{n}}, z)\rangle}{\langle N_g(\hat{\mathbf{n}}, z)\rangle}$$

Real space versus Redshift space

Standard/Newtonian RSD (Redshift Space Distortion)

$$\mathbf{s} = \mathbf{x} + \hat{\mathbf{x}}\,(\mathbf{v}_g\,.\,\hat{\mathbf{x}})/\mathcal{H}$$

$$\delta_g^{\mathrm{Newt}}(\hat{\mathbf{n}}, z) = b_1 D_{\mathrm{m}} - \frac{1}{\mathcal{H}}\frac{\partial \mathbf{v}}{\partial x} \cdot \hat{\mathbf{n}}$$

General Relativistic (GR) Effects

$$\delta_{\rm g}^{\rm rel}(\hat{\mathbf{n}}, z) = b_1 D_{\rm m} - \frac{1}{\mathcal{H}}\frac{\partial \mathbf{v}}{\partial x}\cdot\hat{\mathbf{n}}$$

DRSD

Lensing

$$- (2-5s)\kappa$$

Doppler

$$- \mathcal{A}_1(\mathbf{v}-\mathbf{v}_o)\cdot\hat{\mathbf{n}} + (2-5s)\mathbf{v}_o\cdot\hat{\mathbf{n}}$$

$$+ \mathcal{A}_1(\Psi-\Psi_o) + \left(\mathcal{A}_1\mathcal{H}_0 - \frac{2-5s}{x}\right)V_o - (2-5s)\Phi + \Psi + \frac{1}{\mathcal{H}}\dot{\Phi} + (b_{\rm e}-3)\mathcal{H}V$$

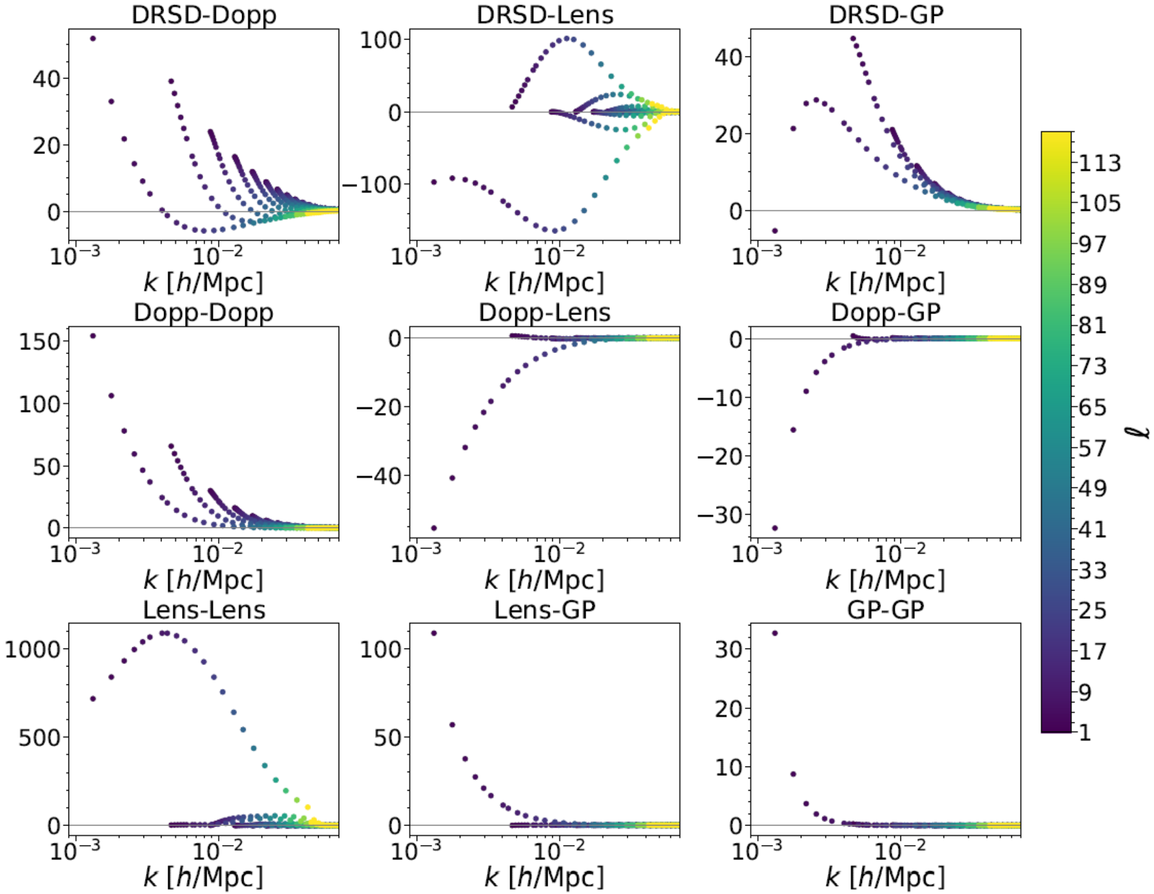
GP

$$- \frac{2-5s}{x}\int_{\tau_0}^{\tau(z)}(\Psi(\tau') + \Phi(\tau'))d\tau'$$

$$- \mathcal{A}_1\int_{\tau_0}^{\tau(z)}(\dot{\Psi}(\tau') + \dot{\Phi}(\tau'))d\tau'.$$

$$\kappa(\hat{n}, z) = \frac{1}{2}\nabla^2_{\hat{\mathbf{n}}'}\psi^{\rm lens} = -\frac{1}{2}\nabla^2_{\hat{\mathbf{n}}}\int_{\tau_0}^{\tau(z)}\frac{\tau' - \tau(z)}{(\tau_0-\tau(z))(\tau_0-\tau')}(\Phi(\tau') + \Psi(\tau'))\,d\tau'$$

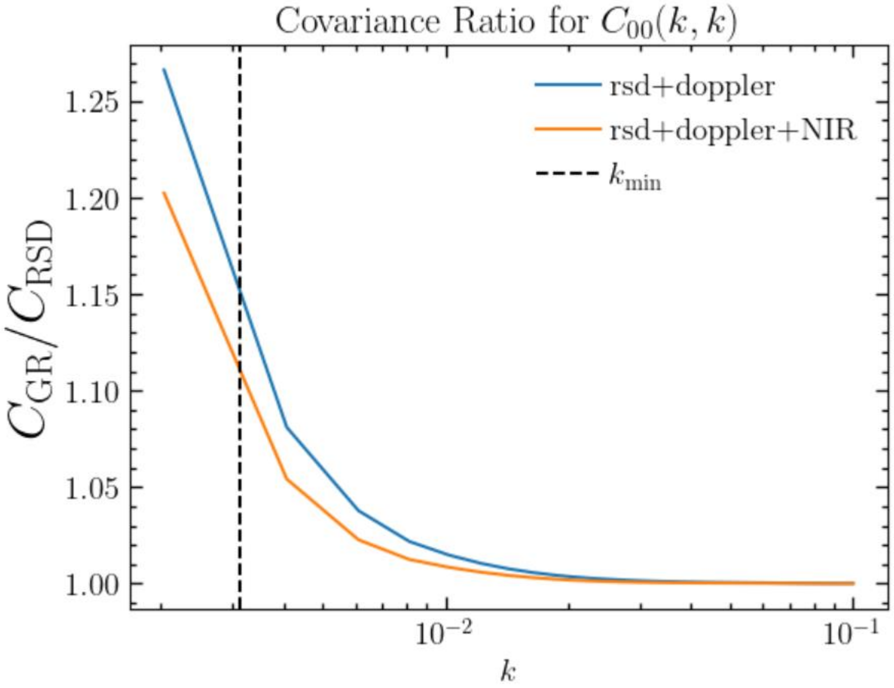
GR Effects in discrete SFB PS



$$z = 1.0 \text{ to } 1.5$$

GR effects only in continuous basis before (Yoo & Desjacques 2014, Semenzato et al. 2024)

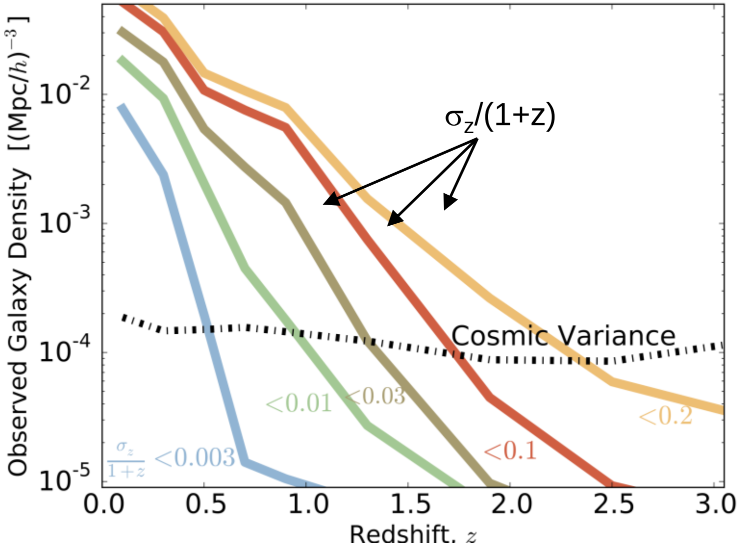
GR Effects in PSM Gaussian Covariance



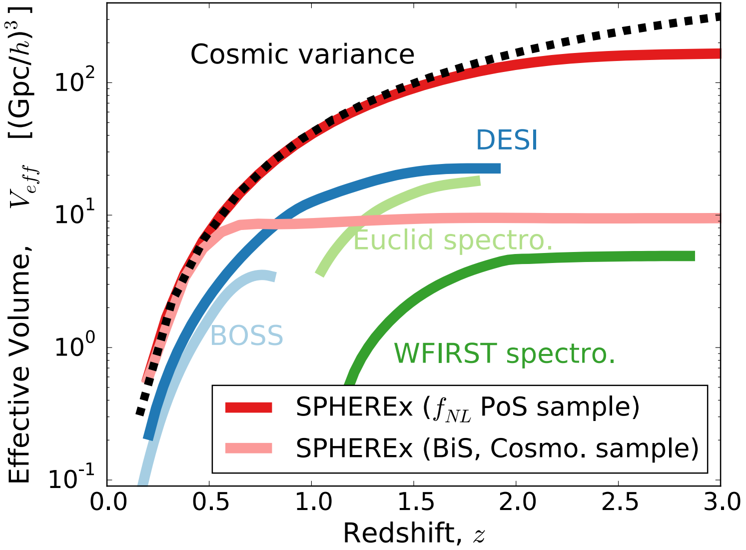
full sky (radial window only, z = 0.2 - 0.5)

SPHERE^x Galaxy Redshift Survey

Catalog Split into Redshift Accuracy Bins



SPHERE^x Surveys Maximum Cosmic Volume

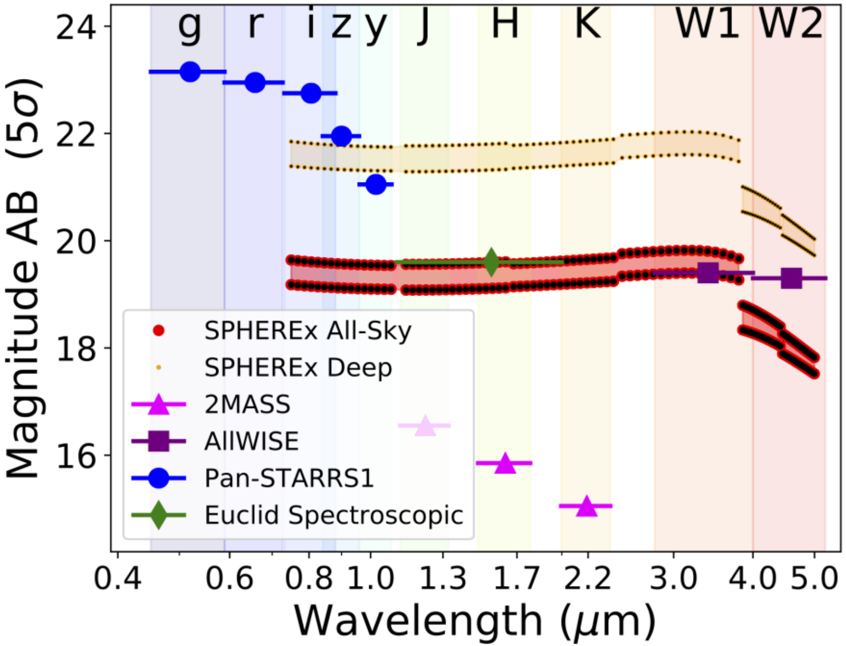


Survey Designed for local PNG

- Probe large spatial modes: wide redshift range, full sky, IR wavelengths, stable
- Large scale power from PS: large number of low-accuracy redshifts
- Modulation of fine-scale power from BS: fewer high-accuracy redshifts

What Can YOU Do With the all-sky survey?

SPHERE^x Point Source Sensitivity



Data are rapidly released to the public

- Calibrated spectral images within 2 months of observation, updated following 2nd and 4th survey recalibration
- High-reliability catalog after 3rd survey
- Core science products at end of mission

Users have access to data exploration, analysis, and visualization tools

- On-the-Fly Mosaics
- Photometry on Known Position
- Spectral Data Cube Extractor
- Variable Source Extractor
- Source Discovery

REDSHIFTS FROM LOW-RESOLUTION SPECTROSCOPY

We extract the spectra from *known* galaxy positions

Controls blending and confusion

We compare each spectrum to a template library:

For each galaxy: redshift, type and redshift error

Many self-consistency tests using SPHEREx data, spectral models, and external redshift catalogs

Detected galaxies	**> 1 billion**
Galaxies Δz/1+z < 10 %	**> 450 million**
Galaxies Δz/1+z < 0.3%	**> 10 million**