Projection effects in likelihood analyses of galaxy clustering

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Outline

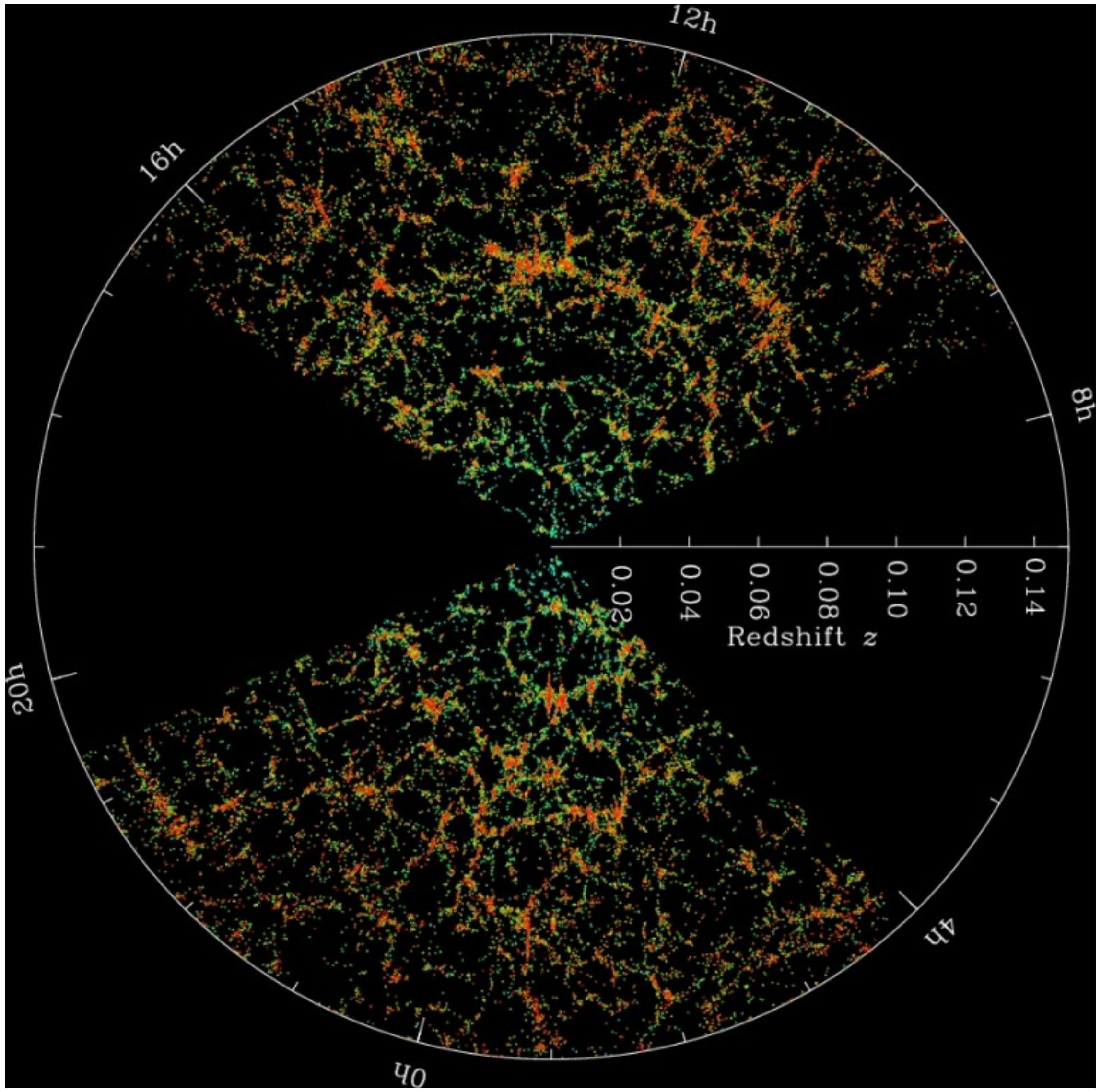
- Cosmology from galaxy clustering
- State-of-art models for galaxy clustering
- Faster model evaluation with evolution mapping and extension to massive neutrinos
- Prior volume effects in cosmological parameter inference
- Conclusions

Galaxy clustering

Why galaxy clustering?

One of the two main cosmological probes to characterise the Large Scale Structure

Galaxy clustering



(SDSS)

Weak gravitational lensing

(HST)

Why galaxy clustering?

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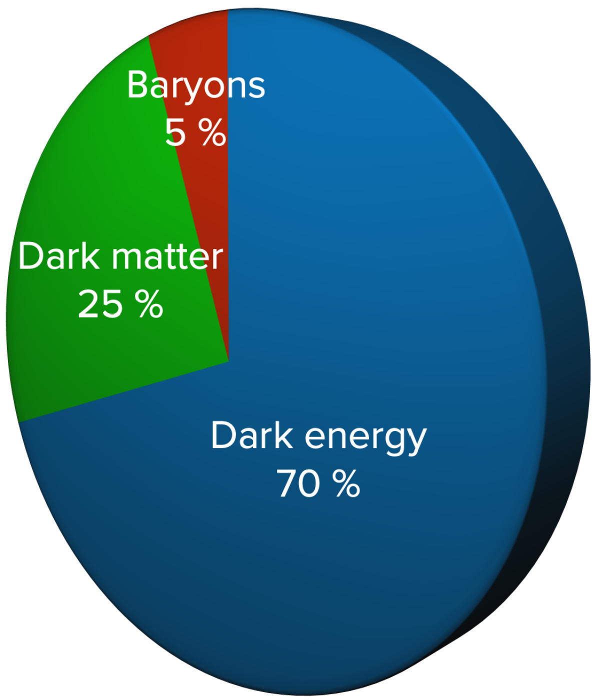
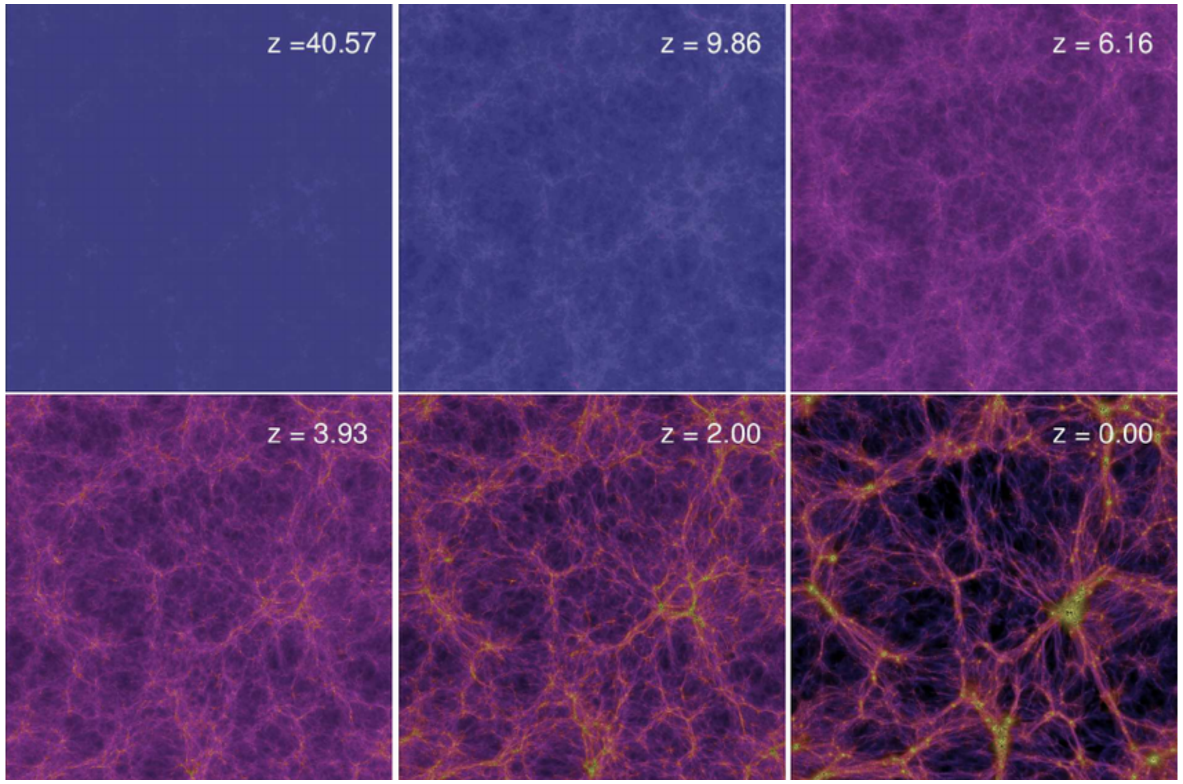
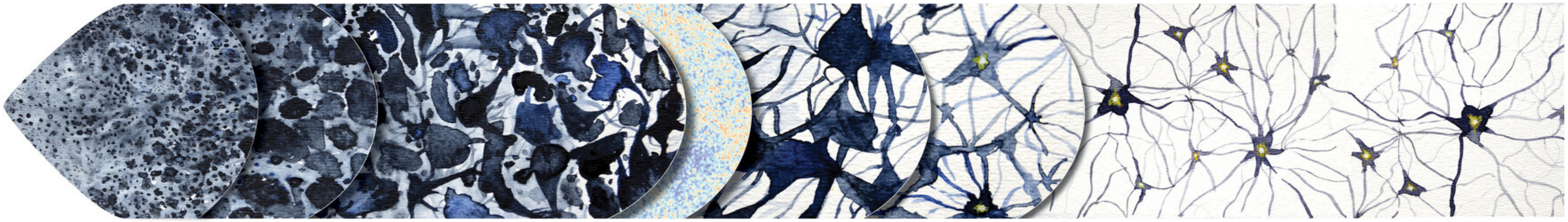
Energy/matter components



Growth of structures



Expansion history



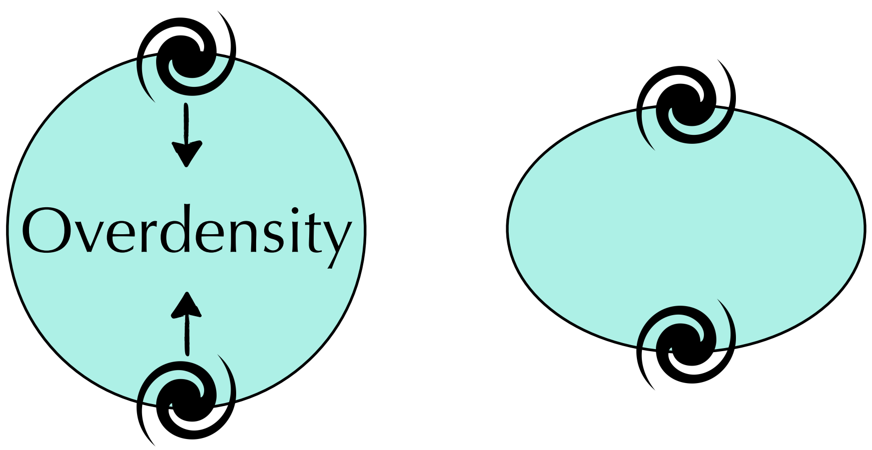
Redshift-space distortions

Peculiar velocities distort the observed distance inferred from the redshift of luminous sources

$$\vec{s} = \vec{r} + \frac{v_{\parallel}}{a\,H(a)}\hat{e}_r$$

Traditionally split into 2 different regimes (but cannot be treated as independent)

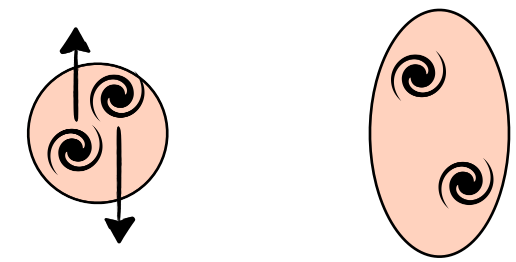
Linear regime ($\gg 1\,h^{-1}\,\mathrm{Mpc}$)



Kaiser effect

Flattening of large overdense regions due to infall towards overdose regions

Nonlinear regime ($\sim 1\,h^{-1}\,\mathrm{Mpc}$)



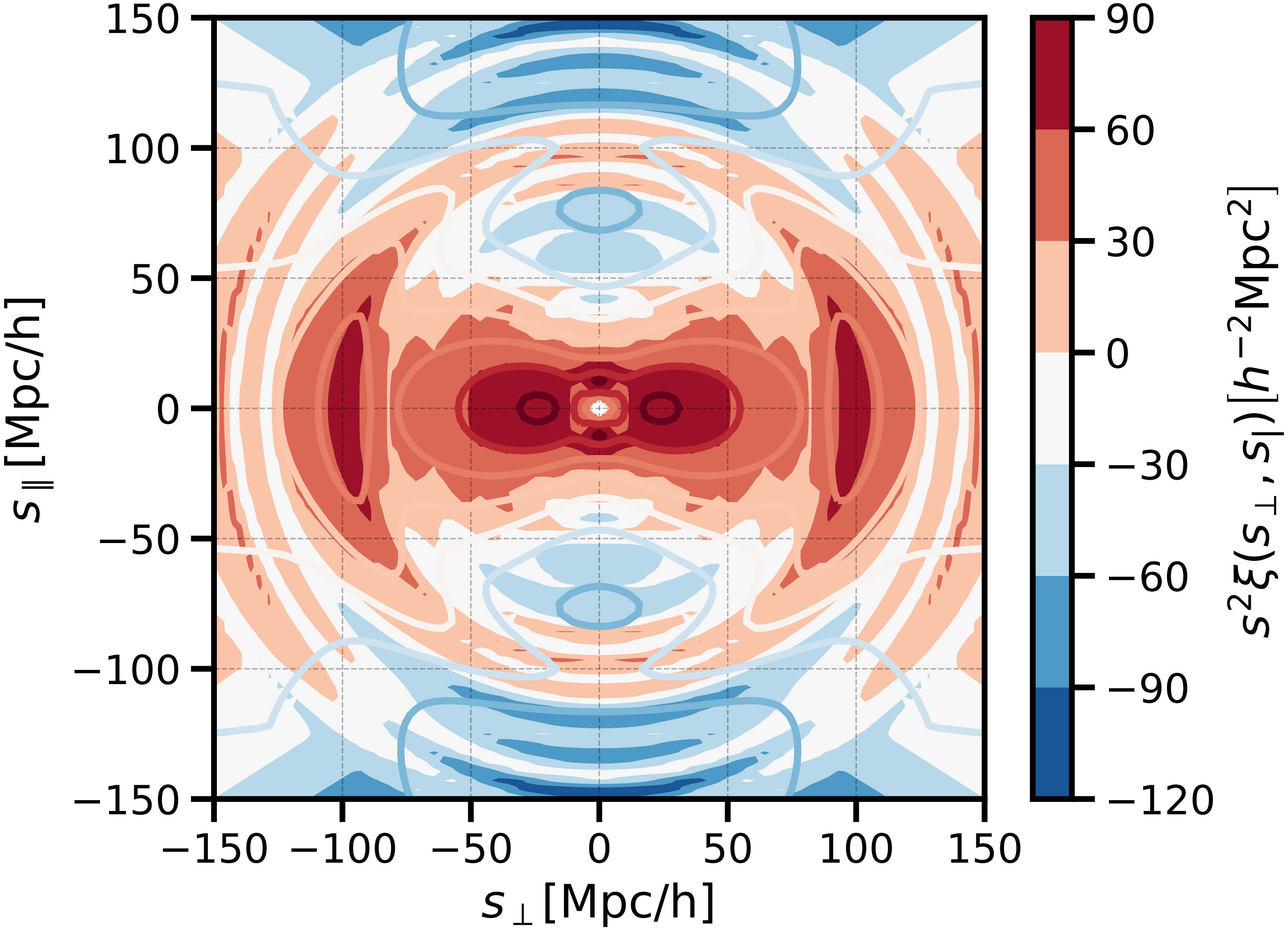
Fingers of God

Stretching of virialised structures along the line-of-sight due to viral velocities

Clustering observables

Anisotropy observable from clustering measurements (e.g. 2PCF, galaxy power spectrum)

eBOSS LRGs (Hou et al. 2020)



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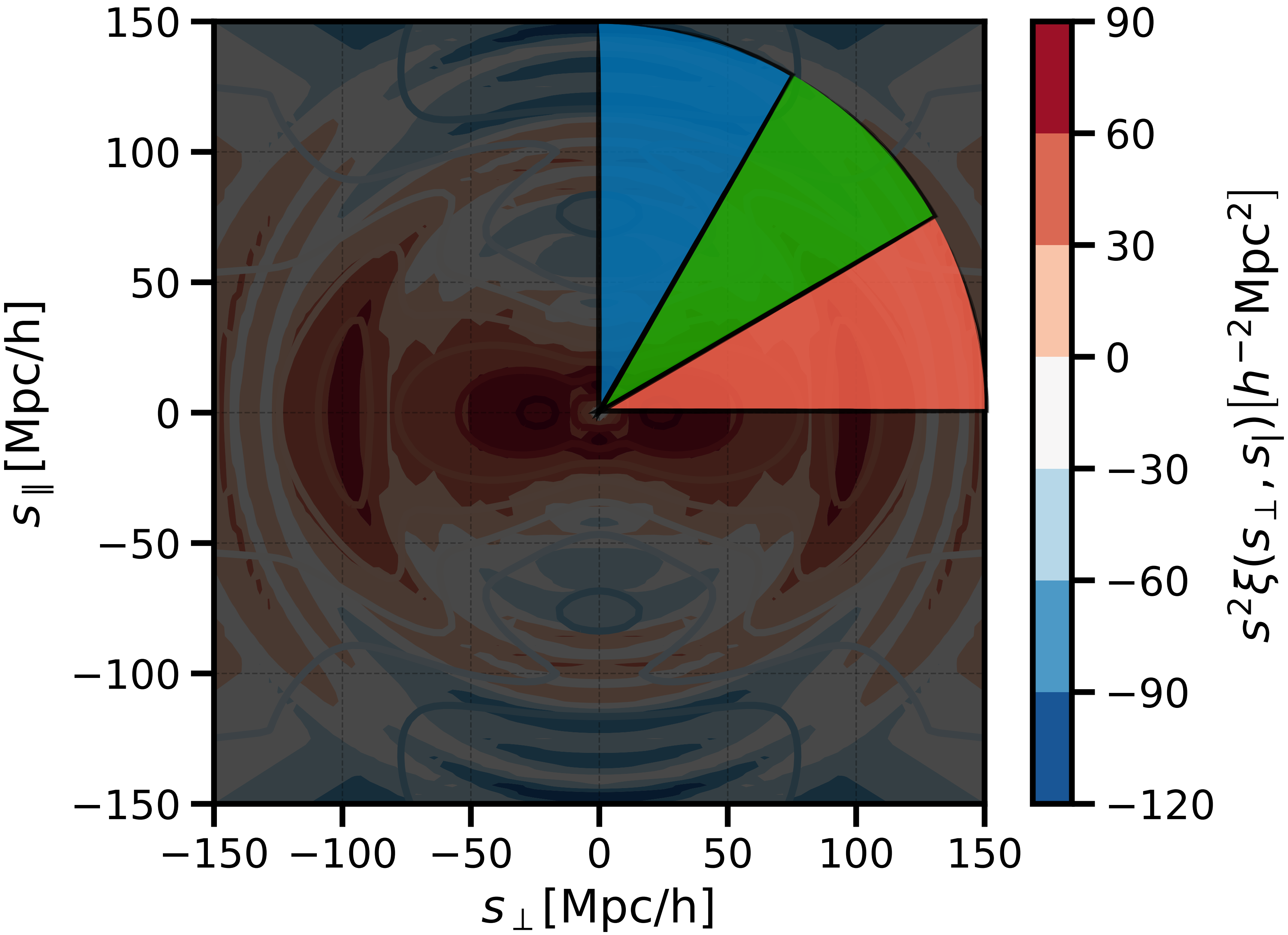
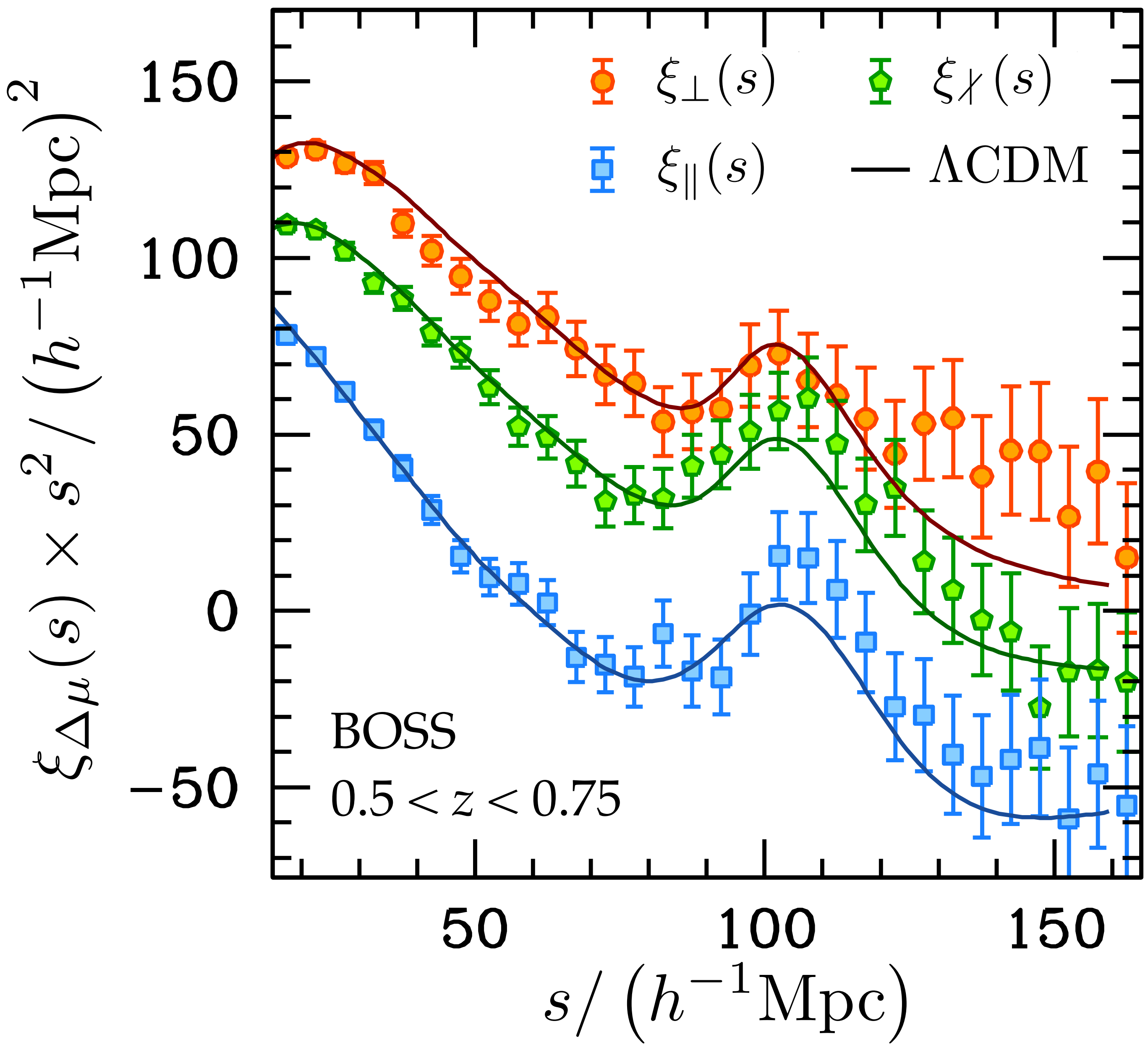


Clustering observables

Standard analyses focus on the projection of anisotropic clustering measurements

(Sánchez et al. 2017a)



- Clustering wedges

$$\xi_{\Delta\mu}(s) = \frac{1}{\Delta\mu}\int_{\Delta\mu} \xi(s,\mu)\,\mathrm{d}\mu$$

- Legendre multipoles

$$\xi_\ell(s) = \frac{2\ell+1}{2}\int_{-1}^{+1} \xi(s,\mu)\,\mathscr{L}_\ell(\mu)\,\mathrm{d}\mu$$

Perturbation theory

Perturbative approaches can be employed to describe clustering measurements

$$\delta_{\rm g} = b_1\,\delta + b_{\nabla^2\delta}\nabla^2\delta + \varepsilon + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\left[\left(\partial_i\,\partial_j\,\Phi\right)^2 - \left(\nabla^2\Phi\right)^2\right] + \ldots$$

- **Nonlinear evolution** of the matter density field
- **Galaxy bias** (relationship between galaxy and matter density field)
- **Redshift-space distortions**

Employed for the analysis of Stage-III data (e.g. BOSS)

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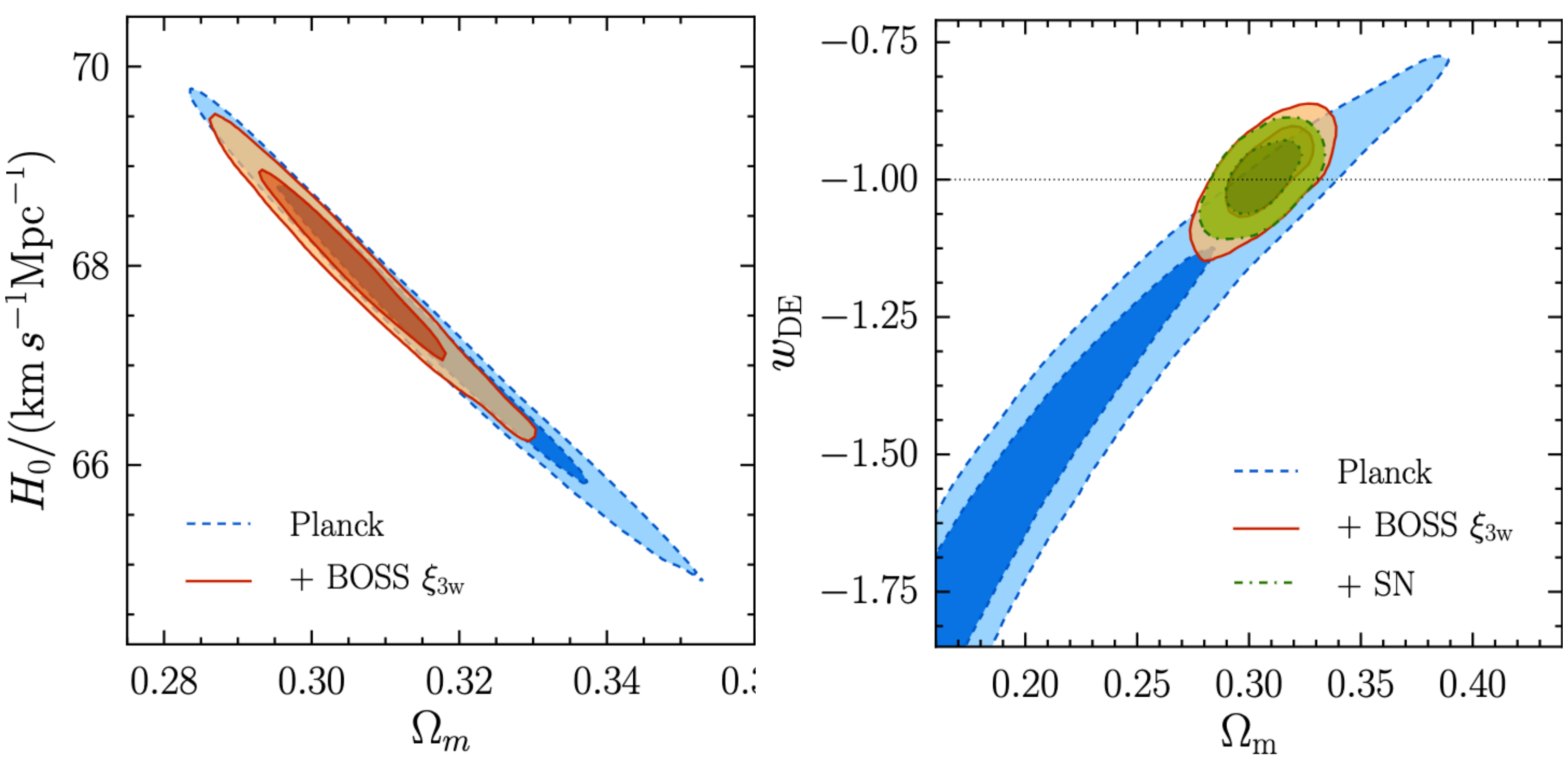

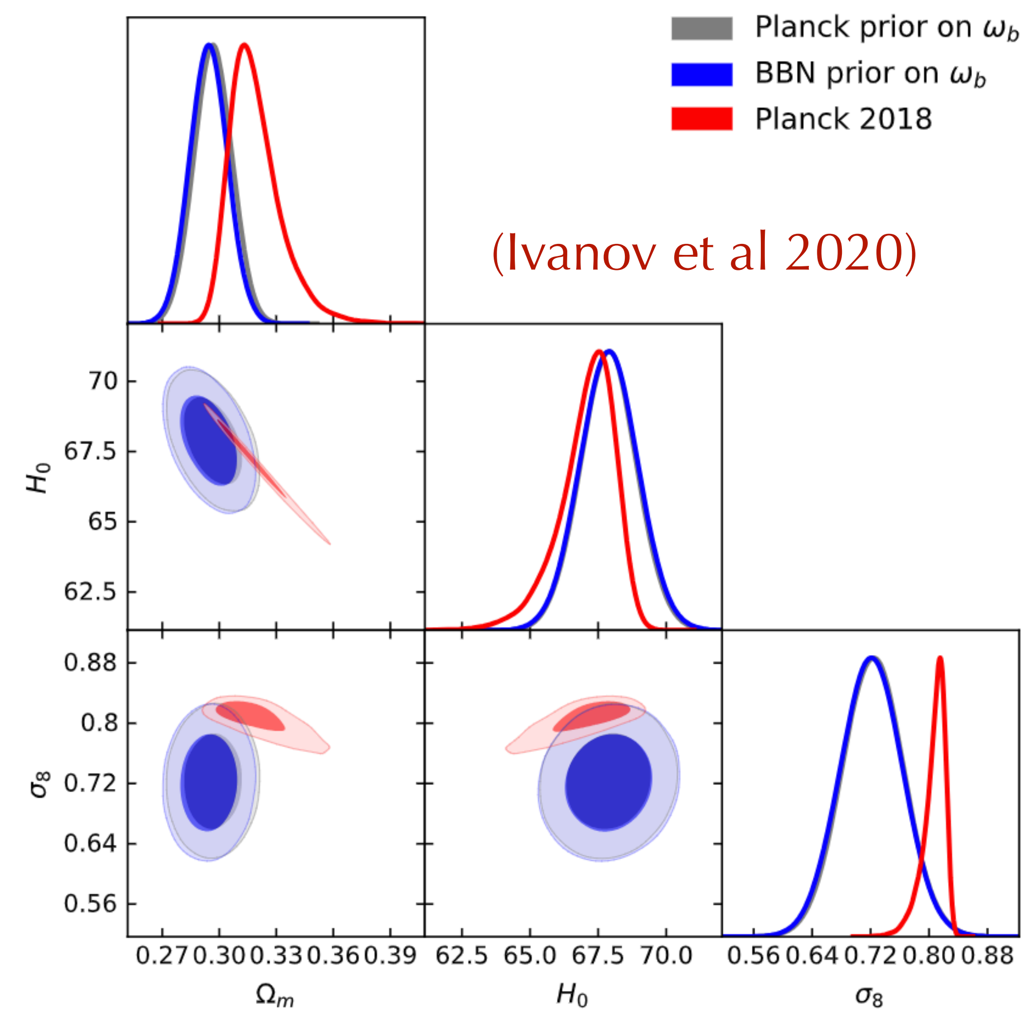

(Ivanov et al 2020)

Perturbation theory

State-of-art modelling of redshift-space galaxy power spectrum: **EFTofLSS**

$$P^{s}_{\rm gg}(k,\mu) = P^{s,\,\rm tree}_{\rm gg}(k,\mu) + P^{s,\,\rm 1\text{-}loop}_{\rm gg}(k,\mu) + P^{s,\,\rm noise}_{\rm gg}(k,\mu) + P^{s,\,\rm ctr}_{\rm gg}(k,\mu)$$

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Tree level
(leading-order)
$\{\, b_1 \,\}$

Loops
(nonlinear corrections)
$\{\, b_2\,,\, b_{\mathcal{G}_2}\,,\, b_{\Gamma_3} \,\}$

Shot-noise
(high-frequency modes)
$\{\, \alpha_P\,,\, \epsilon_0\,,\, \epsilon_2 \,\}$

Counterterms
(FOGs, failure of SPT)
$\{\, c_0\,,\, c_2\,,\, c_4\,,\, c_{\rm nlo} \,\}$

Fully perturbative model including:

- **Local** and **non-local bias**
- **Shot-noise** corrections (constant and scale-dependent)
- **Counterterms** (from expanding pairwise velocity difference, i.e. FOGs; integration of UV modes, higher-derivatives, velocity bias)
- **Resummation of infrared modes** (need recipe to obtain the no-wiggle $P(k)$)
- 11 nuisance parameters, some of them highly degenerate in terms of $P^s_{\rm gg}(k,\mu)$

Speeding up model evaluation

Using PT: is it worth?

IT IS ACCURATE!

- Optimal performances over mildly nonlinear scales
- Flexible enough to accommodate different models
- Extensively employed to derive cosmological constraints from GC data

IT CAN ALSO BE SLOW…

- Standard bayesian inference can request up to $\mathcal{O}(10^6)$ likelihood evaluations
- Evaluation time for a single model can be relevant $\mathcal{O}(\mathrm{s})$
- Even more expensive when considering higher-order statistics
- Additional time lost to account for observational systematics

One solution (out of fews): we can make use of **EMULATORS**

Evolution Mapping (Sánchez et al. 2021)

Based on the split of cosmological parameters into **shape** and **evolution**

$\mathbf{\Theta}_\mathrm{s} = \{\omega_\mathrm{c}, \omega_\mathrm{b}, \omega_\gamma, n_\mathrm{s}, \ldots\}$

Shape of matter **transfer function** and **primordial power spectrum**

$\mathbf{\Theta}_\mathrm{e} = \{H_0, \omega_\mathrm{K}, \omega_\mathrm{DE}, w(a), A_\mathrm{s}, \ldots\}$

Overall **amplitude of power spectrum** at any redshift z

New parameter describing the **degeneracy among evolution parameters** (and redshift):

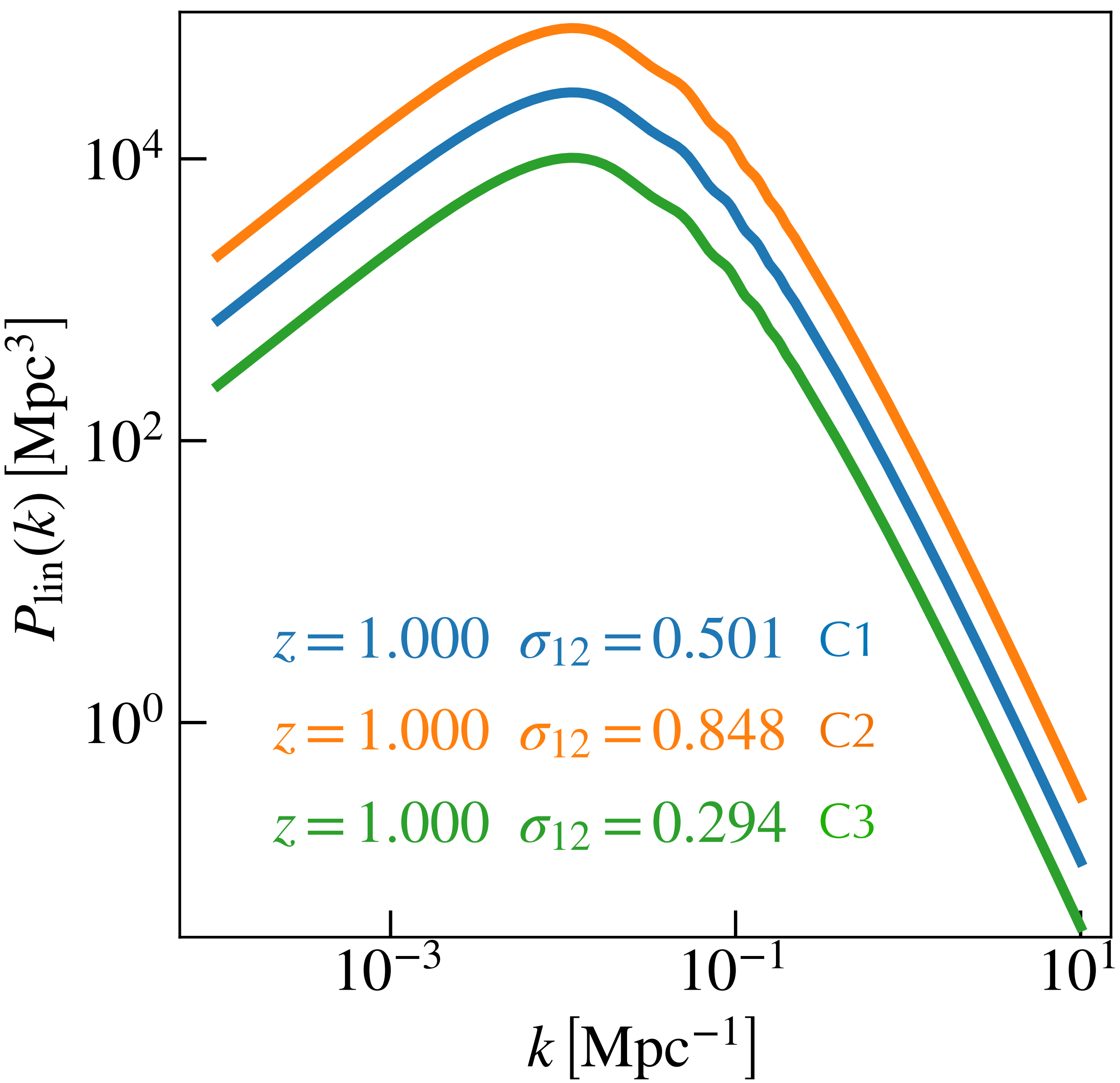
$$\sigma_{12} = \int \mathrm{d}k\, k^2\, P(k)\, W^2_\mathrm{TH}(kR) \Big|_{R=12\mathrm{Mpc}}$$

Same power spectrum when considering models with same shape and σ_{12}

$$P_\mathrm{lin}(k\,|\,\mathbf{\Theta}_\mathrm{s}, \mathbf{\Theta}_\mathrm{e}, z) = P_\mathrm{lin}(k\,|\,\mathbf{\Theta}_\mathrm{s}, \sigma_{12}(\mathbf{\Theta}_\mathrm{s}, \mathbf{\Theta}_\mathrm{e}, z))$$

Evolution Mapping (Sánchez et al. 2021)

Let's see it directly with an example: three cosmologies with same $\boldsymbol{\Theta}_s$ but different $\boldsymbol{\Theta}_e$



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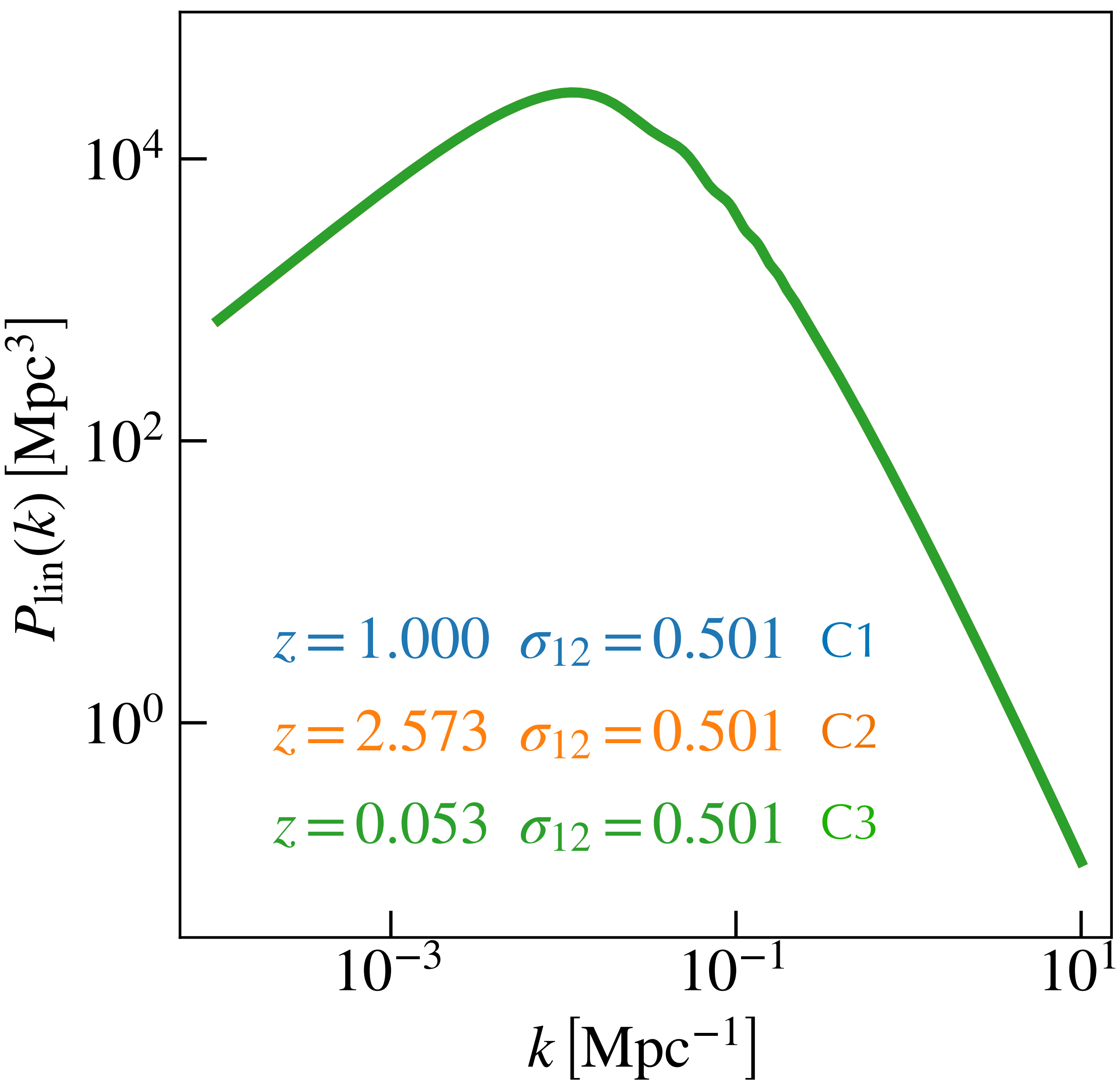

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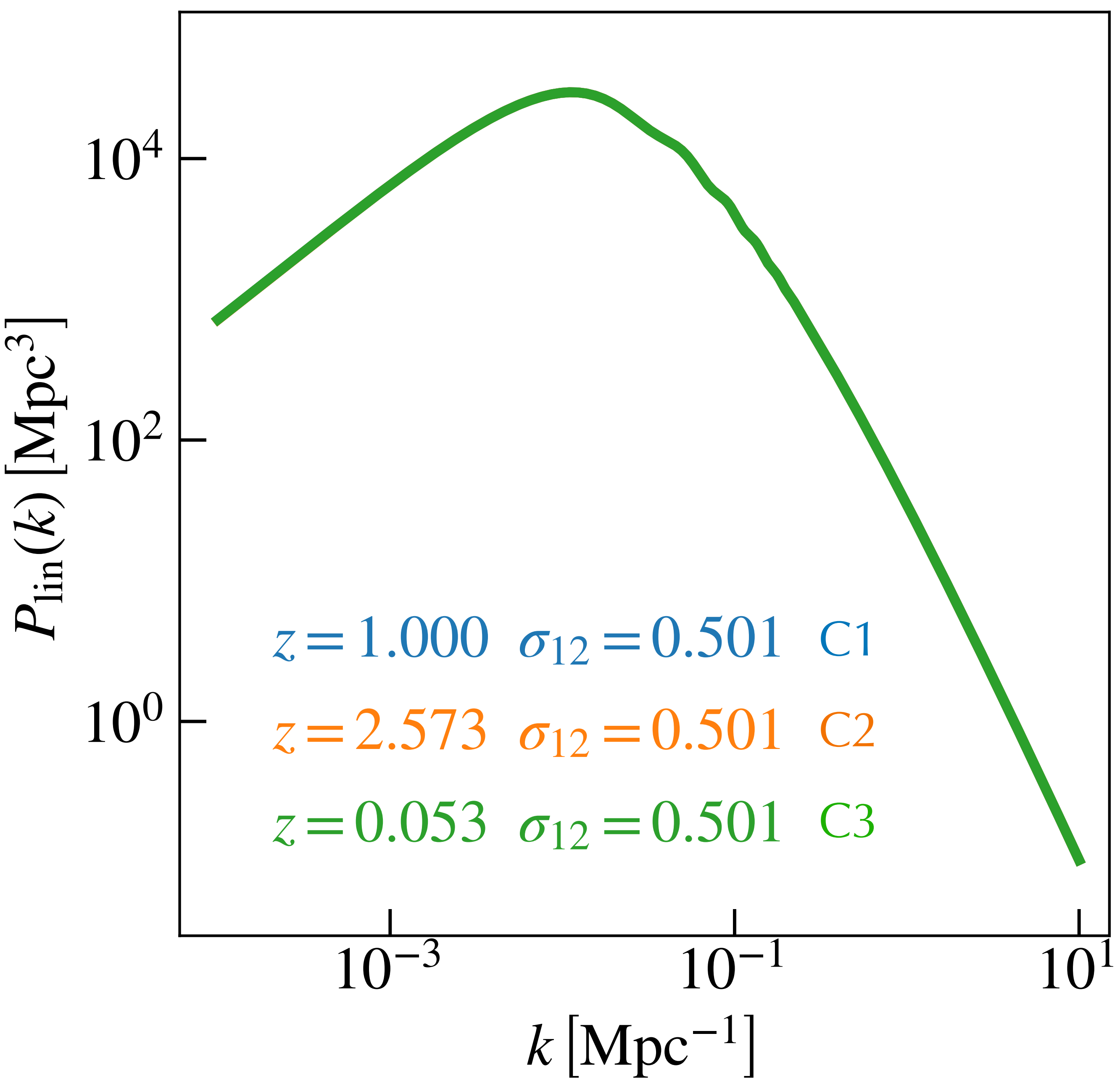
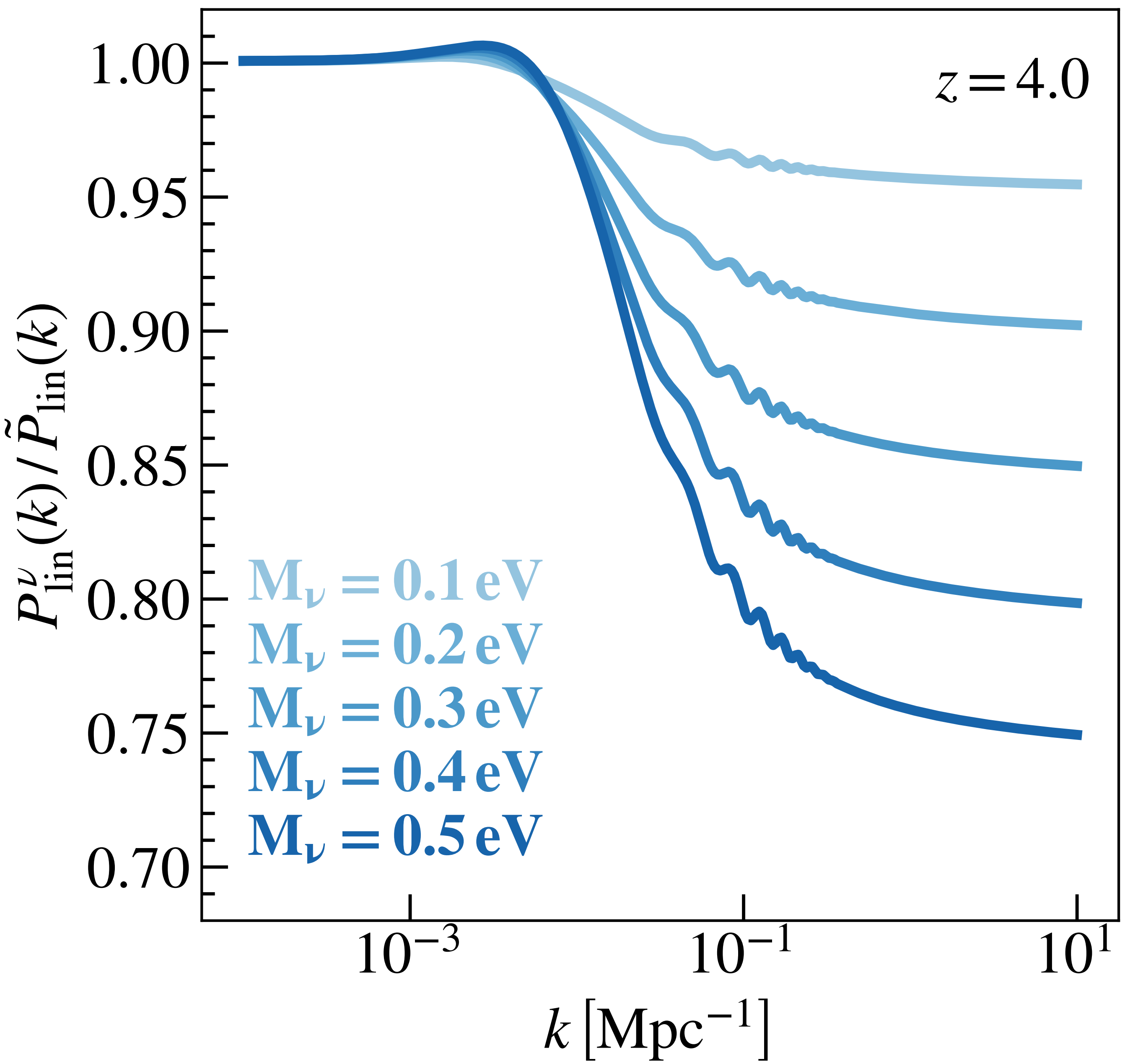


- Mapping is propagated to the non-linear regime (need to correct for different history of structure formation, though)
- Applies to other statistics, like velocity power spectra (Esposito et al. 2024), higher-order statistics, halo mass functions, etc.
- Use of Mpc/h units to express distances breaks the degeneracy

Problems with massive neutrinos

Massive neutrinos break the original formulation of evolution mapping



Redshift evolution induces a modification in the shape of the matter power spectrum

↓

Time variables (z, a) can no longer be described in terms of σ_{12}

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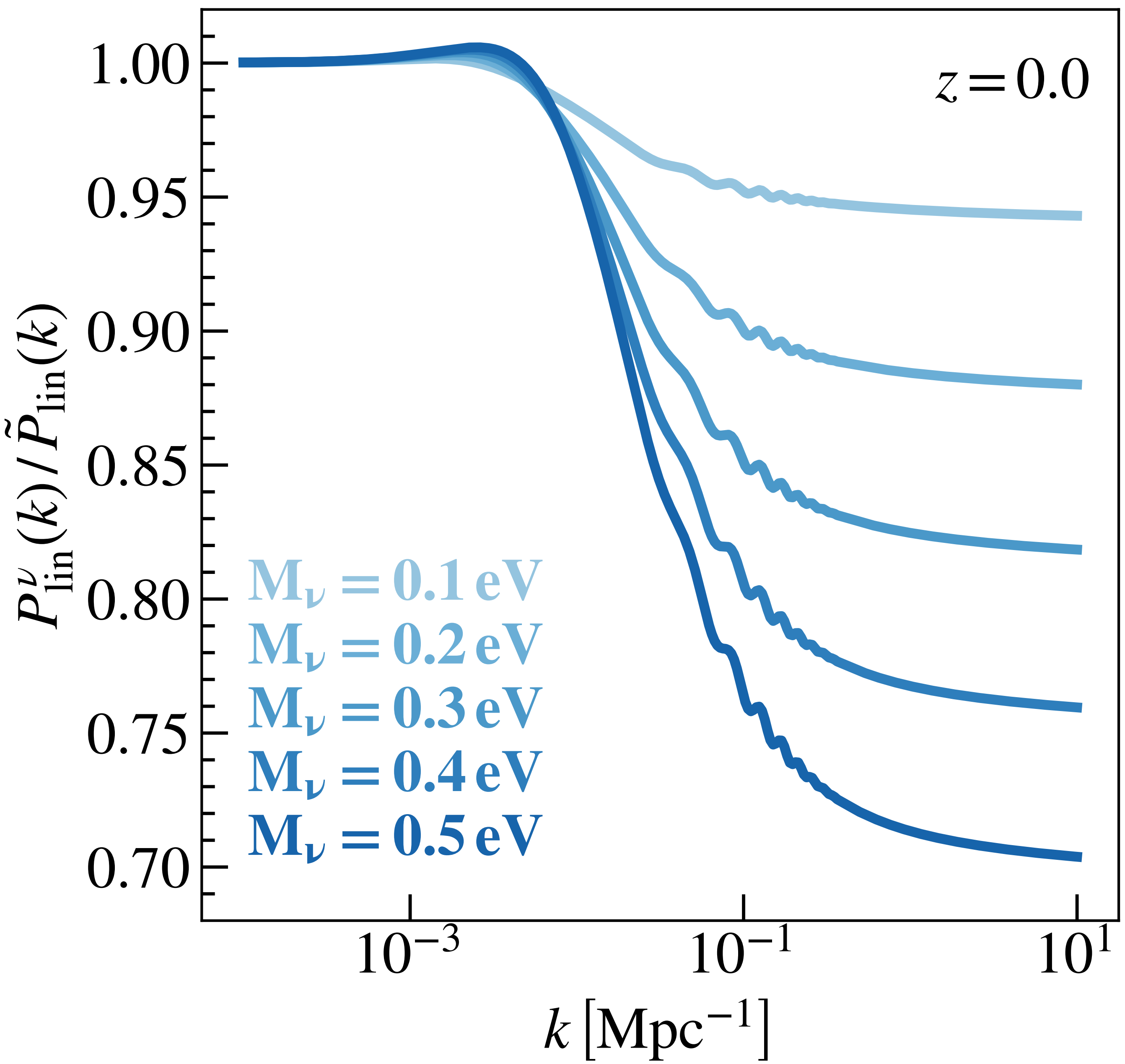

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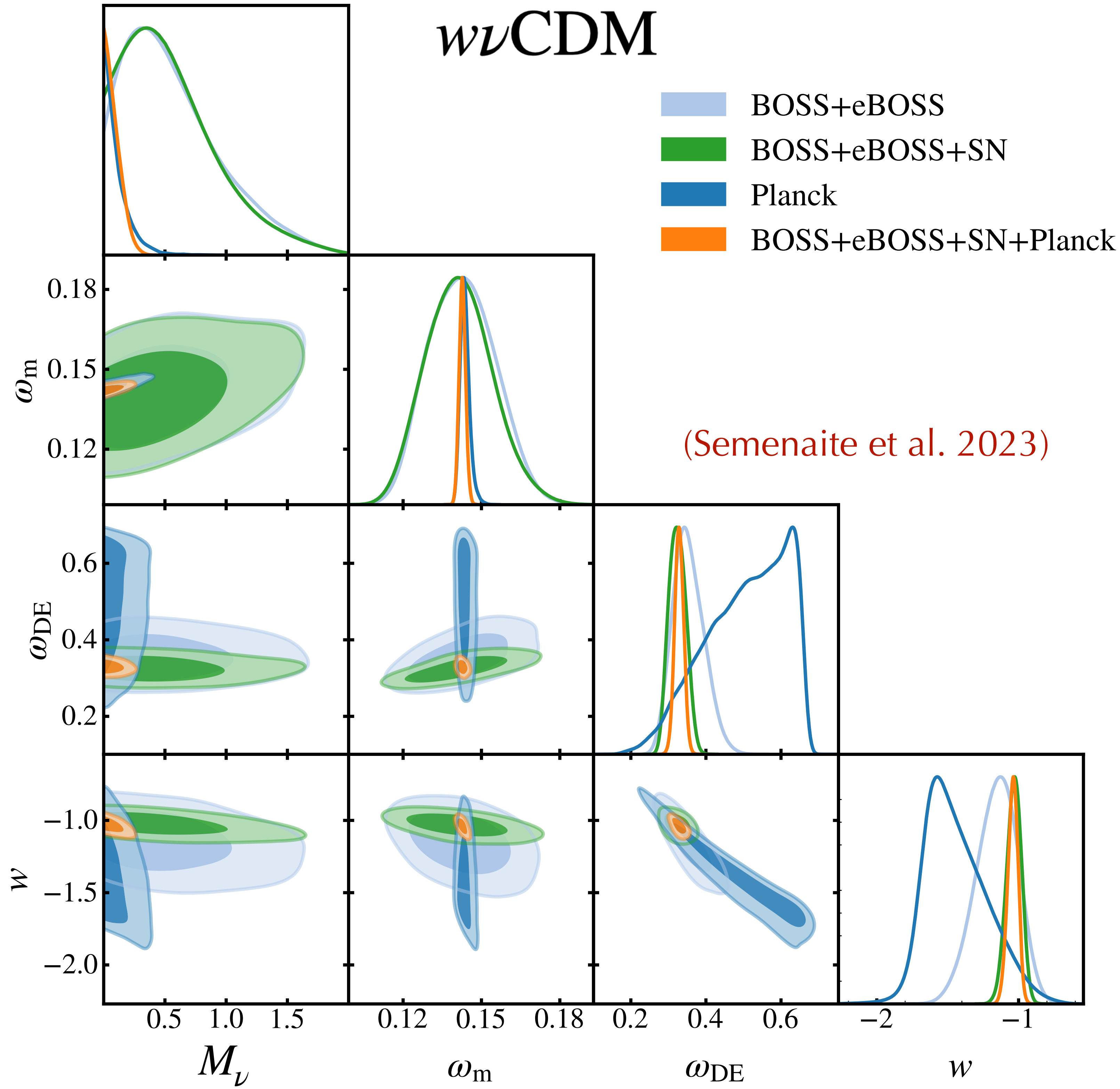
Problems with massive neutrinos

Relevant for Stage-III experiments, since constraining power on M_ν from GC-only data is loose



(Semenaite et al. 2023)

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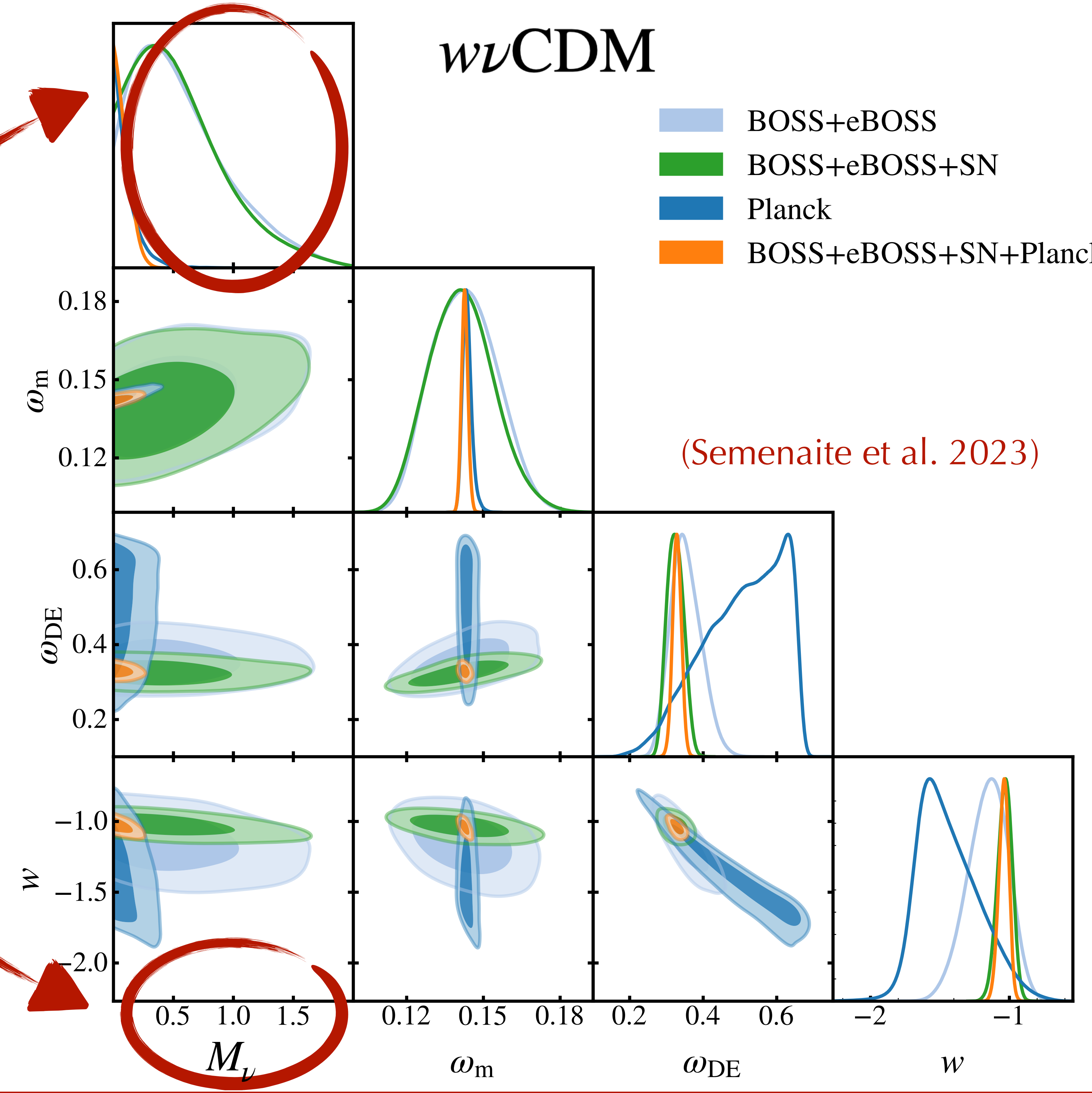

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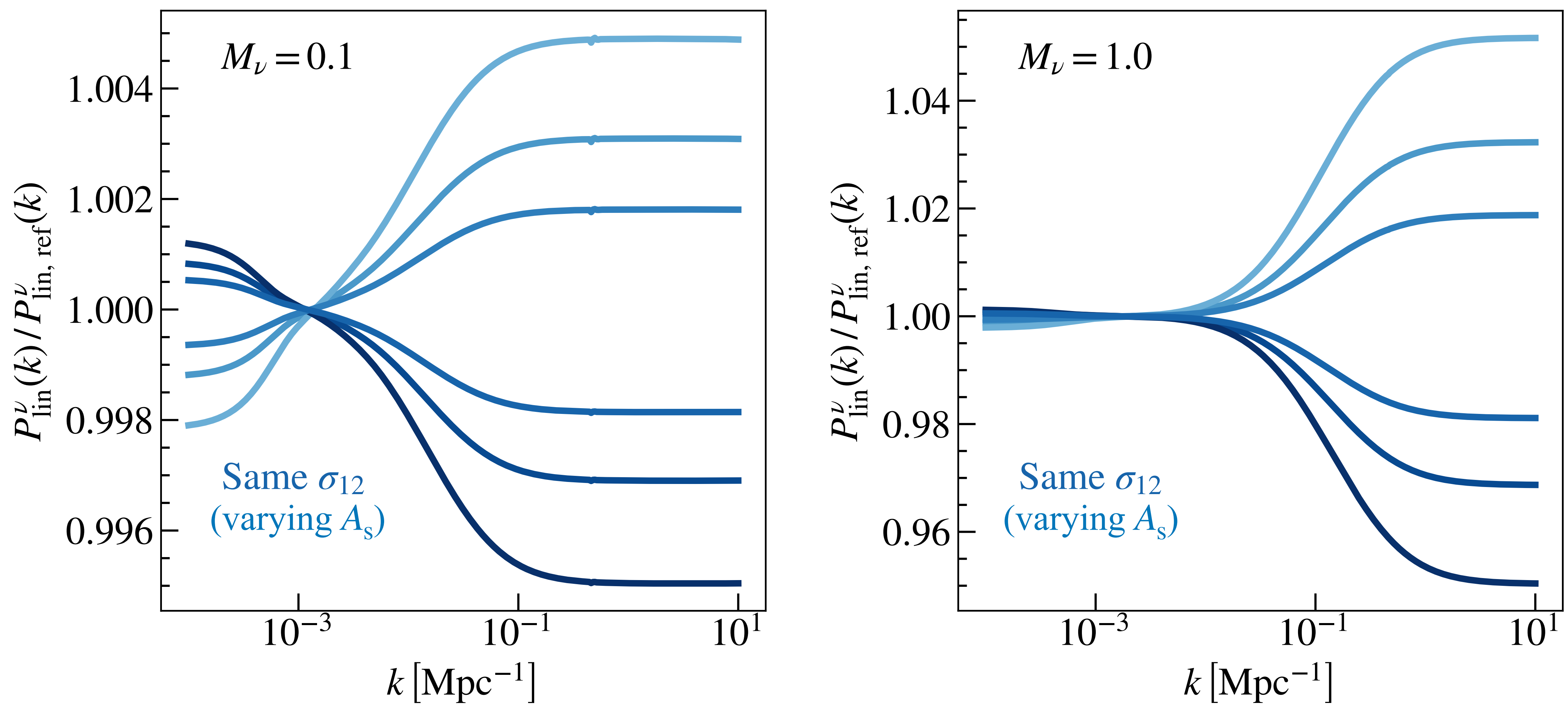
Problems with massive neutrinos

Large-scale amplitude is (partially) recovered only matching σ_{12} of the neutrino-free cosmology ($\omega_c \to \omega_c + \omega_\nu$, $\omega_\nu = 0$), but small-scale differences are important!



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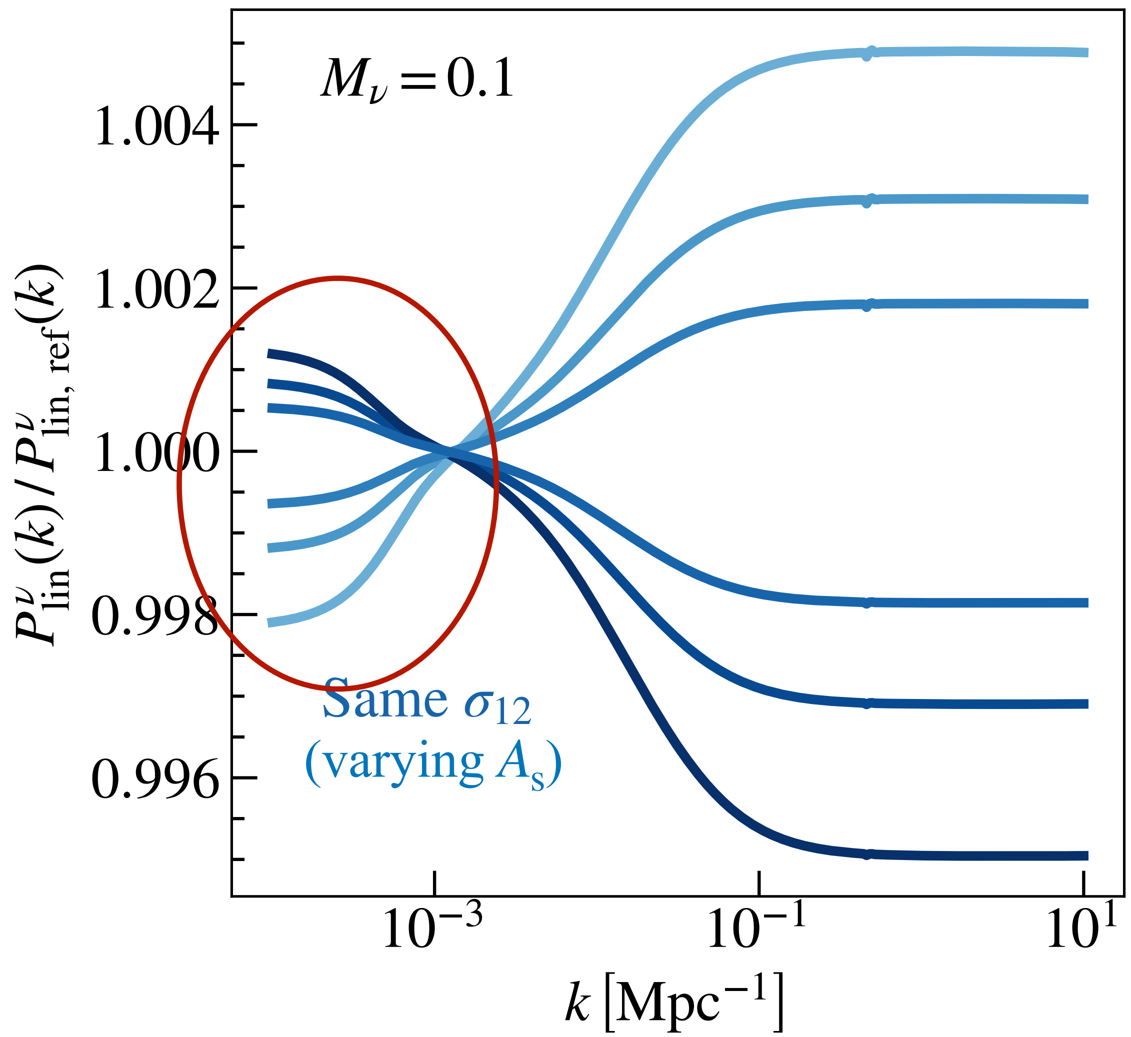

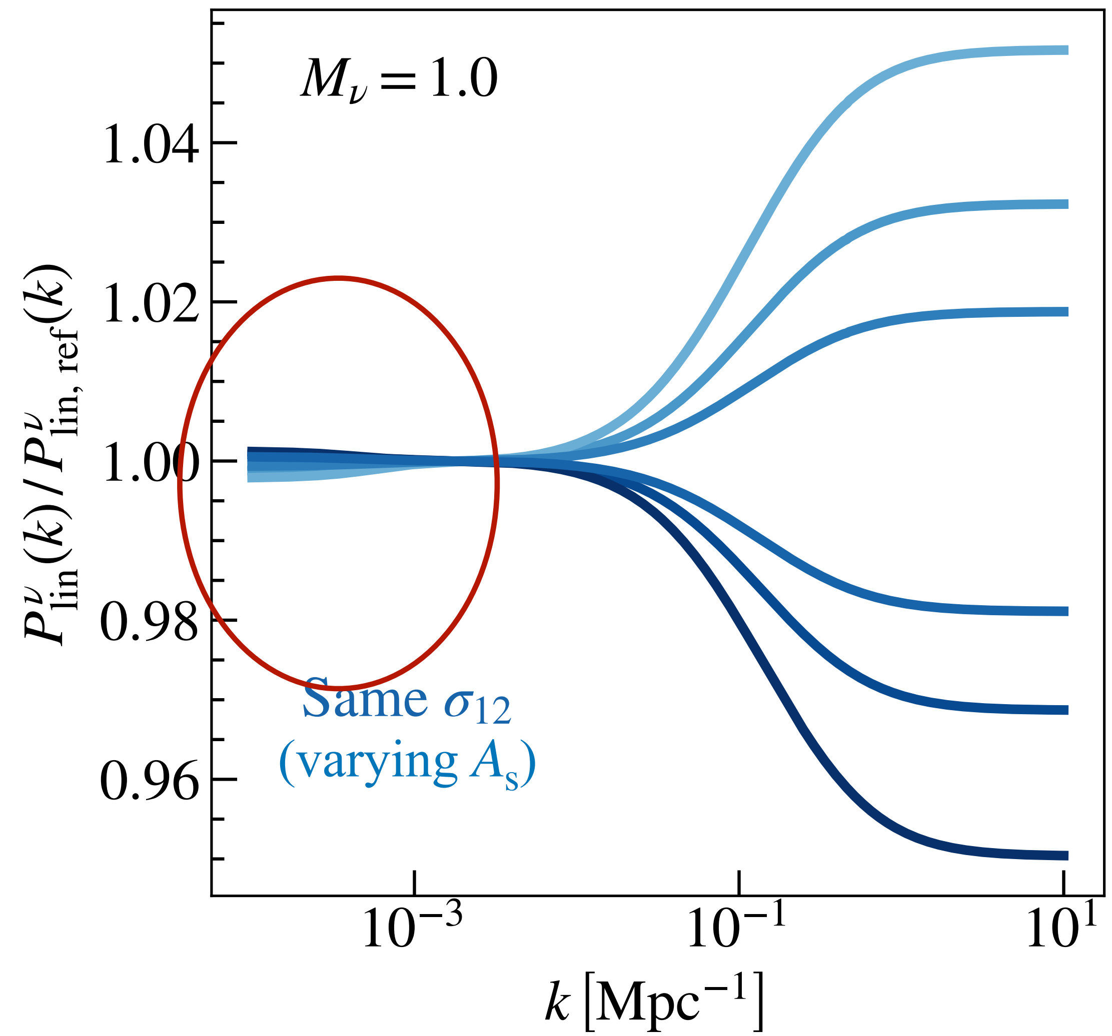

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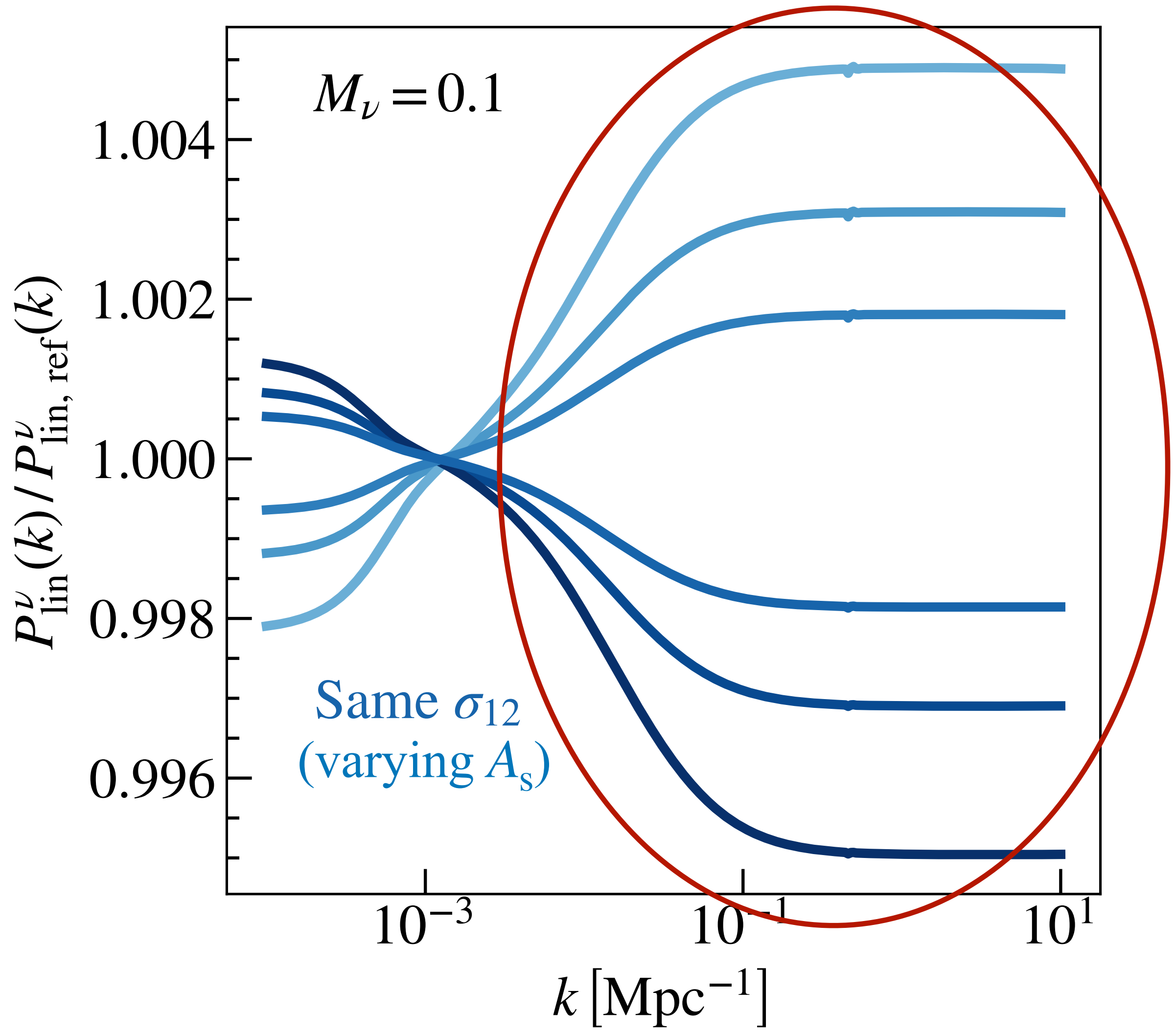

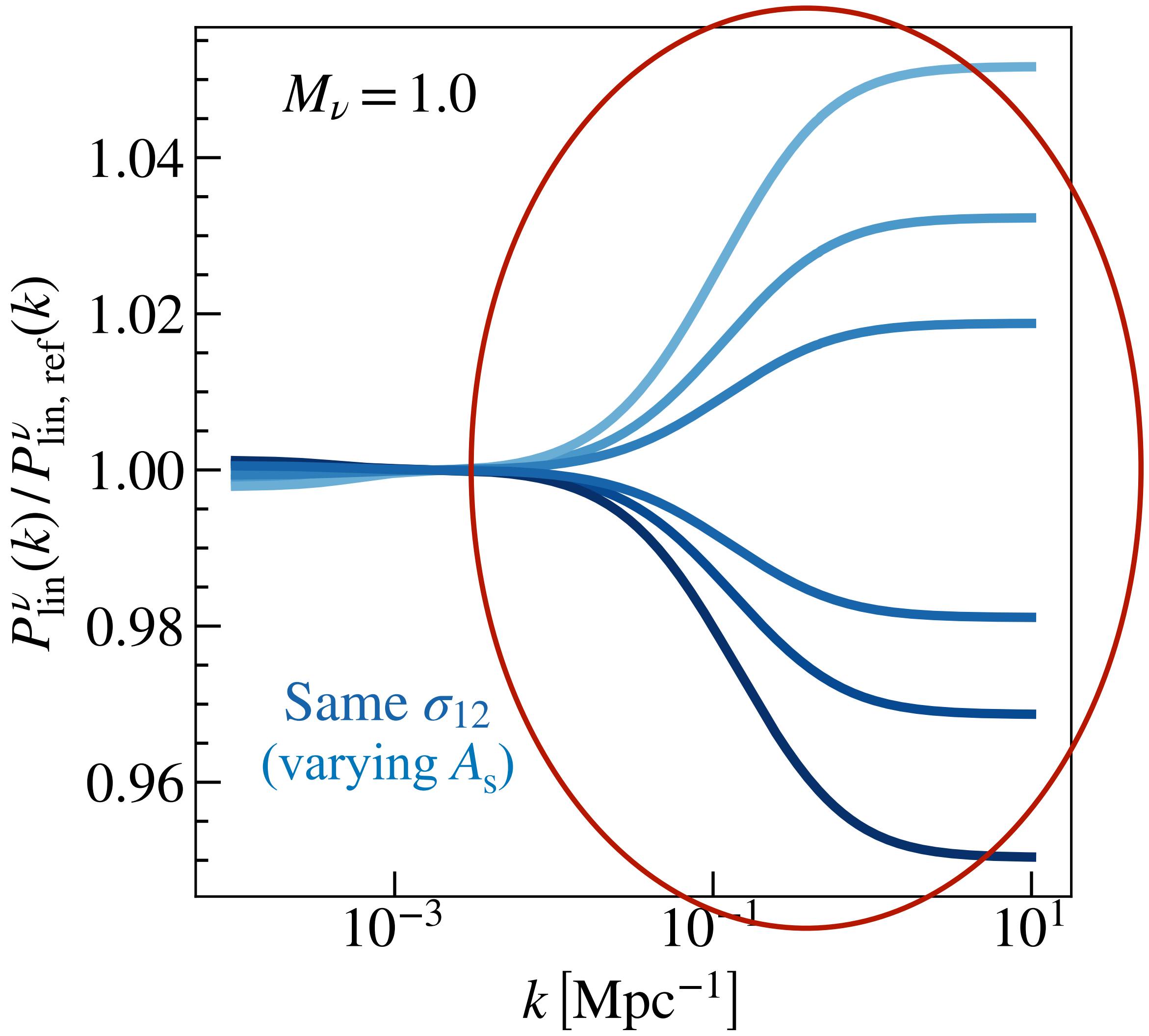

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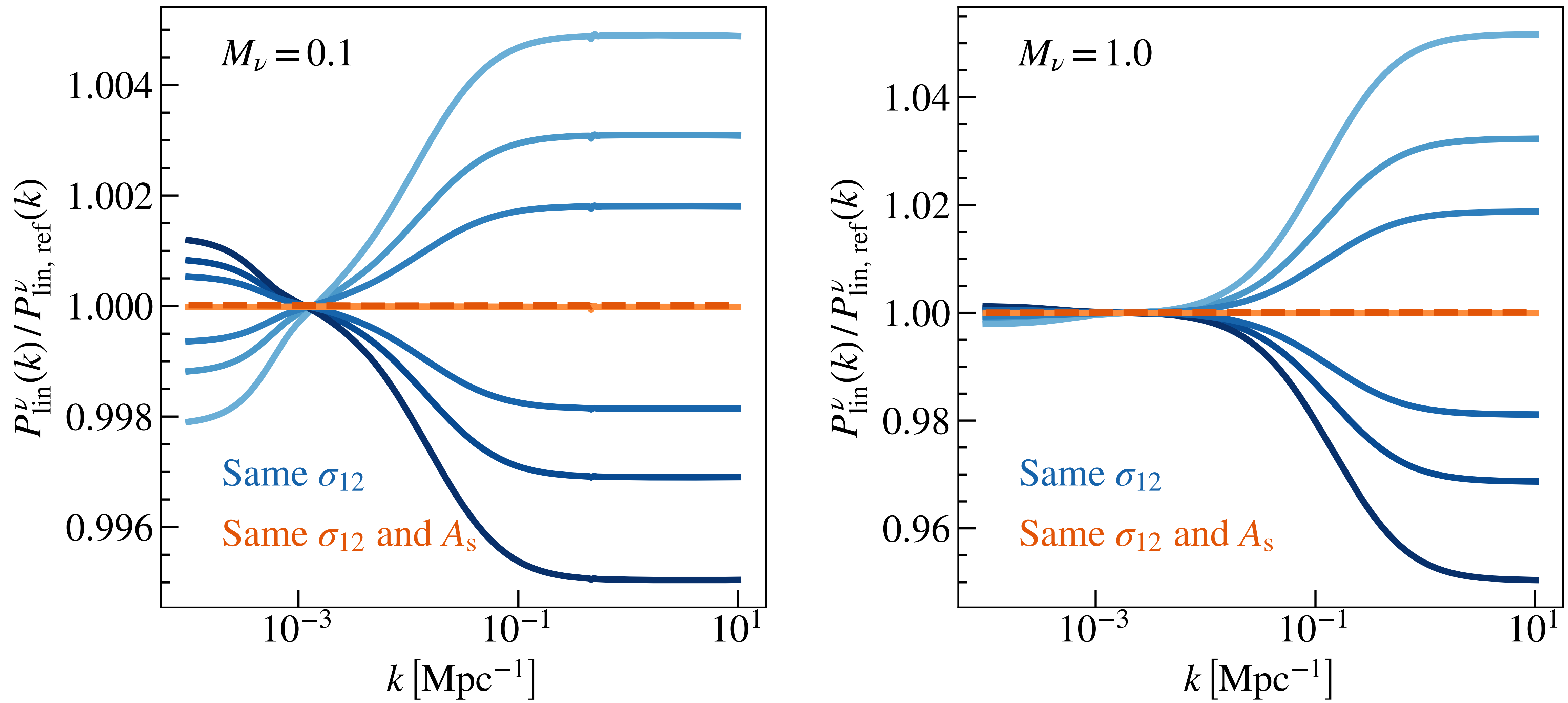

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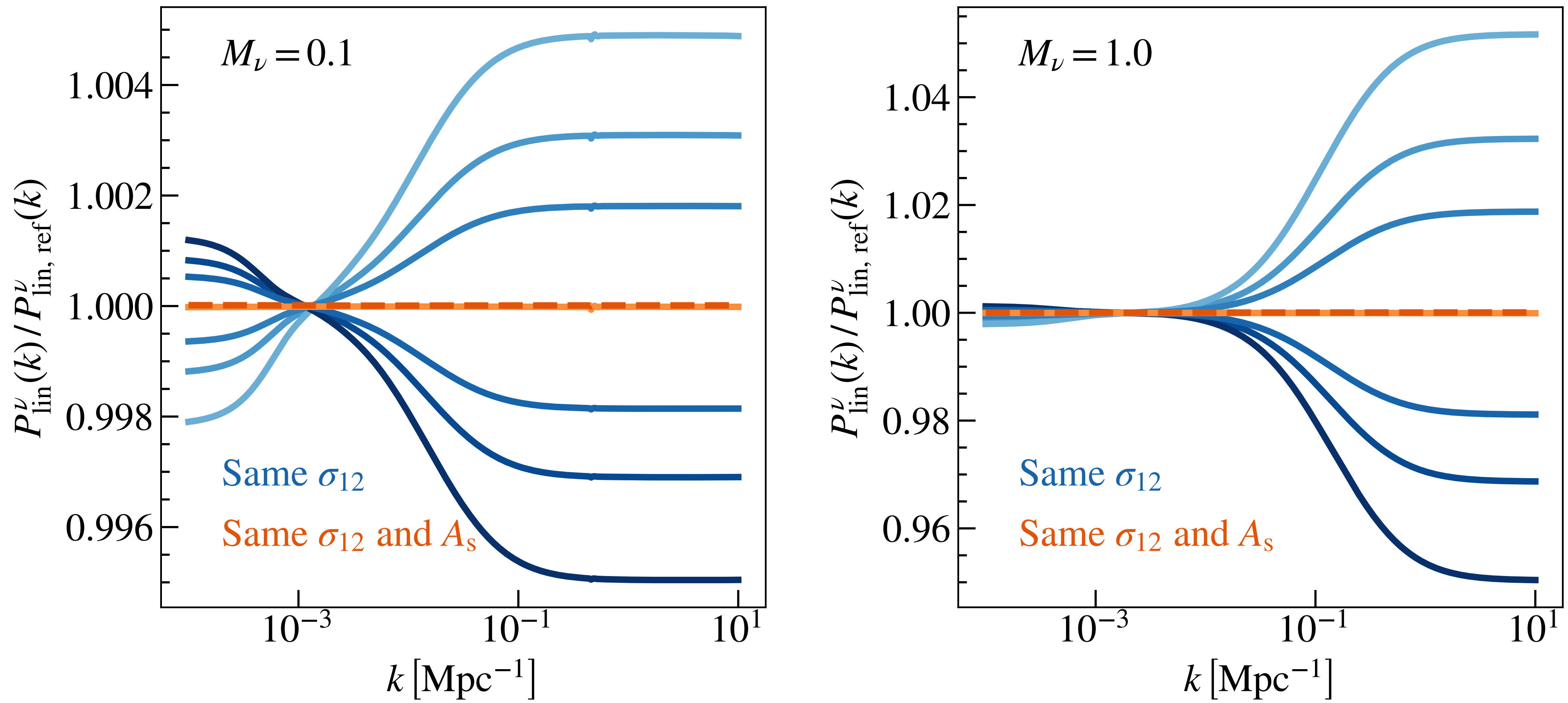

$$P_{\rm lin}(k\,|\,\mathbf{\Theta}_s\,,\mathbf{\Theta}_e\,,z) = P_{\rm lin}(k\,|\,\mathbf{\Theta}_s\,,\sigma_{12}(\mathbf{\Theta}_s\,,\mathbf{\Theta}_e\,,z))$$

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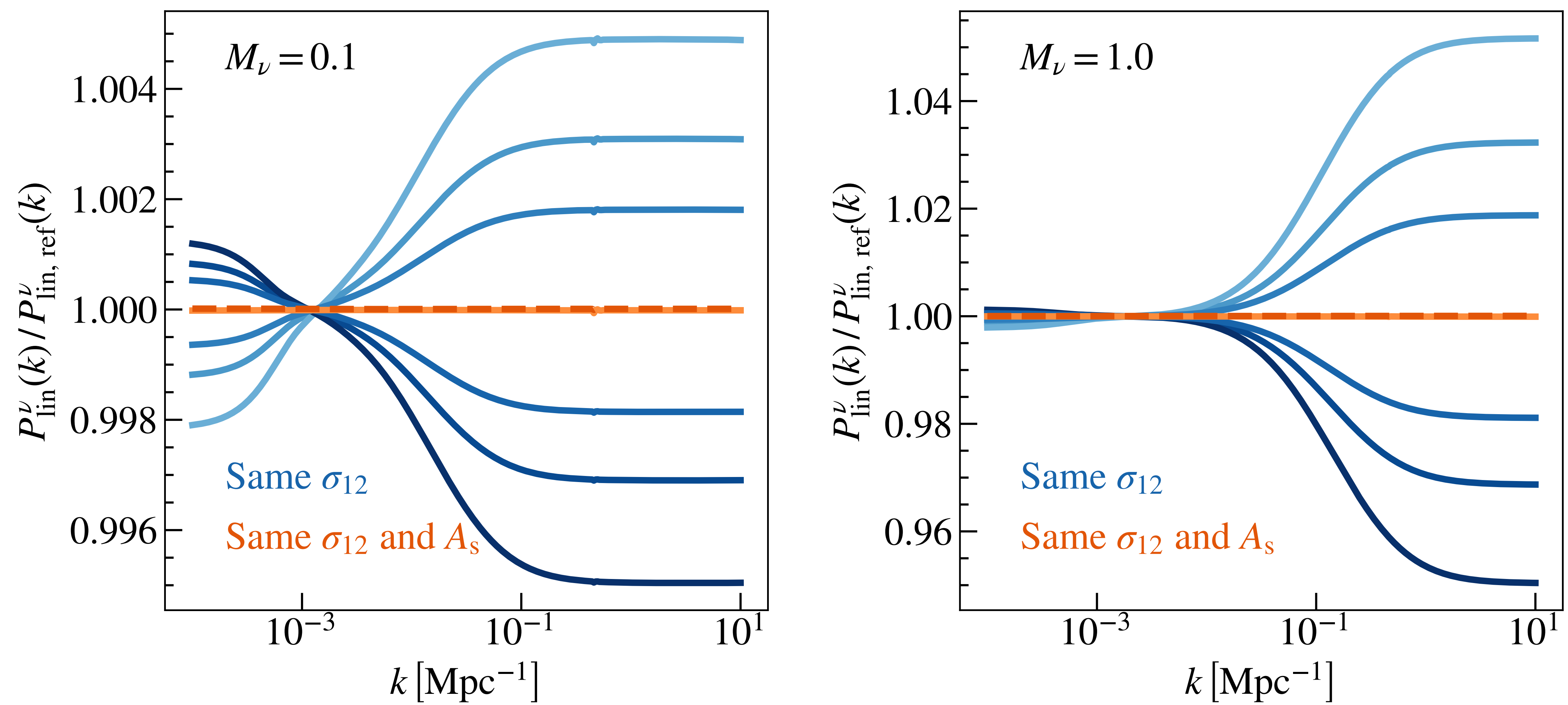


$$P_{\rm lin}(k\,|\,\boldsymbol{\Theta}_s^\star, \boldsymbol{\Theta}_e^\star, z) = P_{\rm lin}(k\,|\,\boldsymbol{\Theta}_s^\star, \tilde{\sigma}_{12}(\boldsymbol{\Theta}_s^\star, \boldsymbol{\Theta}_e^\star, z))$$

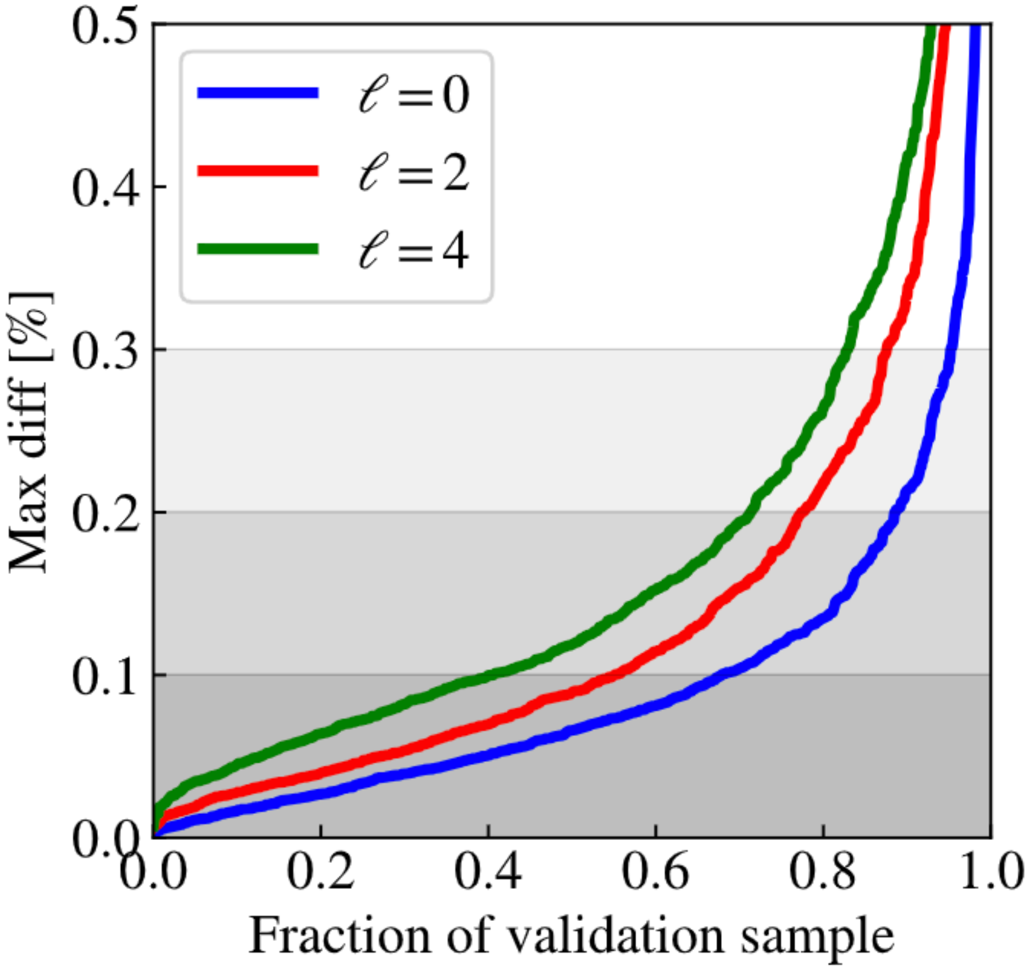
New implementation in COMET

- Emulation of perturbation theory to speed up model evaluation
- Based on evolution mapping to reduce dimensionality of parameter space
- Flexibility to incorporate different PT models (EFT, VDG, …)
- V1 (without massive neutrinos) public (Eggemeier et al. 2023)
  - `git clone git@gitlab.com:aegge/comet-emu.git`
  - `pip install comet-emu`
- Finalising V2 (massive neutrinos + extra features)

$$\{\omega_c, \omega_b, n_s, \sigma_{12}, f\} \rightarrow \{\omega_c, \omega_b, M_\nu, n_s, A_s, \sigma_{12}, f\}$$

Accuracy better than 0.2% for ~90% of the validation sample (**monopole**)

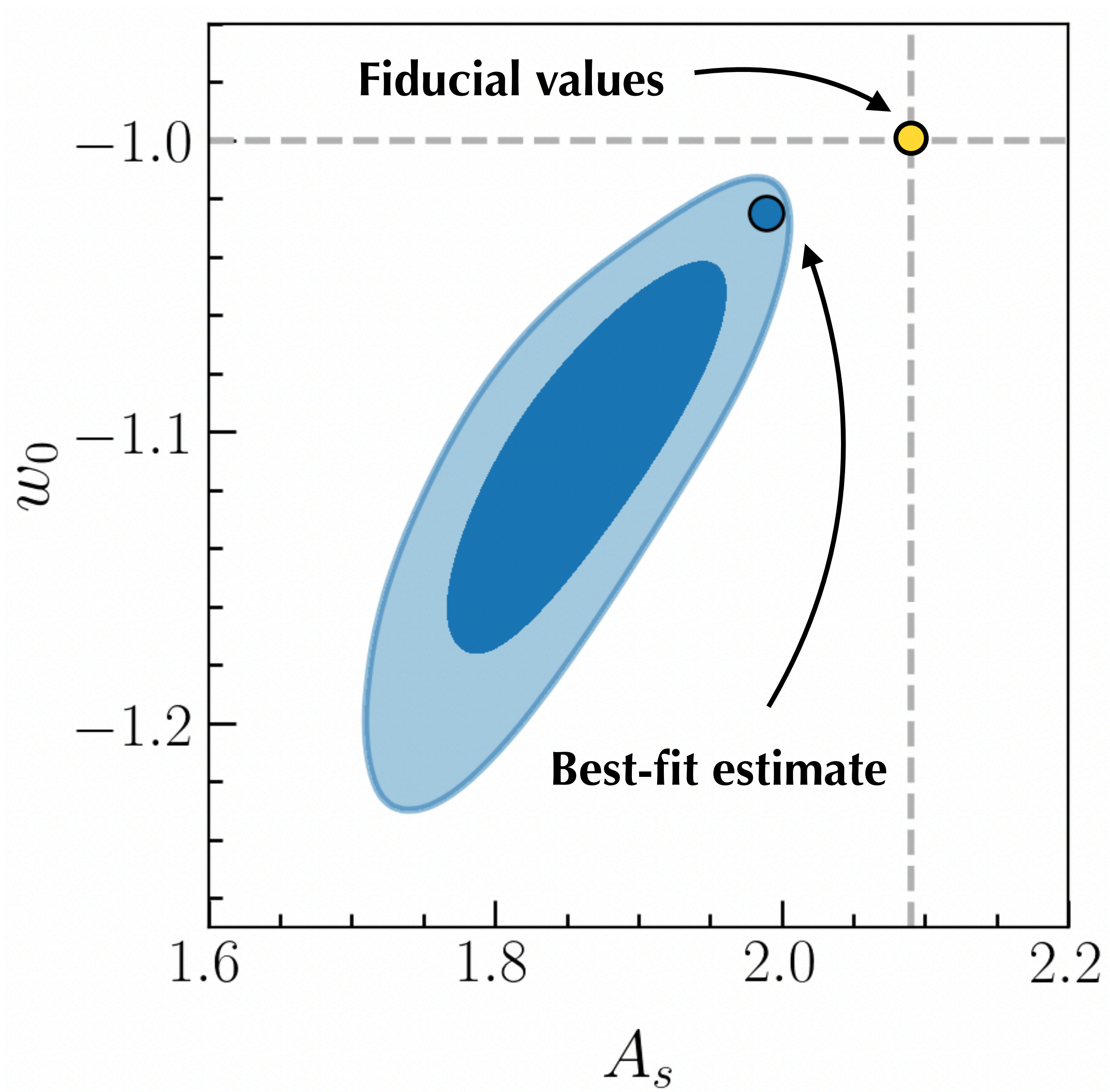
Evaluation time decreases to $\mathcal{O}(10\,\mathrm{ms})$



Prior volume effects

Prior volume effects

Bayesian analyses can be affected by **projection effects** during the marginalisation over nuisance parameters



Test with **synthetic data vectors**

- 4 redshift bins $\bar{z} = \{1, 1.2, 1.4, 1.65\}$
- Final mission sky fraction ($\sim 15000\,\mathrm{deg}^2$)
- wCDM cosmology
- Marginalisation over 6 nuisance parameters per bin $(b_1\,, b_2\,, c_0\,, c_2\,, c_4\,, \alpha_P)$

The importance of projection effects is more relevant when

- exploring high dimensional parameter space
- exploring non-Gaussian likelihoods (which is typically the case for spectroscopic galaxy clustering)

Prior volume effects

Let's assume to have a model (with parameters θ) that describes some data $\mathbf{d}$

$$\vec{\theta} = \{\vec{\Omega}, \vec{n}\}$$

Parameters of interest
(**cosmology**)

Parameters to marginalise over
(**nuisance, bias, RSD, systematics**)

The marginalised posterior distribution of $\vec{\Omega}$ is $\mathbf{P}(\vec{\Omega}\,|\,\mathbf{d}) = \int \mathrm{d}\vec{n}\;\mathbf{P}(\vec{\Omega}, \vec{n}\,|\,\mathbf{d})$

We can expand $\chi^2 \equiv -2\log\mathbf{P}$ around the nuisance parameters $\vec{n}_\star(\vec{\Omega})$ that maximise the posterior

$$\chi^2(\vec{\Omega}, \vec{n}) \simeq \chi^2_\star(\vec{\Omega}) + \Delta\vec{n}^T\mathscr{F}_\star\Delta\vec{n} + \ldots$$

After marginalisation of $\vec{n}$, this reads

$$\chi^2_{\mathrm{marg}}(\vec{\Omega}) \simeq \chi^2_\star(\vec{\Omega}) + \log\{\det[\mathscr{F}_\star(\vec{\Omega})]\}$$

Profile likelihood
(only depends on data)

Laplace term
(prior volume of nuisance parameters)

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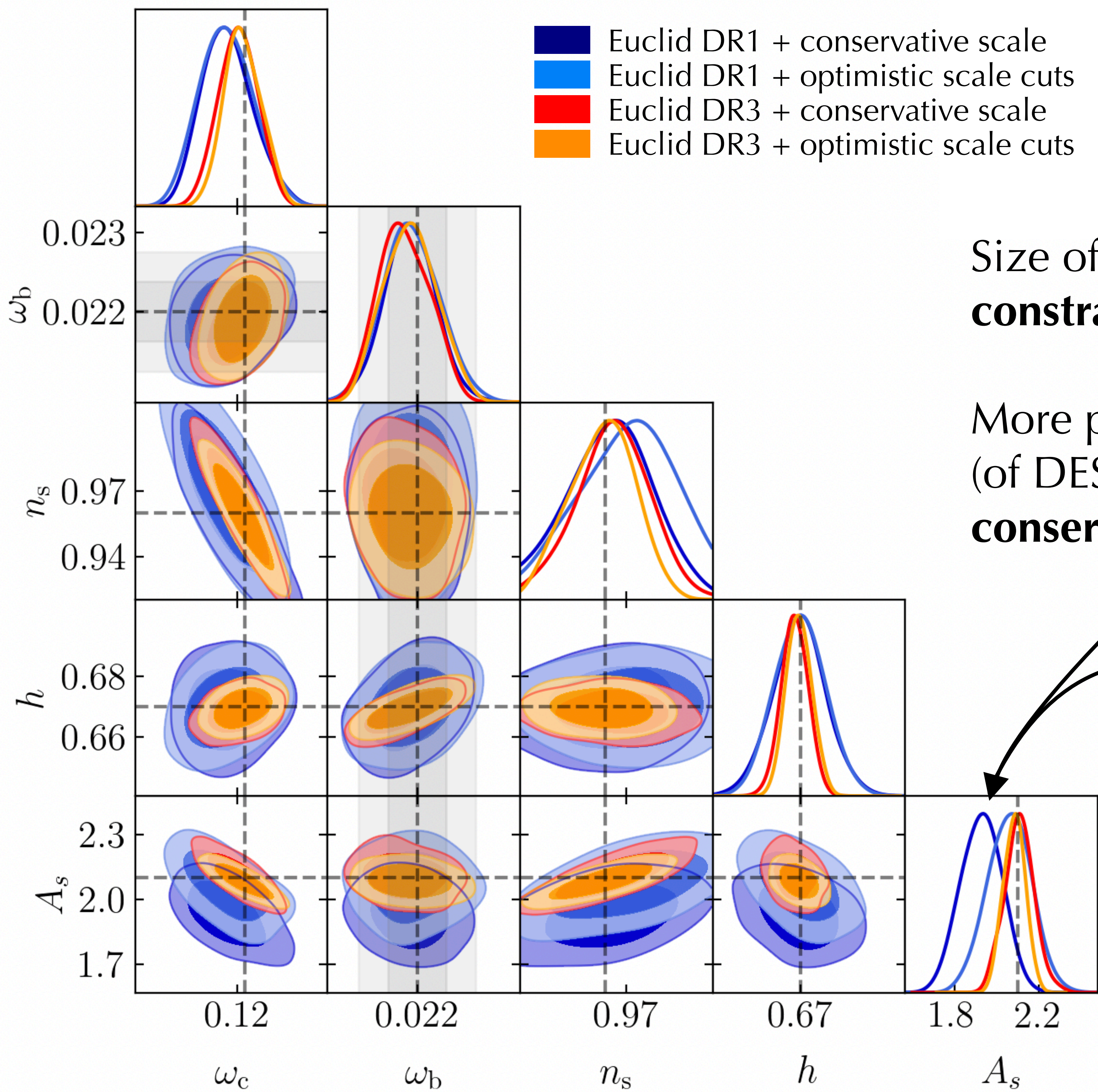
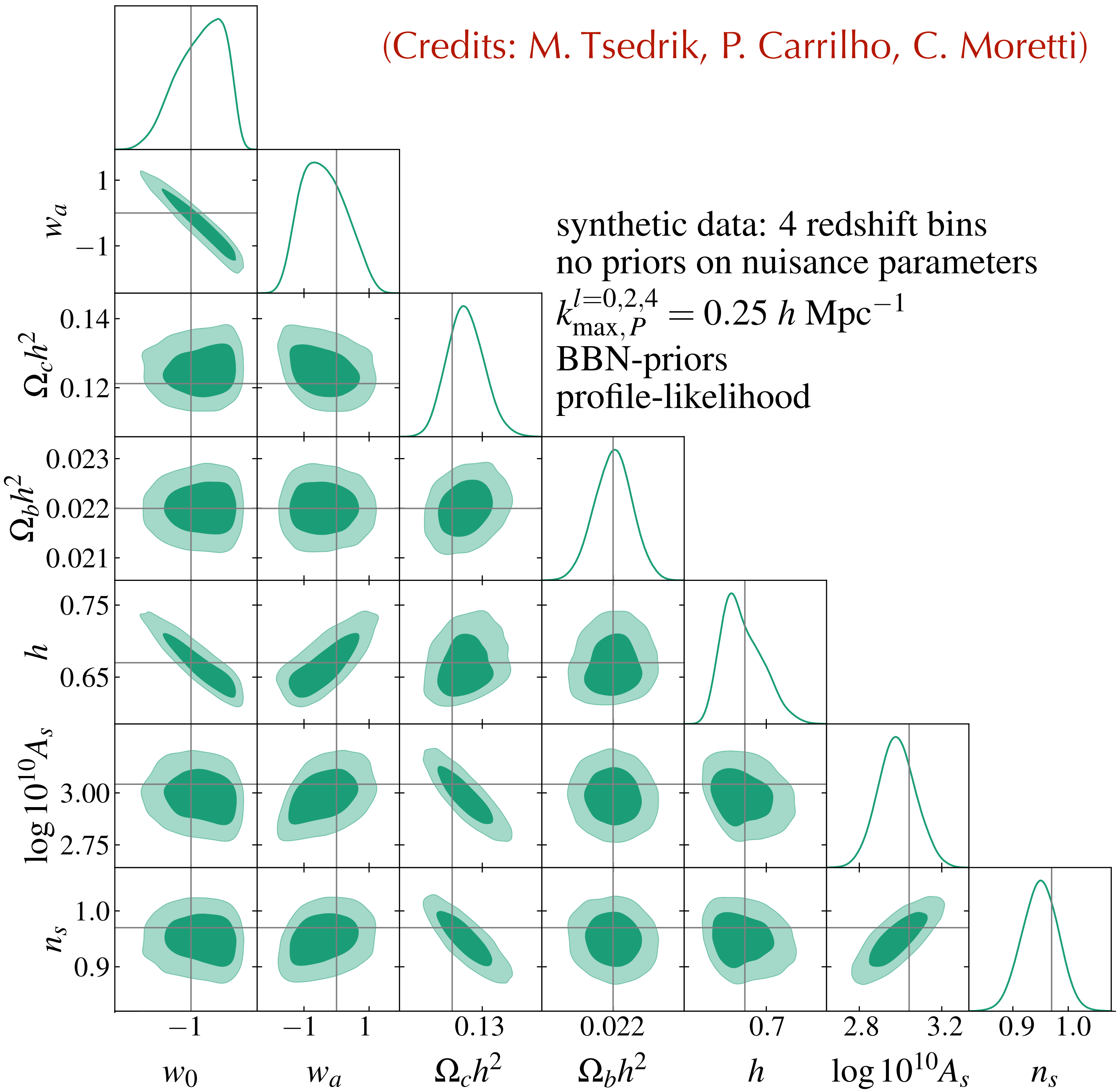


Size of projection effects related to **constraining power** of data vectors

More problematic for **first Data Releases** (of DESI, Euclid, …) and for more **conservative scale cuts**

Some parameters are more affected, i.e. A_s , due to stronger dependence of Laplace term on the nuisance parameters

Potential solutions



1) Using profile likelihood

Removal of Laplace term (the one sourcing the prior volume effects)

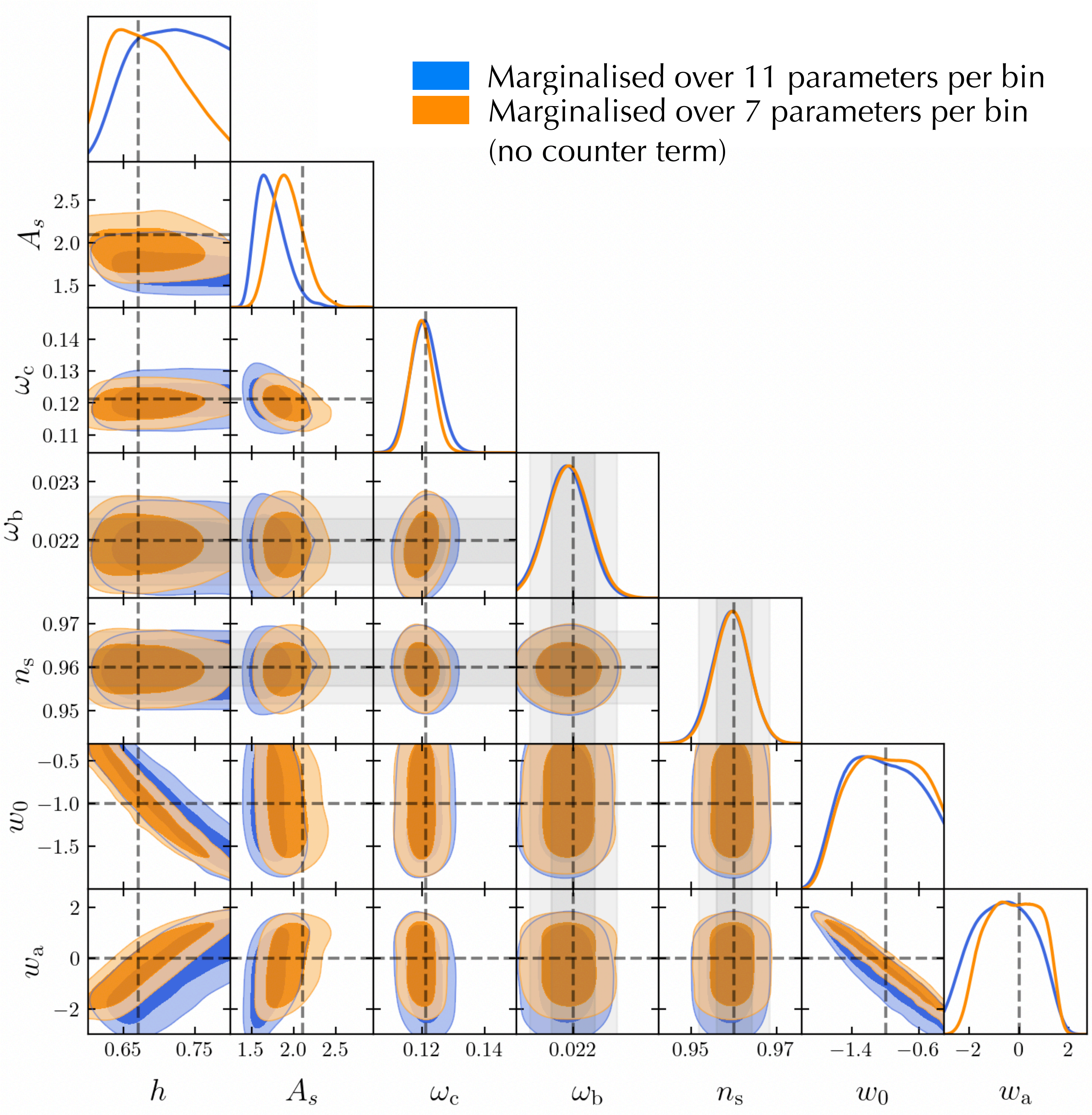
Pros

- Can be done efficiently with optimised tools (e.g. χ^2 minimisers)
- Removes the problem even for extended models (e.g. $w_0 w_a$CDM)

Cons

- Not easy to interpret error bars
- Frequentist approach instead of Bayesian

Potential solutions



2) Using priors on nuisance parameters

Tight priors on parameters that are mostly responsible for appearance of projection effects (e.g. counterterms)

Pros

- Optimal choices such as **Jeffreys priors**
- Encouraged when using RSD models where counterterms are not essential (e.g. VDG)

Cons

- Not in agreement with Bayesian formalism

Conclusions

- Perturbative models will still play a pivotal role in the analysis of Stage-IV data
  - Already reached a sufficient level of maturity
  - Different models to test scale cuts, theory systematics, constraining power, etc
  - Posterior validation thanks to many public codes
- Speeding up the model evaluation is essential
  - Use of theory emulators to maintain accuracy of PT-based models
  - Evolution mapping is an easy way to reduce the number of parameters of the emulators (works with massive neutrinos)
- Prior volume effects as one of the major limitations in the use of these models
  - Due to non-Gaussianity of likelihood
  - Potential ways to reduce their impact requires a questioning of standard Bayesian methods:

    - profile likelihood
    - Jeffrey's priors
    - tight priors on nuisances
    - reparametrization
    - use of alternative models