

COSMOLOGY WITH CLUSTERING OF GALAXY CLUSTERS

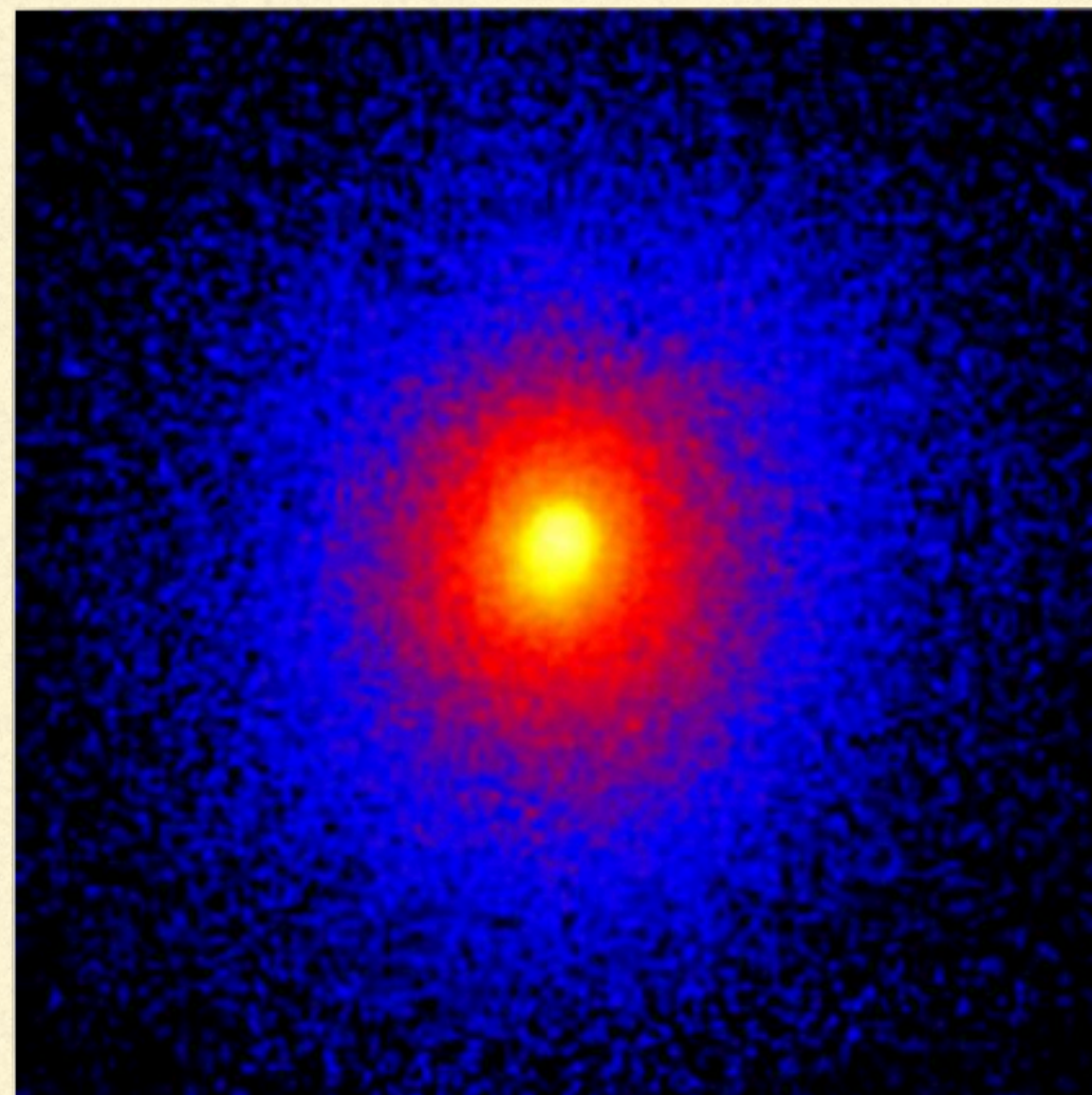
Alessandra Fumagalli

Sexten — 03.07.2024

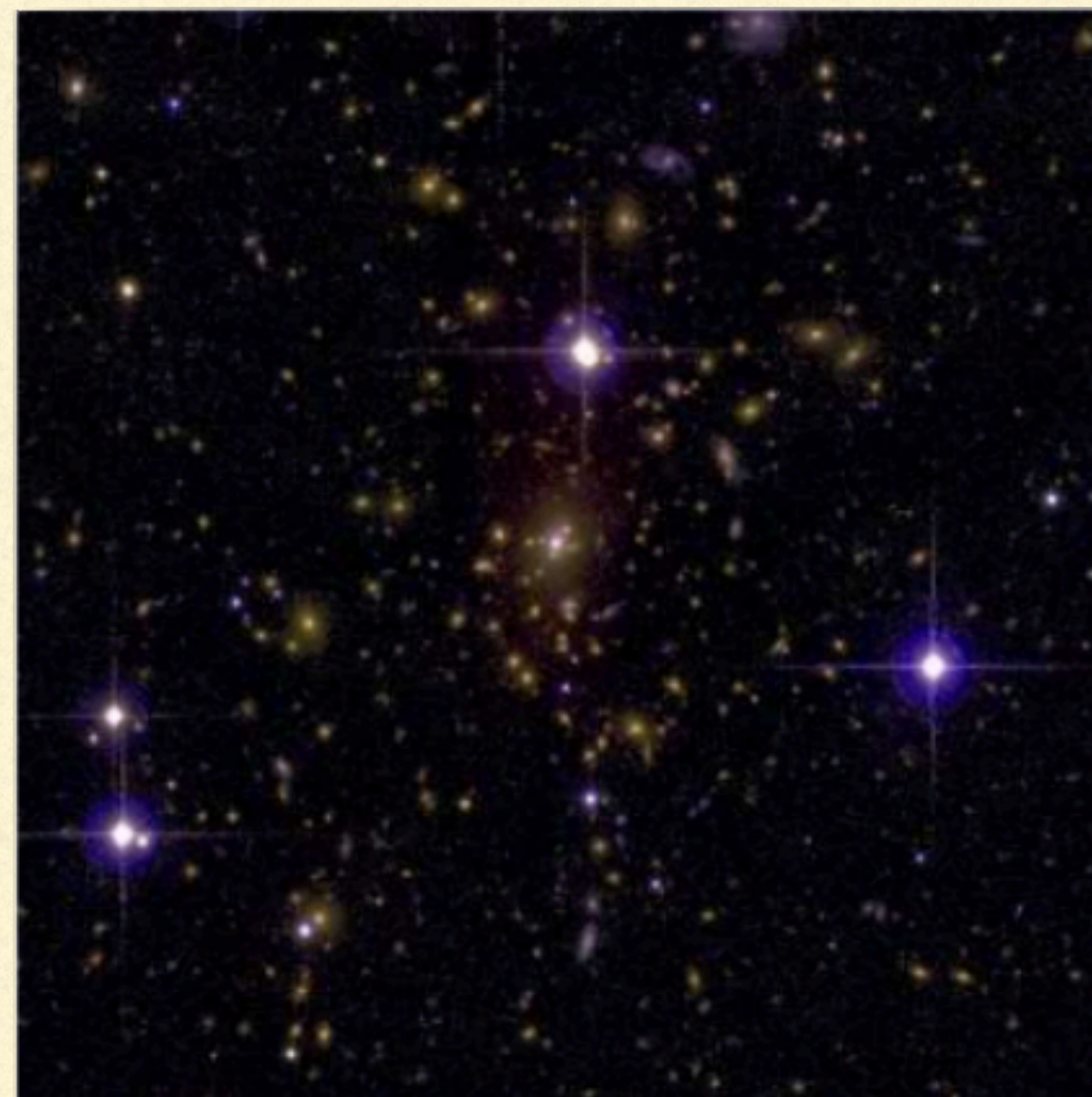
GALAXY CLUSTERS

- most massive gravitationally bound systems in the Universe ($M \sim 10^{14} - 10^{15} M_{\odot}$)
- dark matter (85%) + hot ionised gas (12%) + stars (3%)
- detectable across the entire electromagnetic spectrum

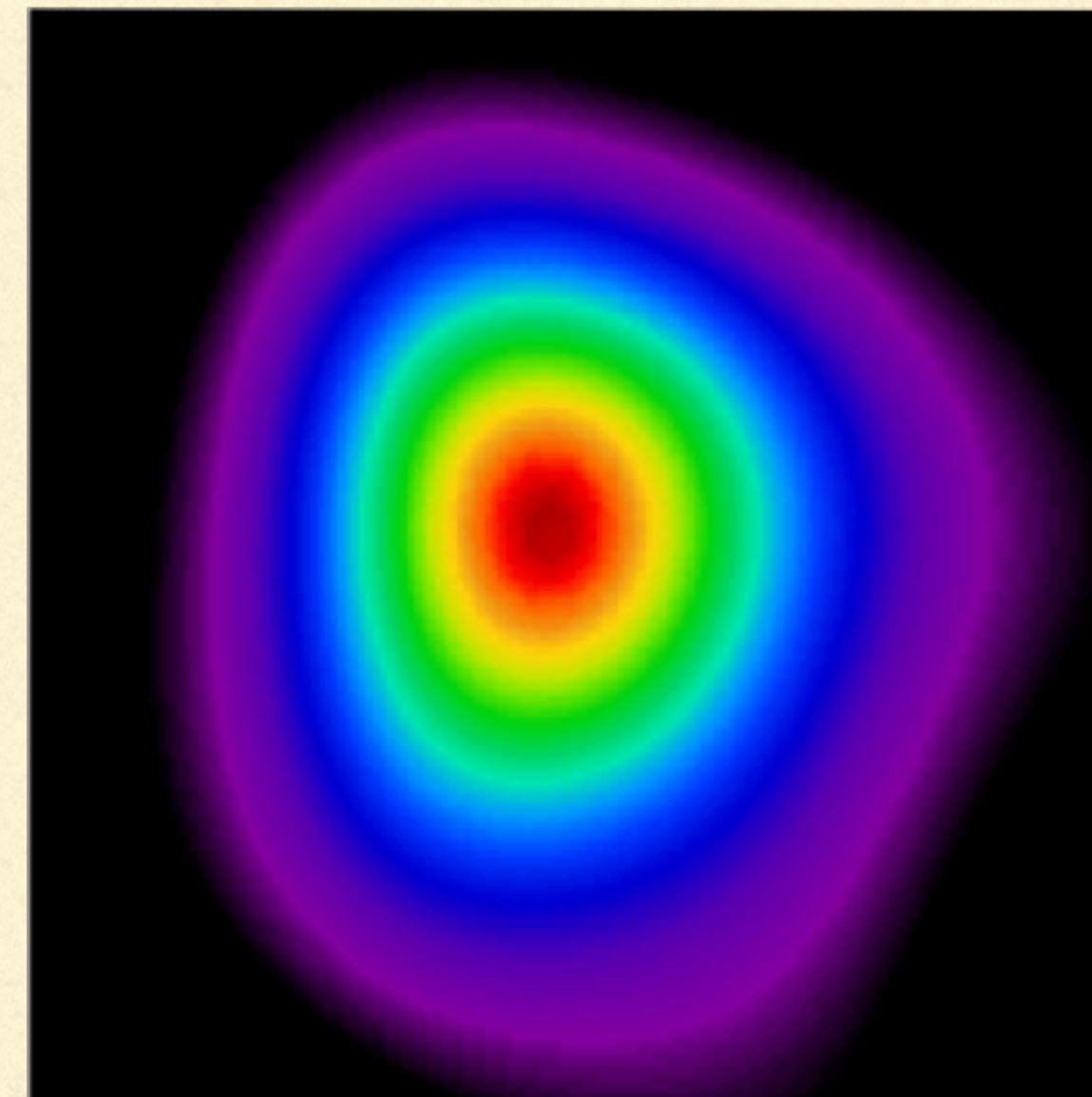
Intracluster gas:
X-ray



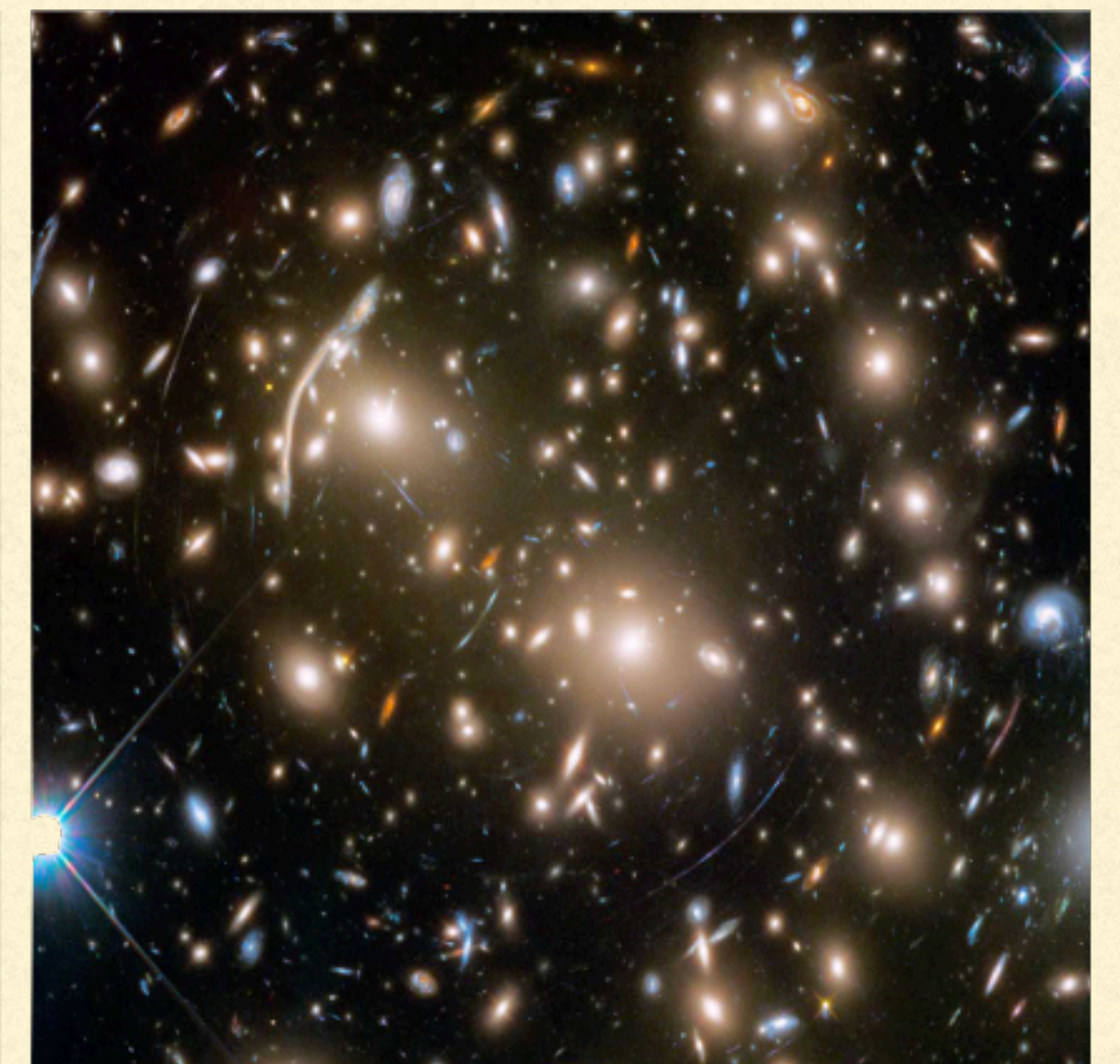
Stars in galaxies:
Optical and infrared



Sunayaev-Zeldovich effect
Millimeter band



Gravitational lensing



CLUSTER COSMOLOGY

Galaxy clusters reside in **dark matter halos**

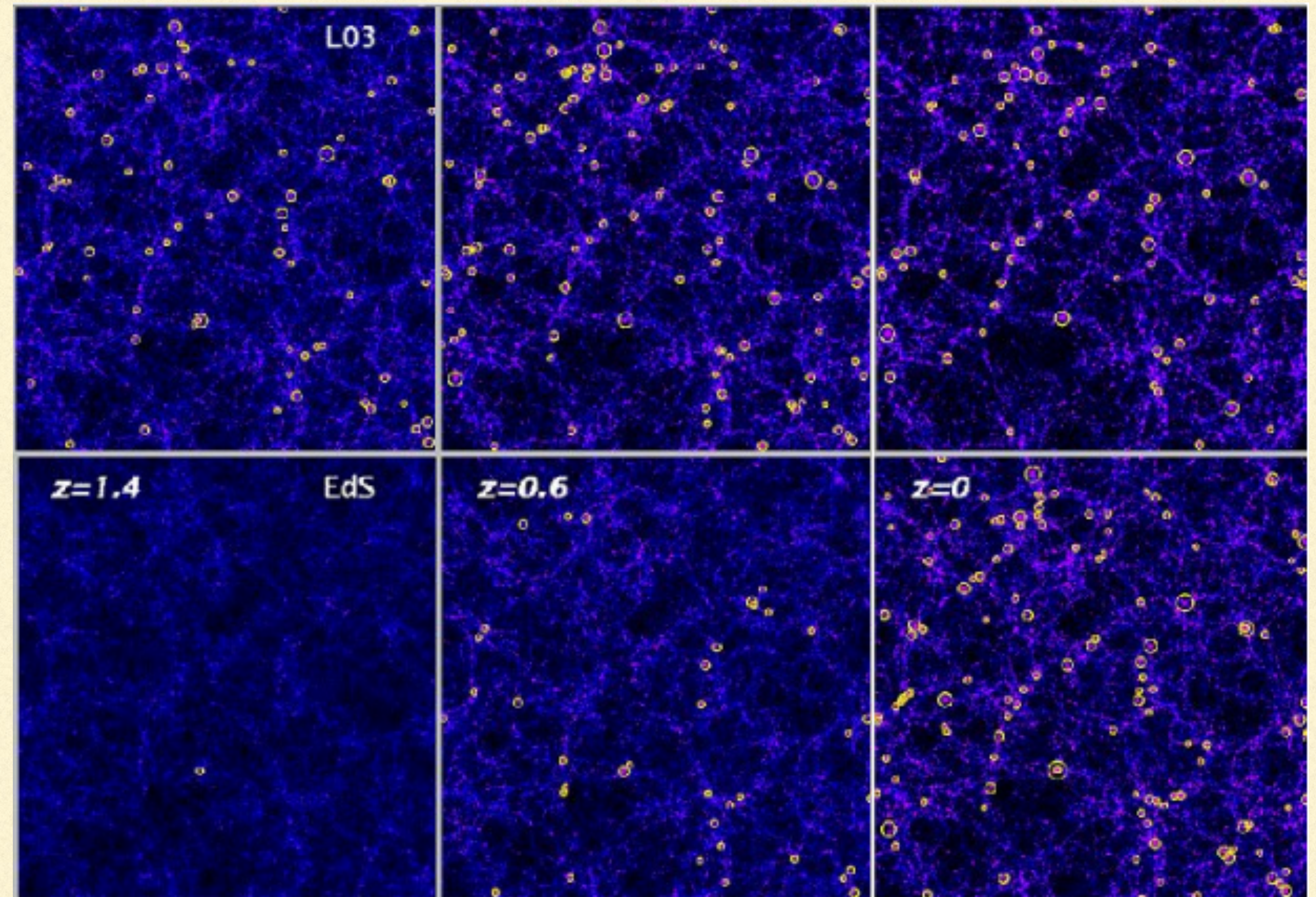


Trace the large-scale structure geometry and evolution



Constrain cosmological parameters,
especially:

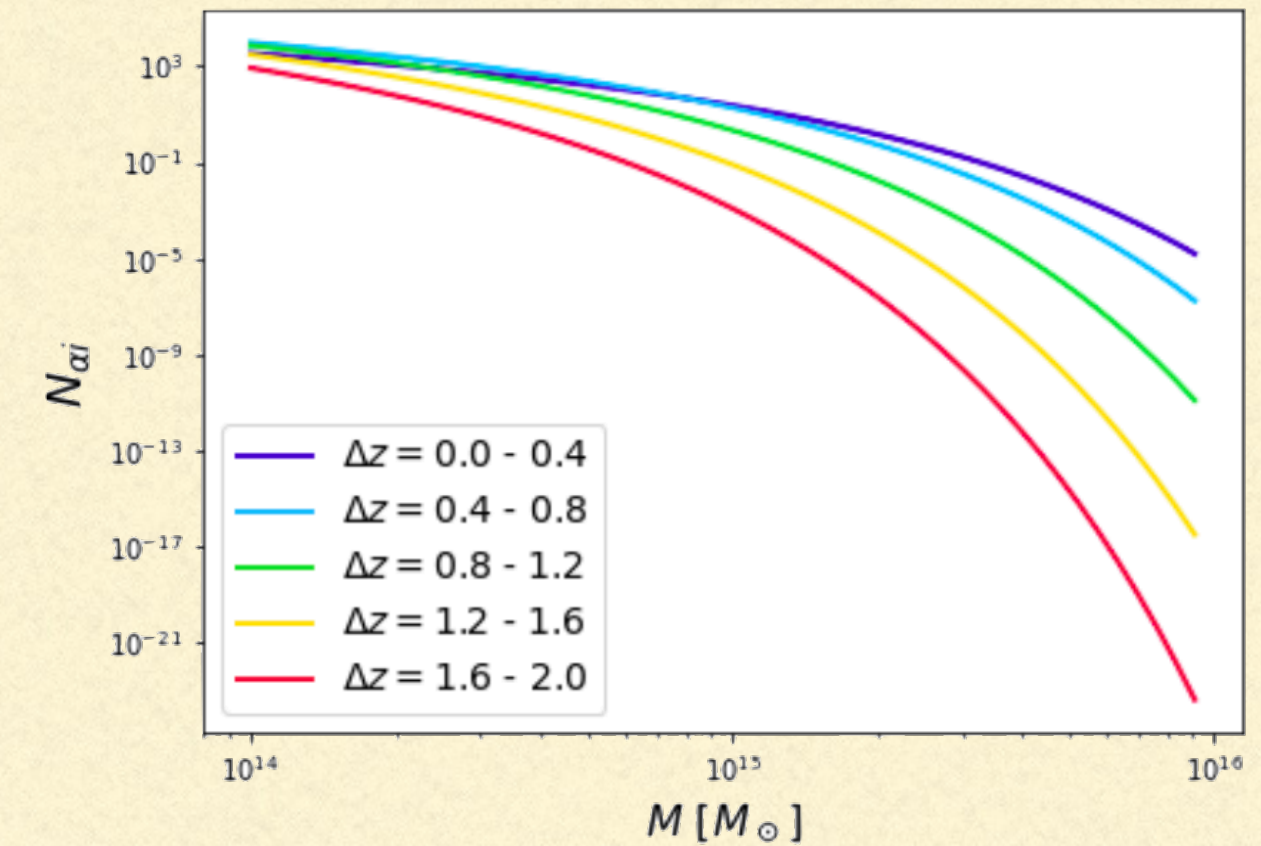
- Ω_m = matter content of the Universe
- σ_8 = amplitude of density fluctuations



CLUSTER COSMOLOGY

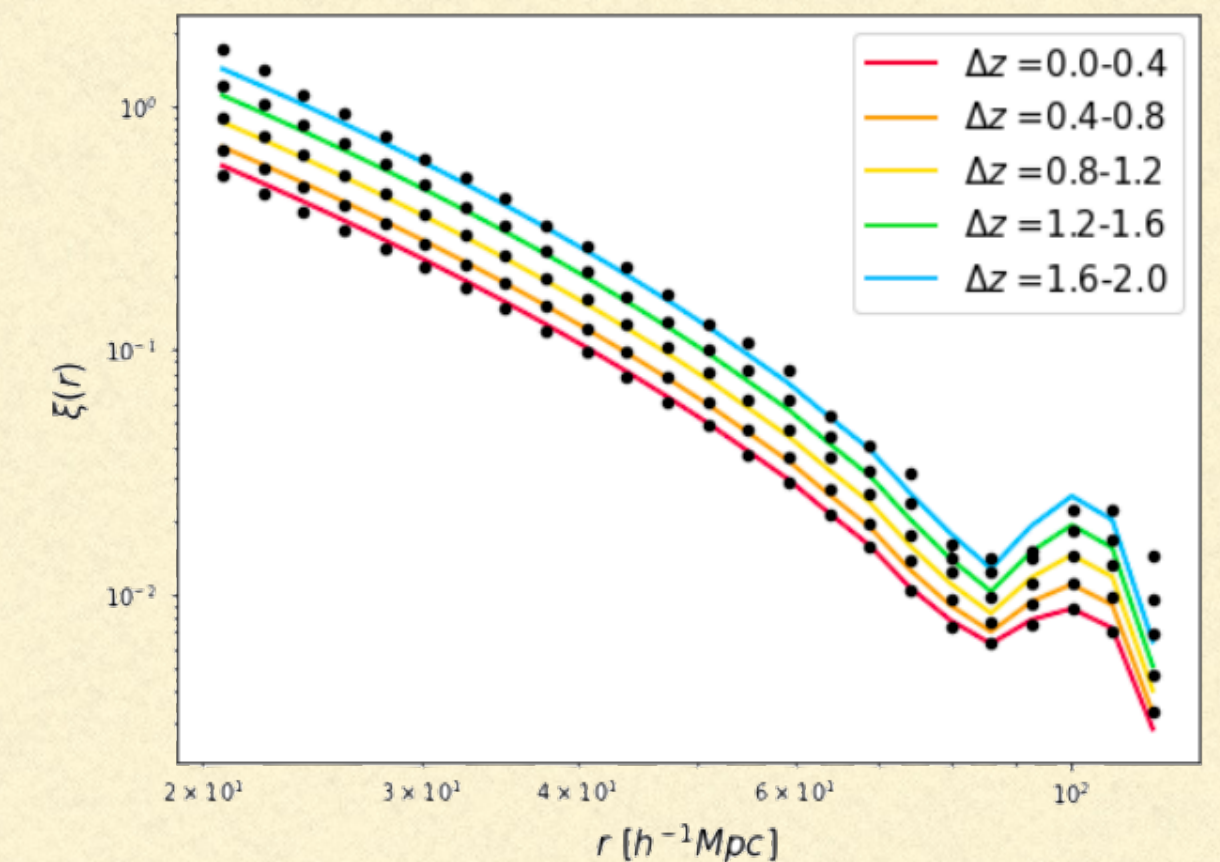
Cosmology with **cluster counts**:

- abundance of clusters in mass and redshift
- most widely used observable in cluster cosmology
- can constrain both cosmological and mass-observable relation parameters



Cosmology with **cluster clustering**:

- spatial distribution of clusters
- less used due to low statistics
- useful to break parameter degeneracies if combined to other probes (e.g., Borgani et al. 1999; Majumdar & Mohr 2004; Mana et al. 2013; Sereno et al. 2015; To et al. 2021; Lesci et al. 2022)



CLUSTER CLUSTERING

3D 2-point correlation function:

$$\xi_h(r, z) = \int \frac{d^3k}{(2\pi)^3} P_h(k, z) e^{i\mathbf{k}\cdot\mathbf{r}}$$

observational uncertainties: **RSD** and **photo-z**

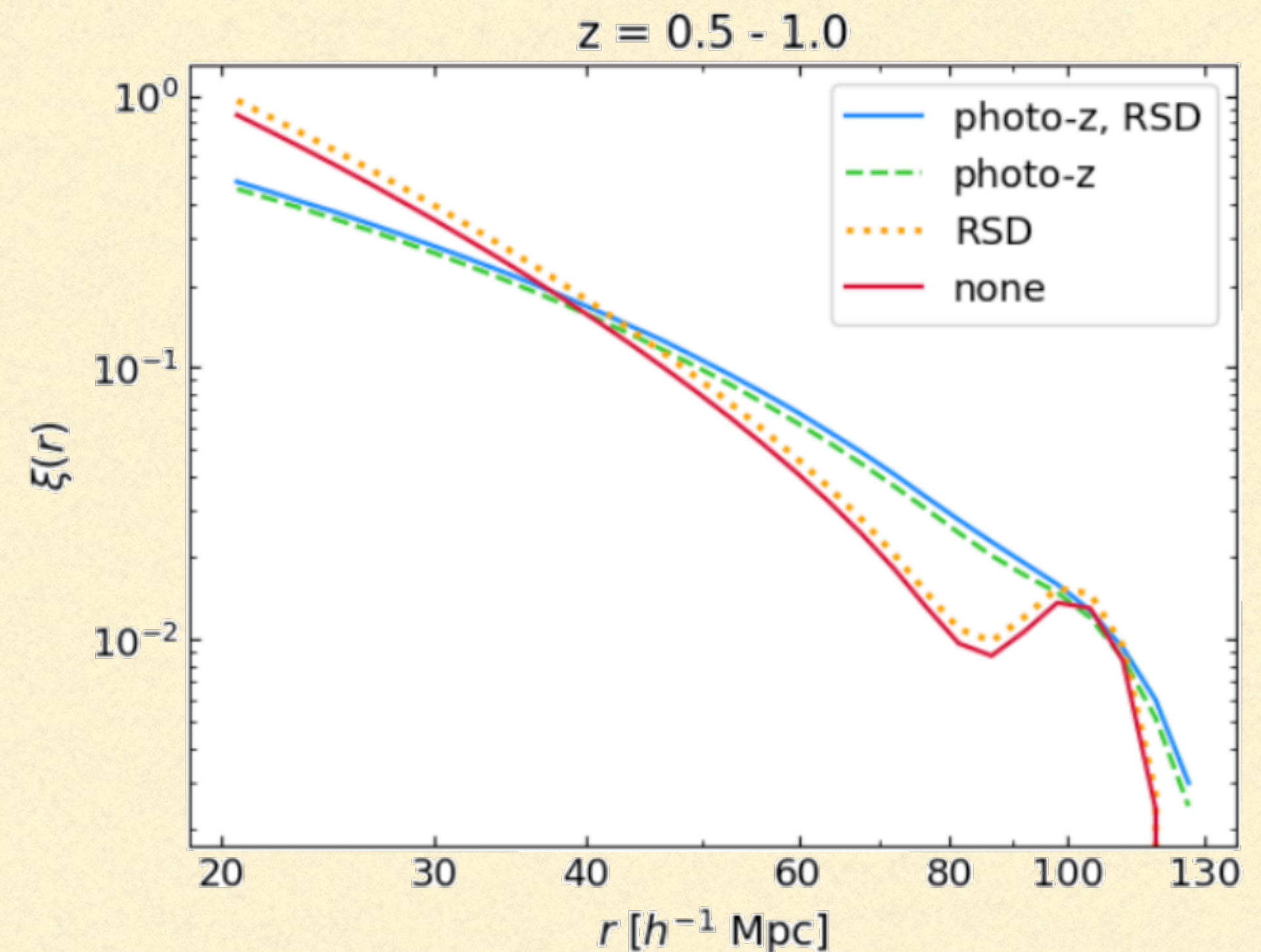
$$z_{\text{ob}} = z + \frac{v_{\parallel}}{c} (1+z) \pm \sigma_z$$

$$P_h^{\text{ob}}(k, \mu) = (1 + \beta \mu^2)^2 P_h(k) e^{-(k\mu\sigma)^2} \quad \sigma = \frac{\sigma_z c}{H(z)}$$

monopole

$$P_h^{\text{ob},0}(k, z) = (A + B\beta + C\beta^2) P_h(k, z)$$

$$A = A(k\sigma) \quad B = B(k\sigma) \quad C = C(k\sigma)$$



HALO BIAS

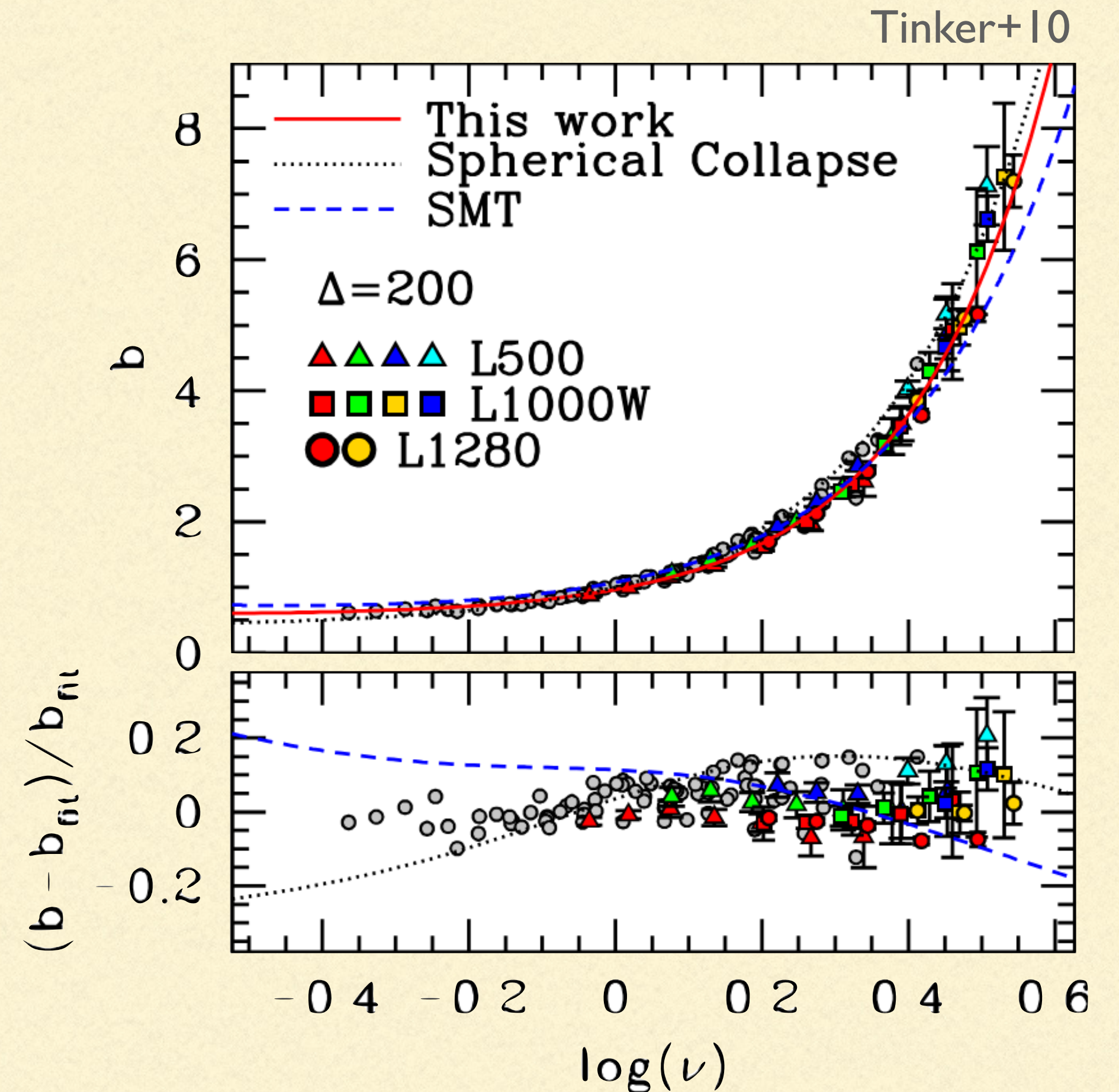
Clusters are **biased tracers** of matter distribution

$$\xi_h(r) = b^2 \xi_m(r)$$



Halo bias:

- More linear and larger than galaxy bias
- Almost scale-independent ($r > 20$ Mpc/h)
- Depends on mass, redshift and cosmology
- Can be fitted from simulations (e.g., Tinker+10, Castro+23)



CLUSTER MASSES

Cluster **masses non directly measurable**

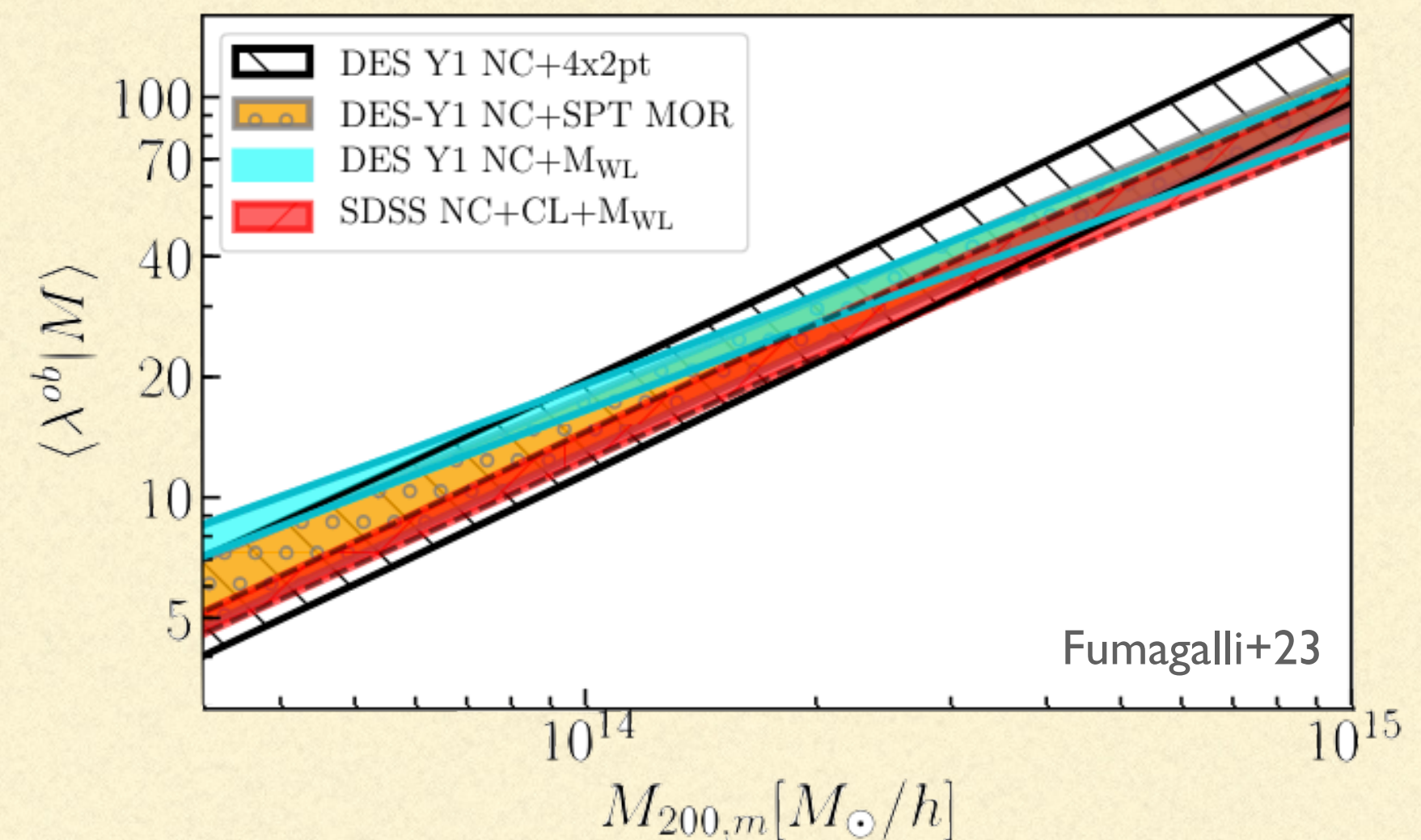


must be inferred through measurable properties of clusters
(**mass proxy** like richness)



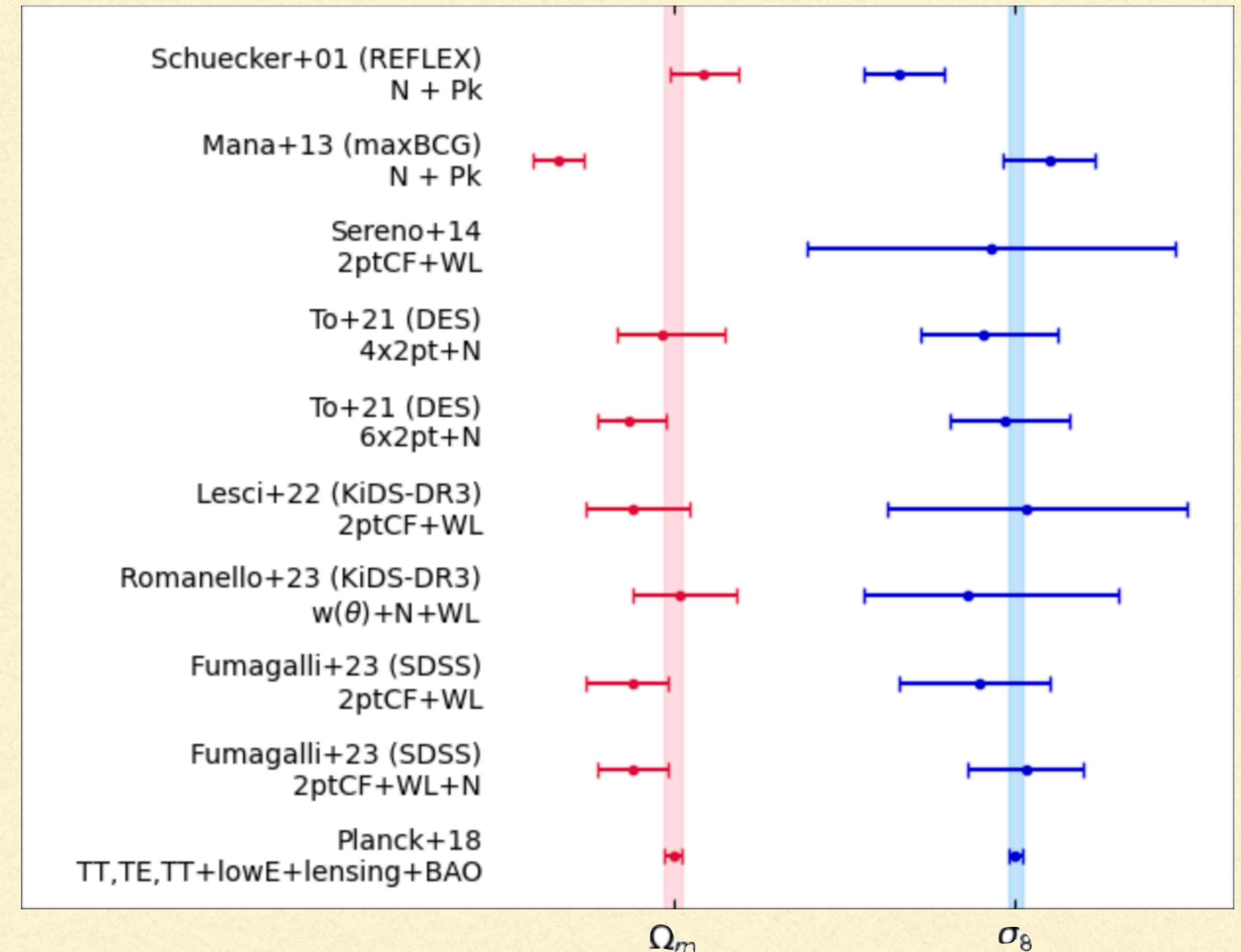
observable-mass relation to be **calibrated**
(usually with the help of weak-lensing masses)

$$\langle \ln \lambda | M_{\text{vir}}, z \rangle = \ln A_{\lambda} + B_{\lambda} \ln \left(\frac{M_{\text{vir}}}{3 \times 10^{14} M_{\odot}} \right) + C_{\lambda} \ln \left(\frac{E(z)}{E(z=0.6)} \right),$$
$$\sigma_{\ln \lambda | M_{\text{vir}}, z}^2 = D_{\lambda}^2.$$



STATE-OF-THE-ART

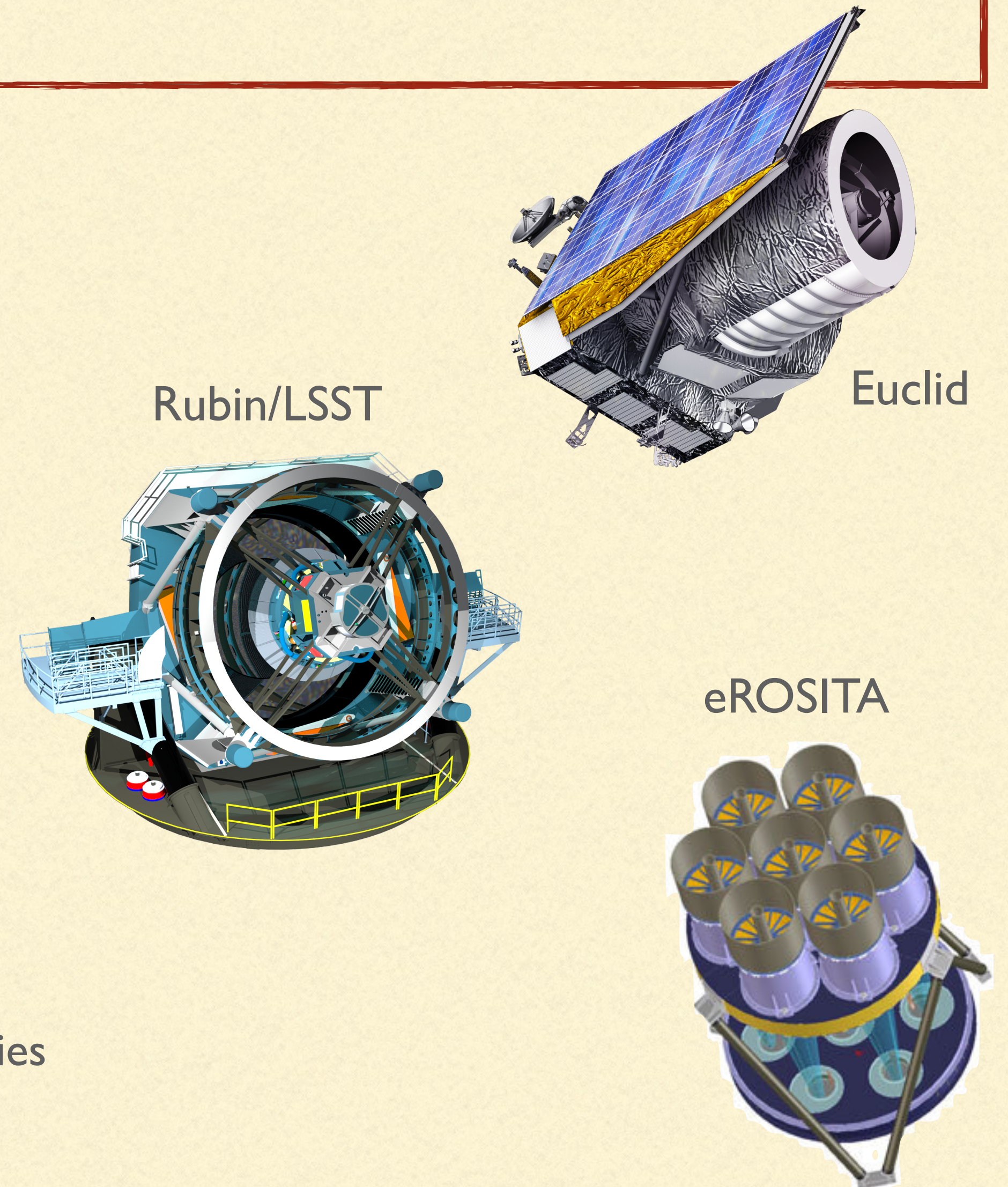
- Cluster clustering (CC) first used by Borgani+99 to constrain σ_8 ($0.8 < \sigma_8 < 2.0$ for $\Omega_m = 0.3$)
- CC first combined with cluster counts by Schuecker+03 to constrain Ω_m and σ_8
- CC useful to help constraining scaling relations and cosmology (e.g., Majumdar&Mohr04; Mana+13; Sereno+14, To+21; Lesci+22, Romanello+23, Fumagalli+23)
- CC useful to identify BAO (Miller+01; Angulo+05; Huetsi+10; Veropalumbo+14; Moresco+21)



CLUSTER SURVEYS

- Recent cluster surveys (e.g., **SDSS, DES, SPT, KiDS, ...**) have significantly expanded our catalogs of galaxy clusters
- Upcoming cluster surveys (e.g., **Euclid, LSST, eROSITA, ...**) will provide cluster catalogs with **unprecedented statistics**:
 - Increase number of clusters (up to $\sim 10^5$)
 - Wider redshift ranges (up to $z \approx 2$)
 - Large survey volumes
 - Multi-wavelength surveys to be combined

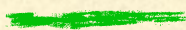



→ main problem in cluster cosmology: **control of systematic** uncertainties



SYSTEMATICS AND UNCERTAINTIES

$$\xi(\Delta r_a, \Delta z_i^{\text{ob}}) = \frac{1}{N_i^2} \int_0^\infty dz_1 dz_2 \frac{dV}{dz}(z_1) \frac{dV}{dz}(z_2) \bar{n}(z_1) \bar{n}(z_2) \bar{b}(z_1) \bar{b}(z_2) \bar{\xi}_m(\Delta r_a, z_1, z_2) \int_{\Delta z_i^{\text{ob}}} dz_1^{\text{ob}} dz_2^{\text{ob}} P(z_1^{\text{ob}} | z_1) P(z_2^{\text{ob}} | z_2)$$

$$\bar{b}(z | \lambda^{\text{ob}}) = \frac{1}{\bar{n}} \int_0^\infty dM n(M, z) b(M, z) \int_0^\infty d\lambda \int_{\lambda_{\text{min}}}^\infty d\lambda^{\text{ob}} P(\lambda^{\text{ob}} | \lambda) P(\lambda | M, z),$$

-  cosmology-dependent quantities \Rightarrow constrain cosmological parameters
 -  semi-analytical models \Rightarrow calibrate on numerical simulations
 -  mass-observable relation \Rightarrow cluster mass not directly observable, to be inferred through mass proxies (e.g. richness)
 -  selection functions \Rightarrow observational inaccuracy (photo-z error, projection effects, ...)
- + cluster detection \Rightarrow catalog's completeness and purity
- + covariance matrix \Rightarrow statistical errors (shot-noise, sample variance, ...).

COVARIANCE MATRIX

Inclusion of uncertainties of statistical quantities fundamental to constrain cosmological parameters

- **Numerical matrix** from a large set of simulations

- + all the contributes are included

- noisy due to finite number of simulations / high computational resources

- cosmology-independent matrix

$$\hat{C}_{ij} = \frac{1}{N-1} \sum_{a=1}^N \left(\hat{d}_i^a - \langle \hat{d}_i \rangle \right) \left(\hat{d}_j^a - \langle \hat{d}_j \rangle \right)$$

- **(Semi-)analytical models**

- + noise free

- + cosmology-dependent

- difficult to include all the terms (non-linearities, non-Gaussianity, window functions...)

$$C_{ij} = C_{ij}(\Theta), \quad \Theta = \{ \Omega_m, \sigma_8, \dots \}$$



to be validated/calibrated with simulations

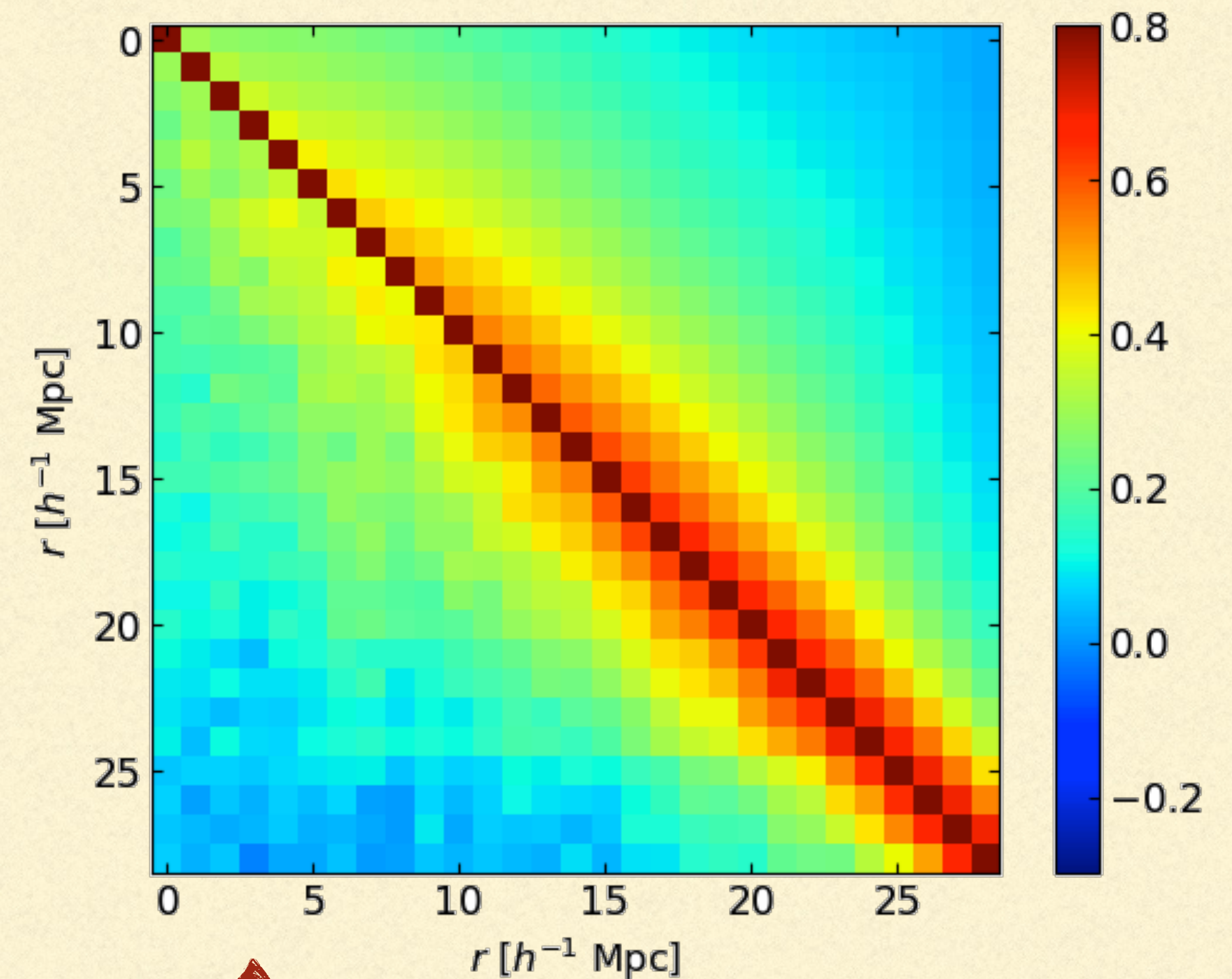
COVARIANCE MATRIX MODEL

Euclid Collaboration: Fumagalli+22 ([arXiv:2211.12965](https://arxiv.org/abs/2211.12965))

$$\begin{aligned}
 C(\Delta z_a, \Delta r_i, \Delta r_j) = & \frac{2}{V_{\Delta z}} \int \frac{dk k^2}{2\pi^2} \left\langle \bar{b}^2 P_m(k) + \frac{1}{\bar{n}} \right\rangle_{\Delta z_a}^2 W_i(k) W_j(k) \\
 & + \frac{2}{V_{\Delta z_a} V_i} \int \frac{dk k^2}{2\pi^2} \left\langle \bar{b}^2 P_m(k) \left(\frac{1}{\bar{n}} \right)^2 \right\rangle_{\Delta z_a} W_j(k) \delta_{ij}^D \\
 & + \text{high order terms}(\propto B, T)
 \end{aligned}$$

Validated against **1000 Euclid-like lightcones** generated with LPT-based **PINOCCHIO** algorithm (Monaco et al. 2021)

Analytical model



Numerical matrix from simulations

COVARIANCE MATRIX MODEL

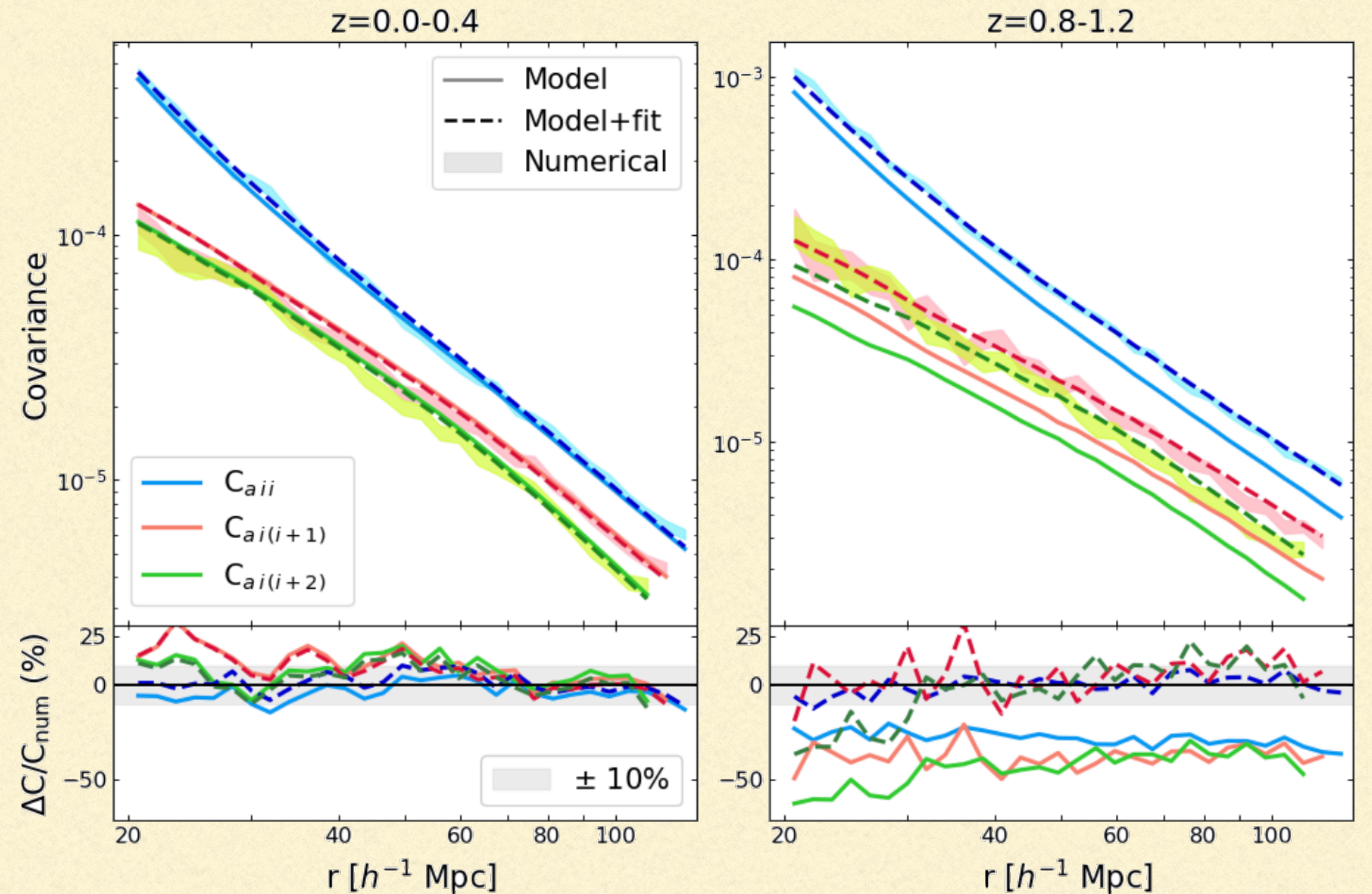
Euclid Collaboration: Fumagalli+22 ([arXiv:2211.12965](https://arxiv.org/abs/2211.12965))

$$C(\Delta r_i, \Delta r_j) = \frac{2}{V_{\Delta z}} \int \frac{dk k^2}{2\pi^2} \left\langle (\beta \bar{b})^2 P_m(k) + \frac{1+\alpha}{\bar{n}} \right\rangle_{\Delta z} W_i(k) W_j(k) \\ + \frac{2}{V_{\Delta z} V_i} \int \frac{dk k^2}{2\pi^2} \left\langle (\beta \bar{b})^2 P_m(k) \left(\frac{1+\gamma}{\bar{n}} \right)^2 \right\rangle_{\Delta z} W_j(k) \delta_{ij}^D$$

α, β, γ fitted from few ($\sim 10^2$) simulations following Fumagalli+22 ([arXiv:2206.05191v2](https://arxiv.org/abs/2206.05191v2))



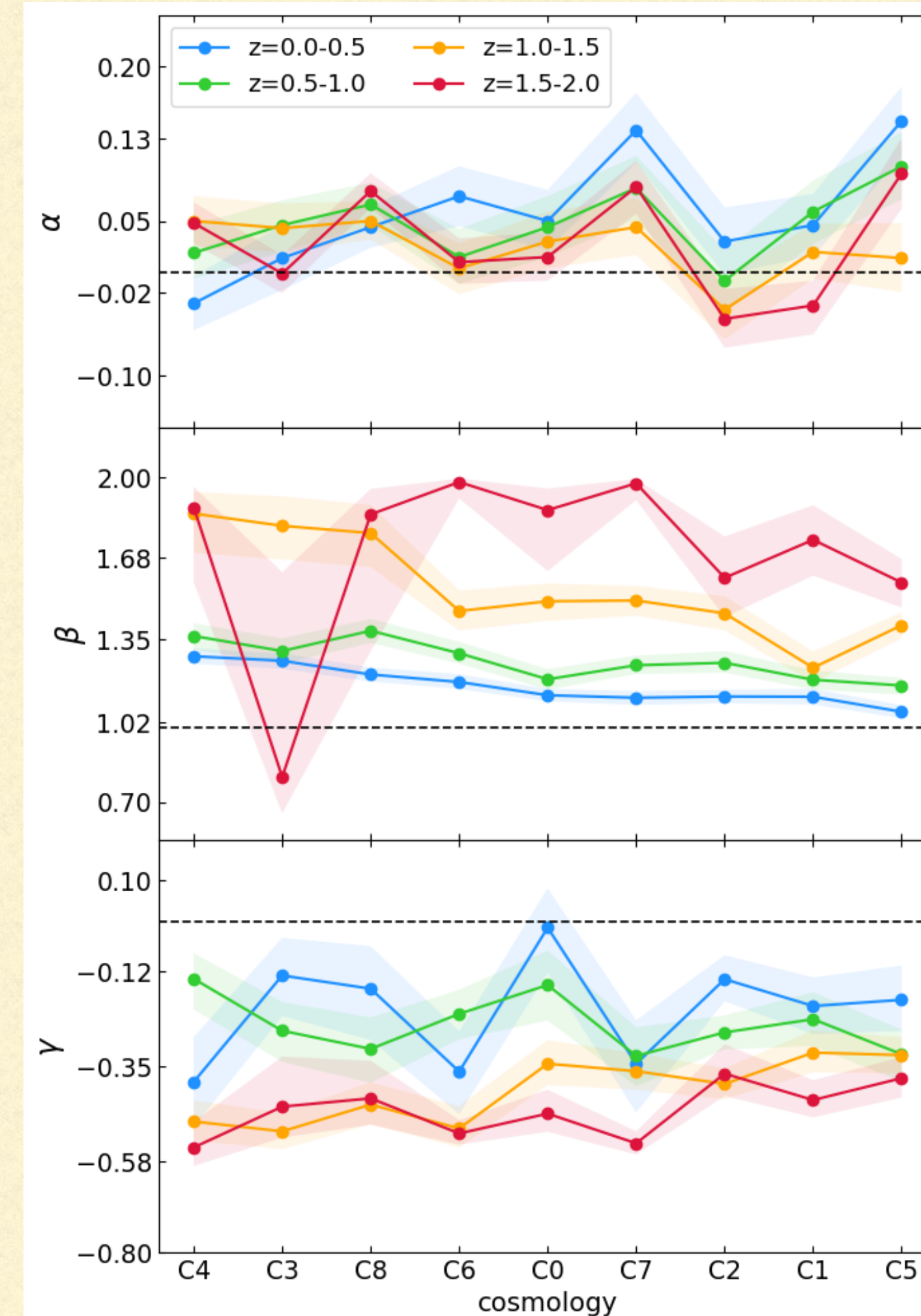
accurate, noise-free, cosmo-dependent covariance matrix



COVARIANCE FIT PARAMETERS

$$C(\Delta r_i, \Delta r_j) = \frac{2}{V_{\Delta z}} \int \frac{dk k^2}{2\pi^2} \left\langle (\beta \bar{b})^2 P_m(k) + \frac{1+\alpha}{\bar{n}} \right\rangle_{\Delta z}^2 W_i(k) W_j(k) \\ + \frac{2}{V_{\Delta z} V_i} \int \frac{dk k^2}{2\pi^2} \left\langle (\beta \bar{b})^2 P_m(k) \left(\frac{1+\gamma}{\bar{n}} \right)^2 \right\rangle_{\Delta z} W_j(k) \delta_{ij}^D$$

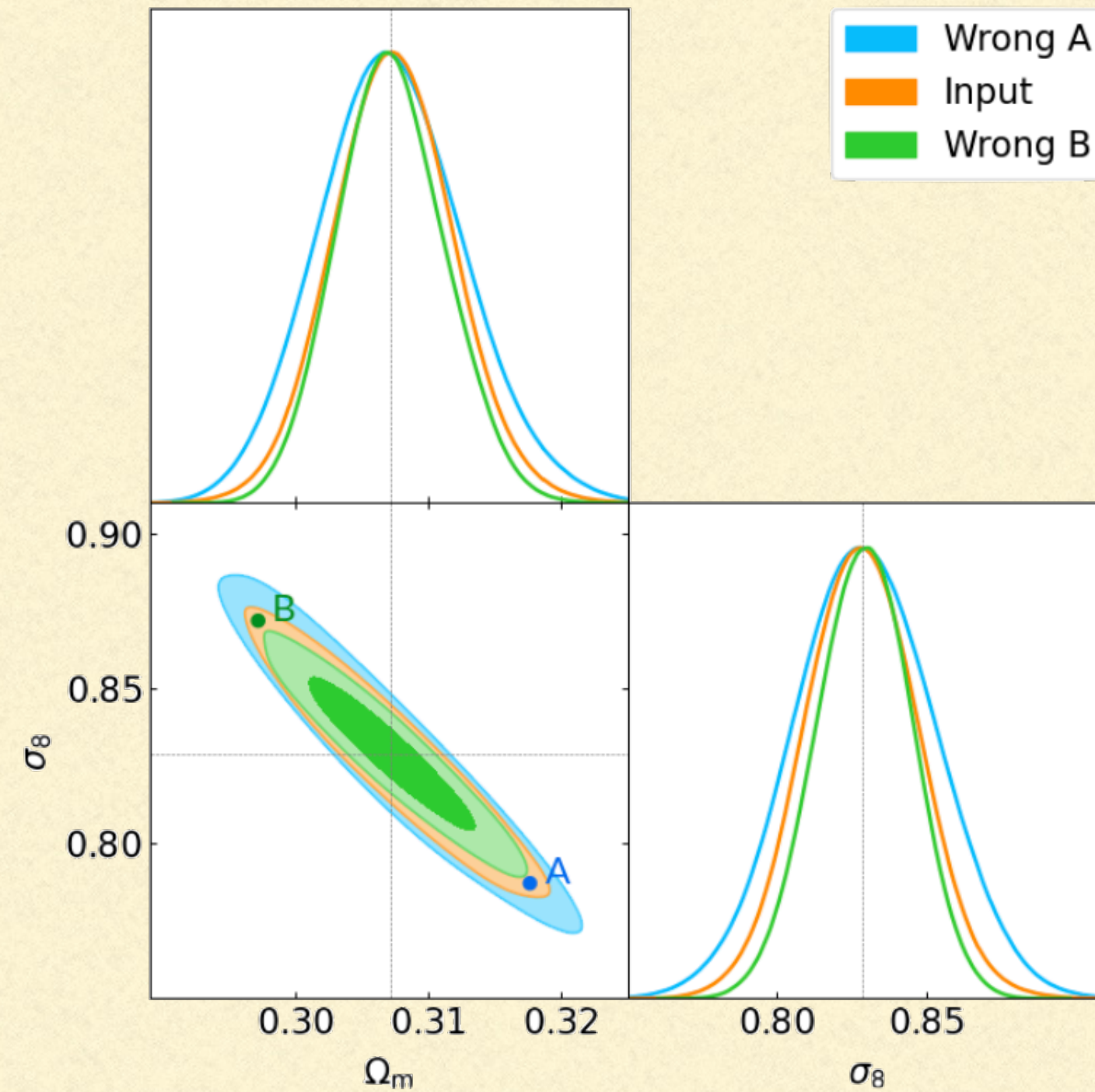
α, β, γ independent on cosmology



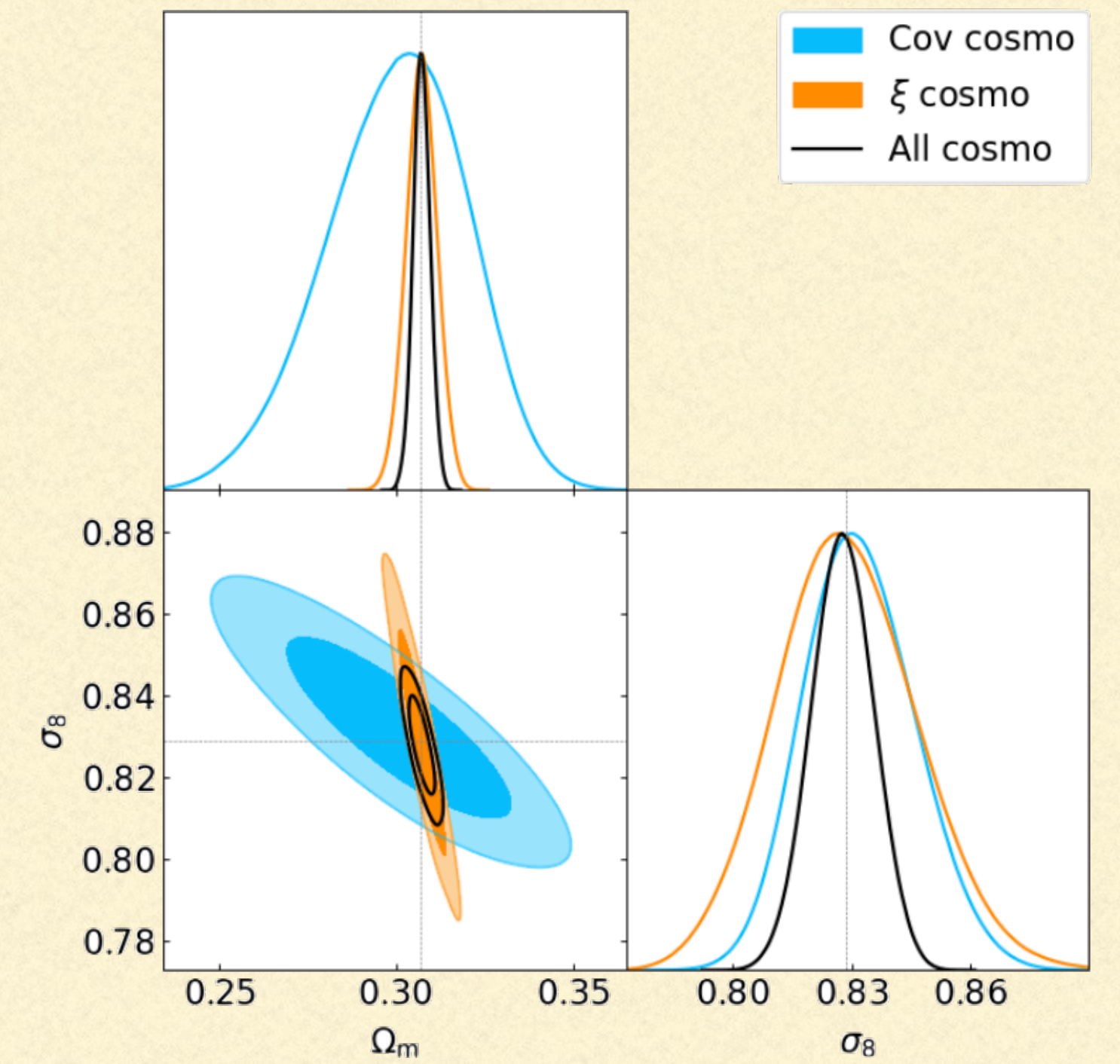
COSMO-DEPENDENT COVARIANCE

Covariance with different degeneracy on parameters w.r.t. ξ due to shot-noise \propto mass function

$$C(\Delta r_i, \Delta r_j) = \frac{2}{V_{\Delta z}} \int \frac{dk k^2}{2\pi^2} \left\langle (\beta \bar{b})^2 P_m(k) + \frac{1 + \alpha}{\bar{n}} \right\rangle_{\Delta z} W_i(k) W_j(k) + \frac{2}{V_{\Delta z} V_i} \int \frac{dk k^2}{2\pi^2} \left\langle (\beta \bar{b})^2 P_m(k) \left(\frac{1 + \gamma}{\bar{n}} \right)^2 \right\rangle_{\Delta z} W_j(k) \delta_{ij}^D$$



Euclid Collaboration: Fumagalli+22



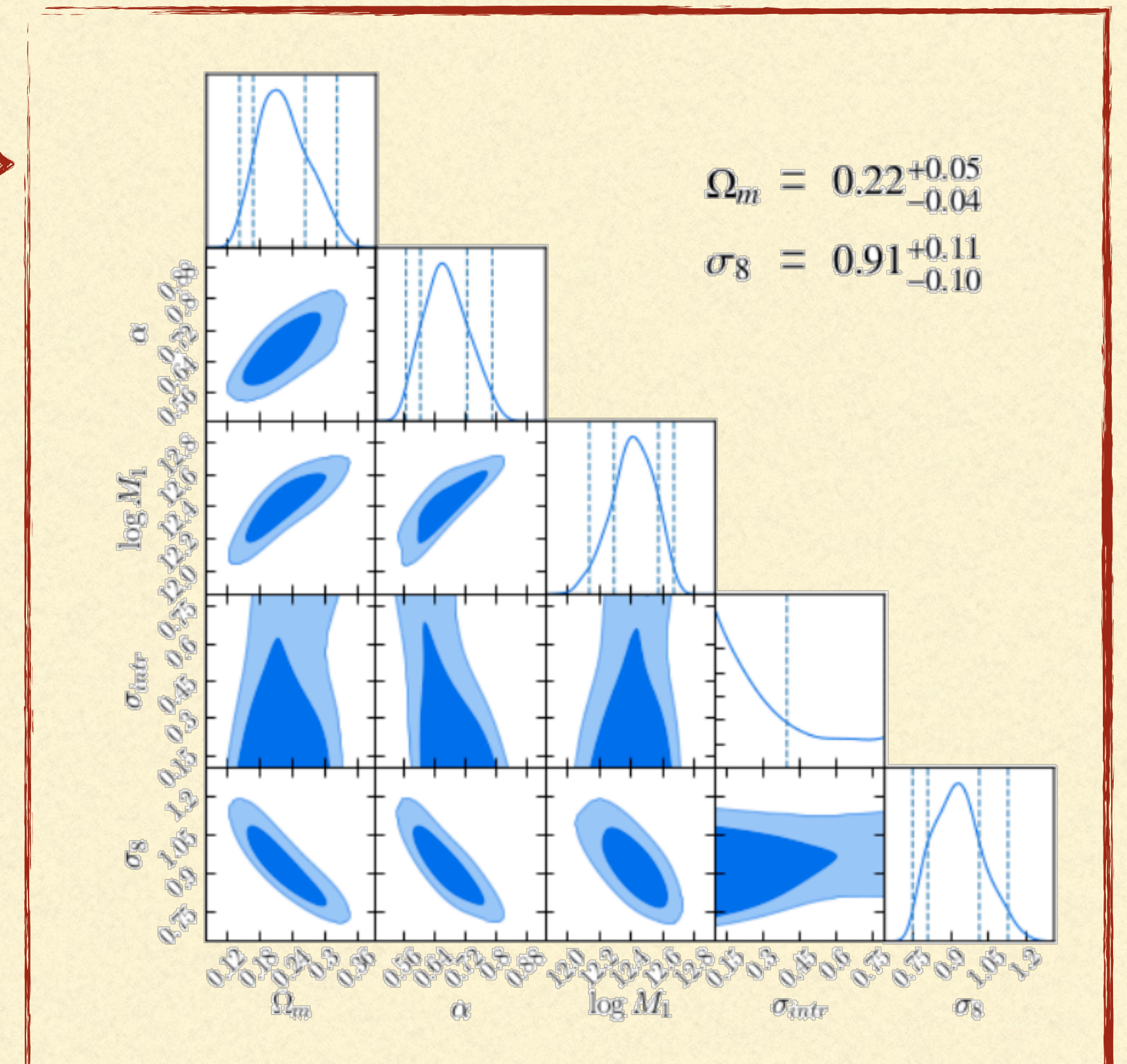
CLUSTER CLUSTERING ON REAL DATA: SDSS

Cosmological and richness-mass relation constraints from:

- cluster abundance and weak lensing mass (Costanzi et al. 2019)
- cluster abundance and weak lensing mass
+ 3D cluster clustering

Cosmological constraints from abundance, weak-lensing and clustering of galaxy clusters: application to the SDSS, [arXiv:2310.09146](https://arxiv.org/abs/2310.09146)

A. Fumagalli, M. Costanzi, A. Saro, T. Castro, S. Borgani



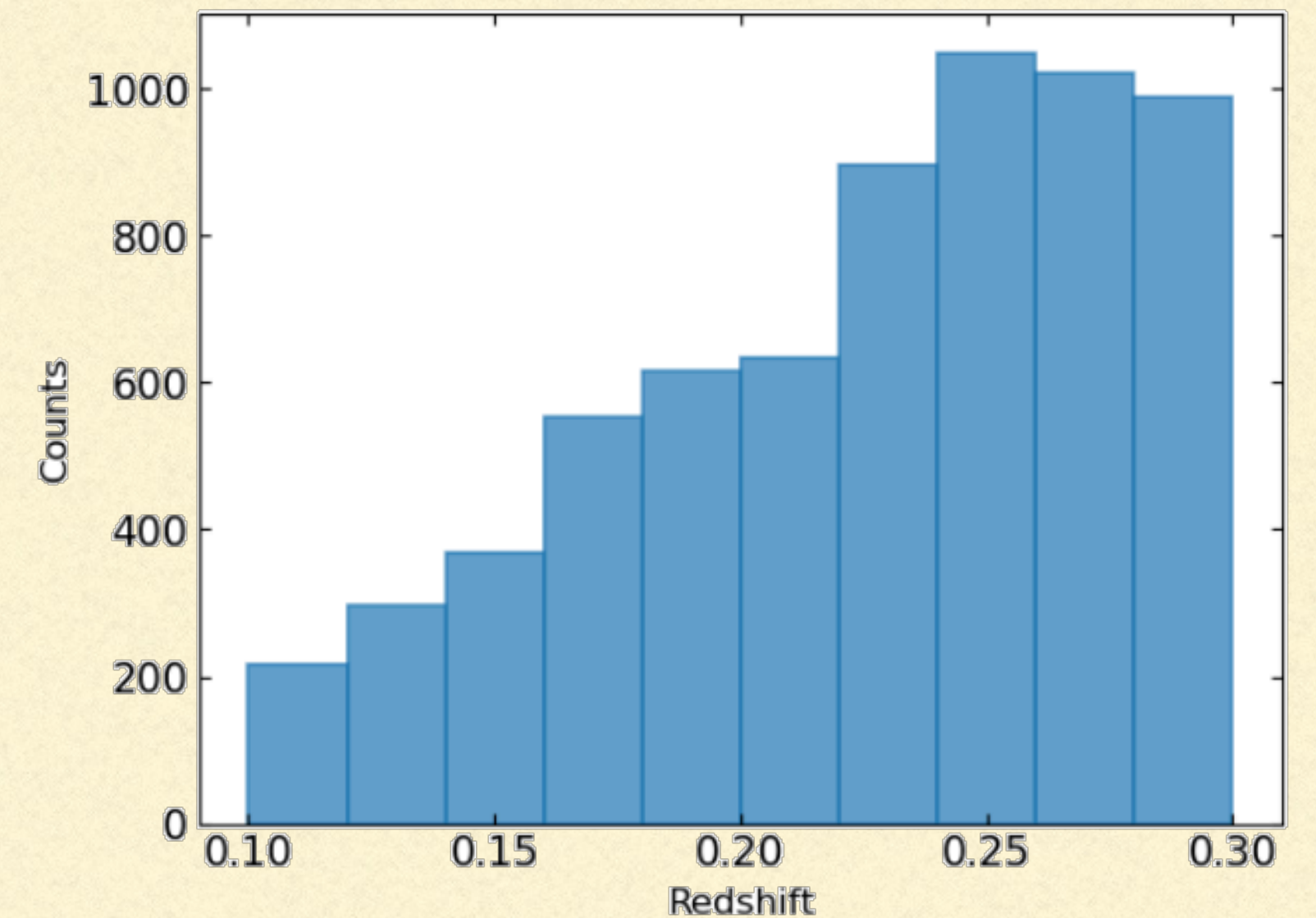
DATASET

Data: redMaPPer cluster catalog (Rozo+15) from **SDSS DR8** (Aihara+11)

- **6964** photometrically-selected clusters
- Sky area of **10 000 deg²**
- Richness range $\lambda \geq 20$
- Redshift range $z = 0.1 - 0.3$
- Photo-z uncertainty $\sigma_z/(1+z) \lesssim 0.01$
($\sigma_z \approx 0.005$ at $z = 0.15$ to $\sigma_z \approx 0.014$ at $z = 0.3$)

Stacked WL mass profiles from Simet+17

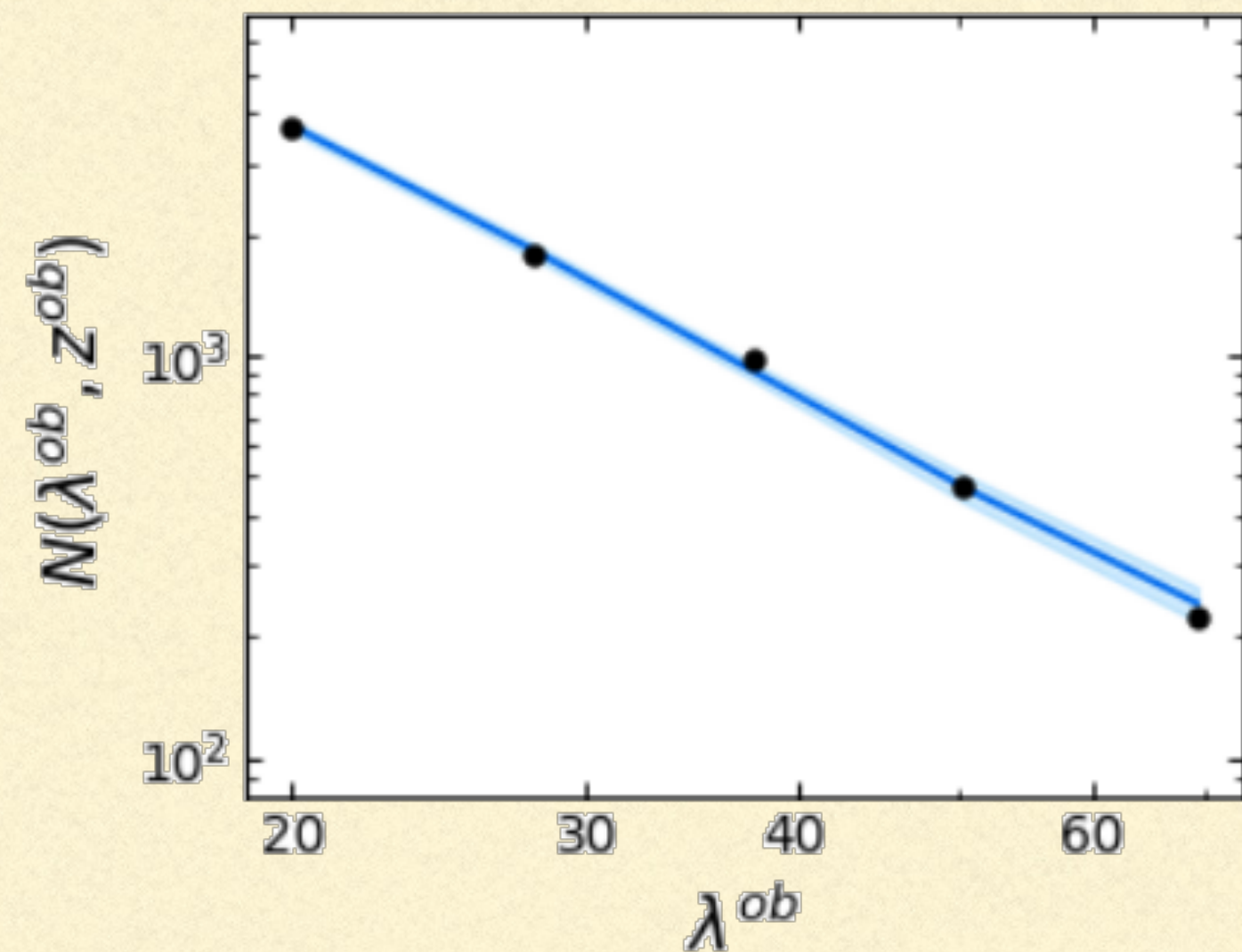
- ~39 million galaxies
- ~9000 deg² of SDSS footprint (Reyes+12) from SDSS DR8



COSMOLOGICAL PROBES

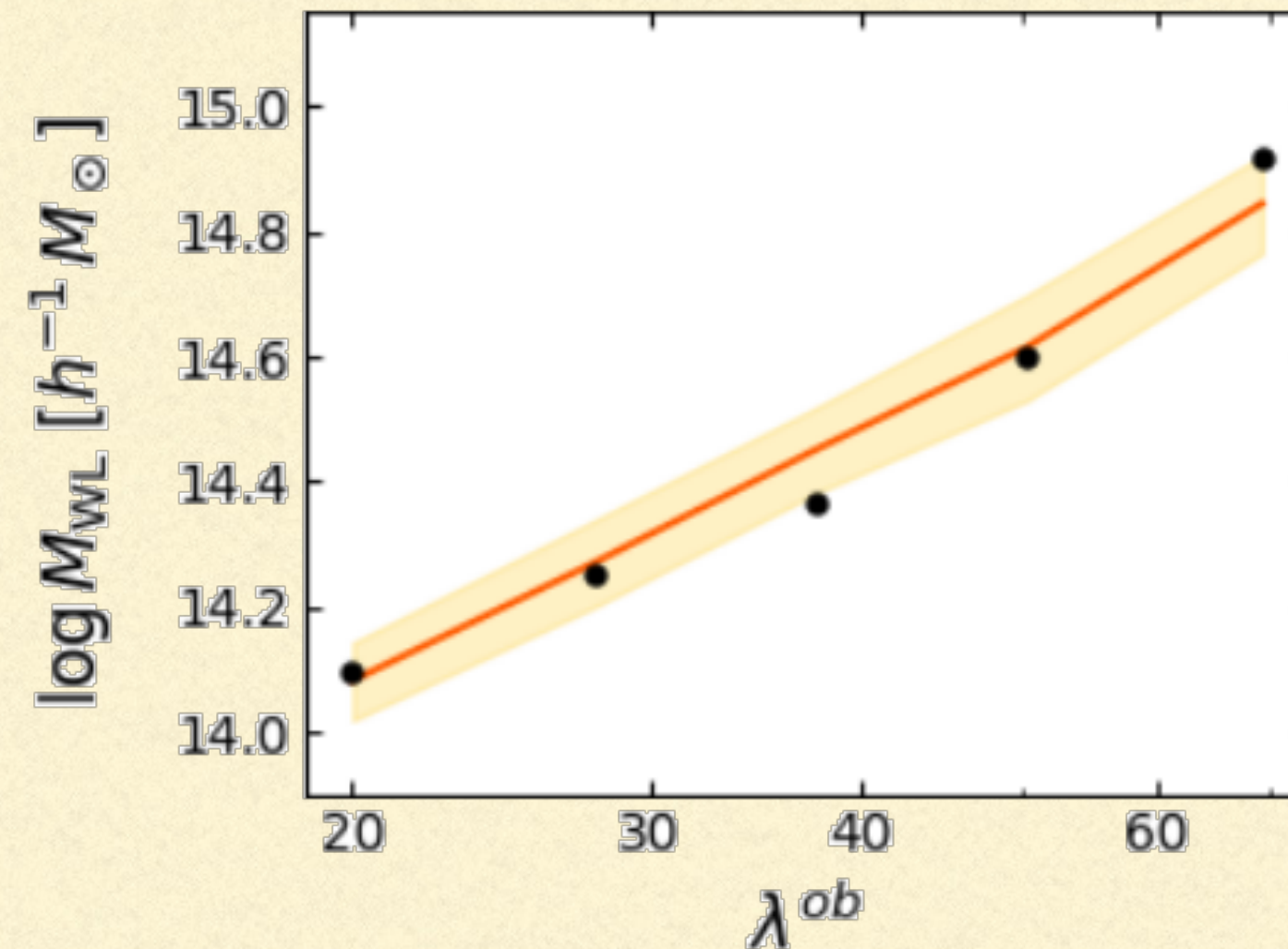
Cluster abundance

$$N(\Delta\lambda_j^{\text{ob}}) = \int dz^{\text{tr}} \Omega_{\text{mask}}(z^{\text{tr}}) \frac{dV}{dz d\Omega}(z^{\text{tr}}) \times \langle n(z^{\text{tr}}, \Delta\lambda_j^{\text{ob}}) \rangle \int_{\Delta z} dz^{\text{ob}} P(z^{\text{ob}} | z^{\text{tr}})$$



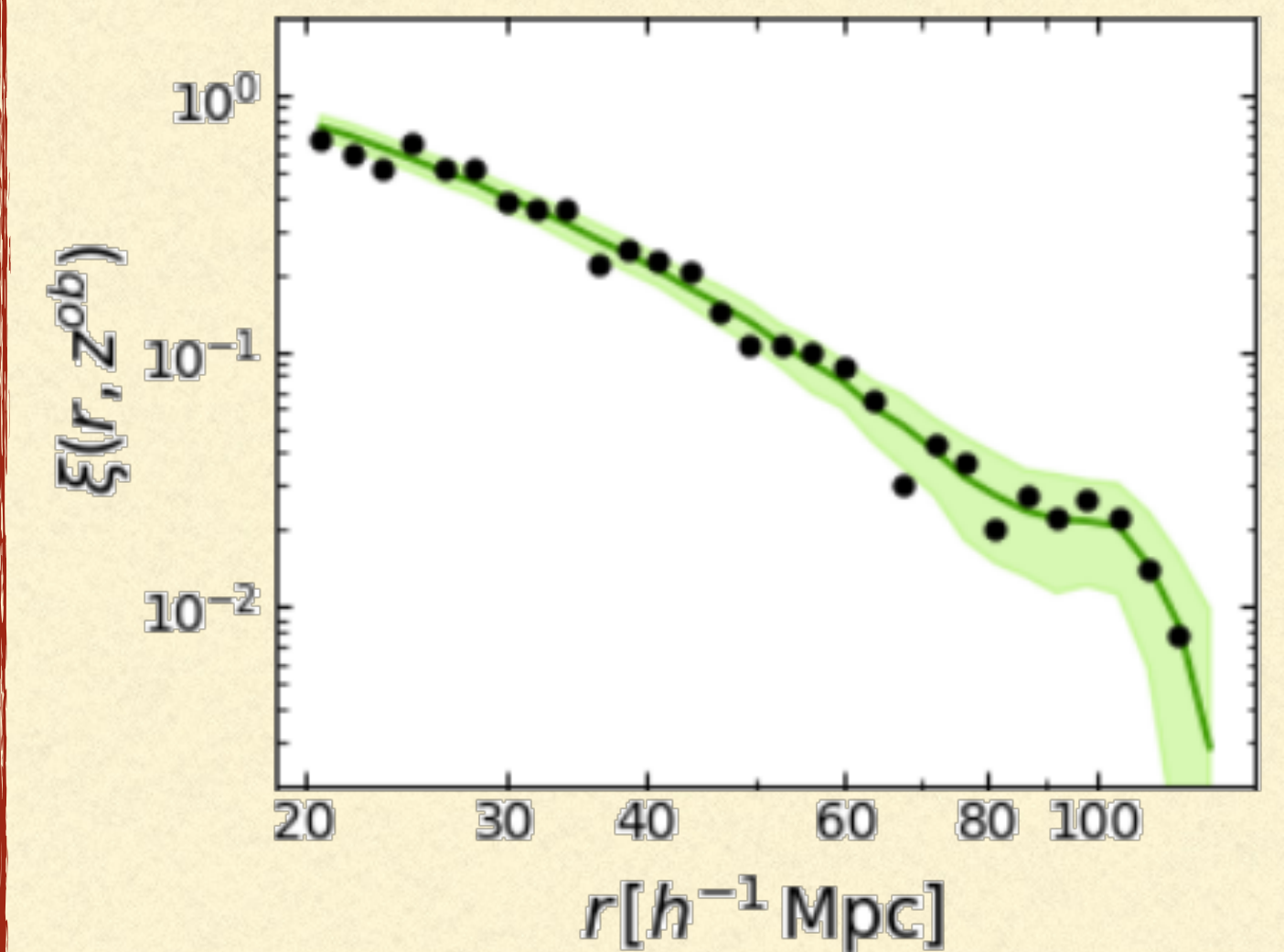
Weak lensing mass

$$\bar{M}(\Delta\lambda_j^{\text{ob}}) = \frac{M^{\text{tot}}(\Delta\lambda_j^{\text{ob}})}{N(\Delta\lambda_j^{\text{ob}})}$$



3D cluster clustering

$$\xi_{\text{h}}(\Delta r_i) = \int \frac{dk k^2}{2\pi^2} \langle \bar{b}^2 P_{\text{m}}(k) \rangle_{\Delta z} W_i(k)$$



LIKELIHOOD SETUP

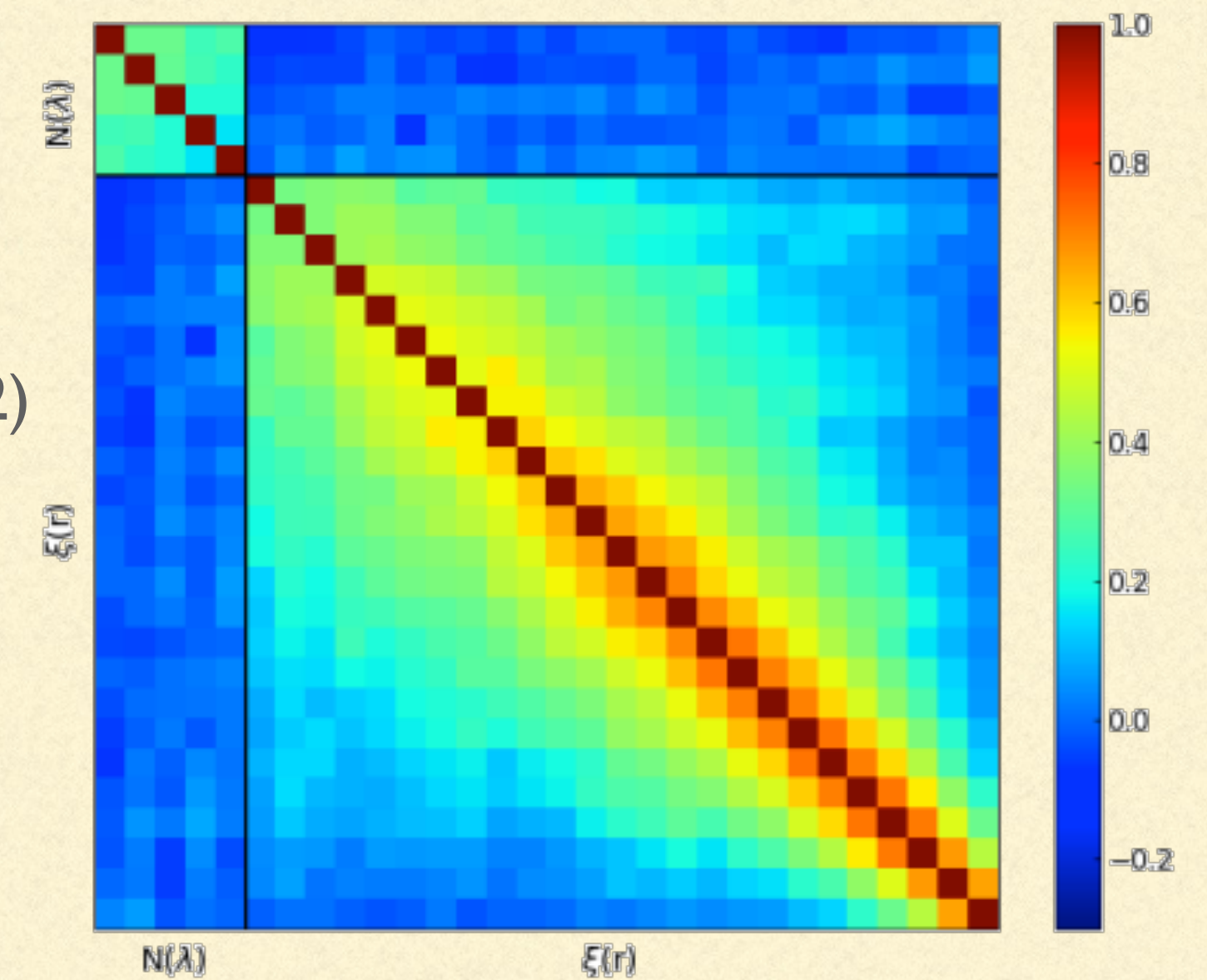
$$\mathcal{L}(\mathbf{d} | \boldsymbol{\theta}) = \frac{\exp \left\{ -\frac{1}{2} [\mathbf{d} - \mathbf{m}(\boldsymbol{\theta})]^T \mathbf{C}^{-1} [\mathbf{d} - \mathbf{m}(\boldsymbol{\theta})] \right\}}{\sqrt{(2\pi)^N |\mathbf{C}|}}$$

Likelihood:

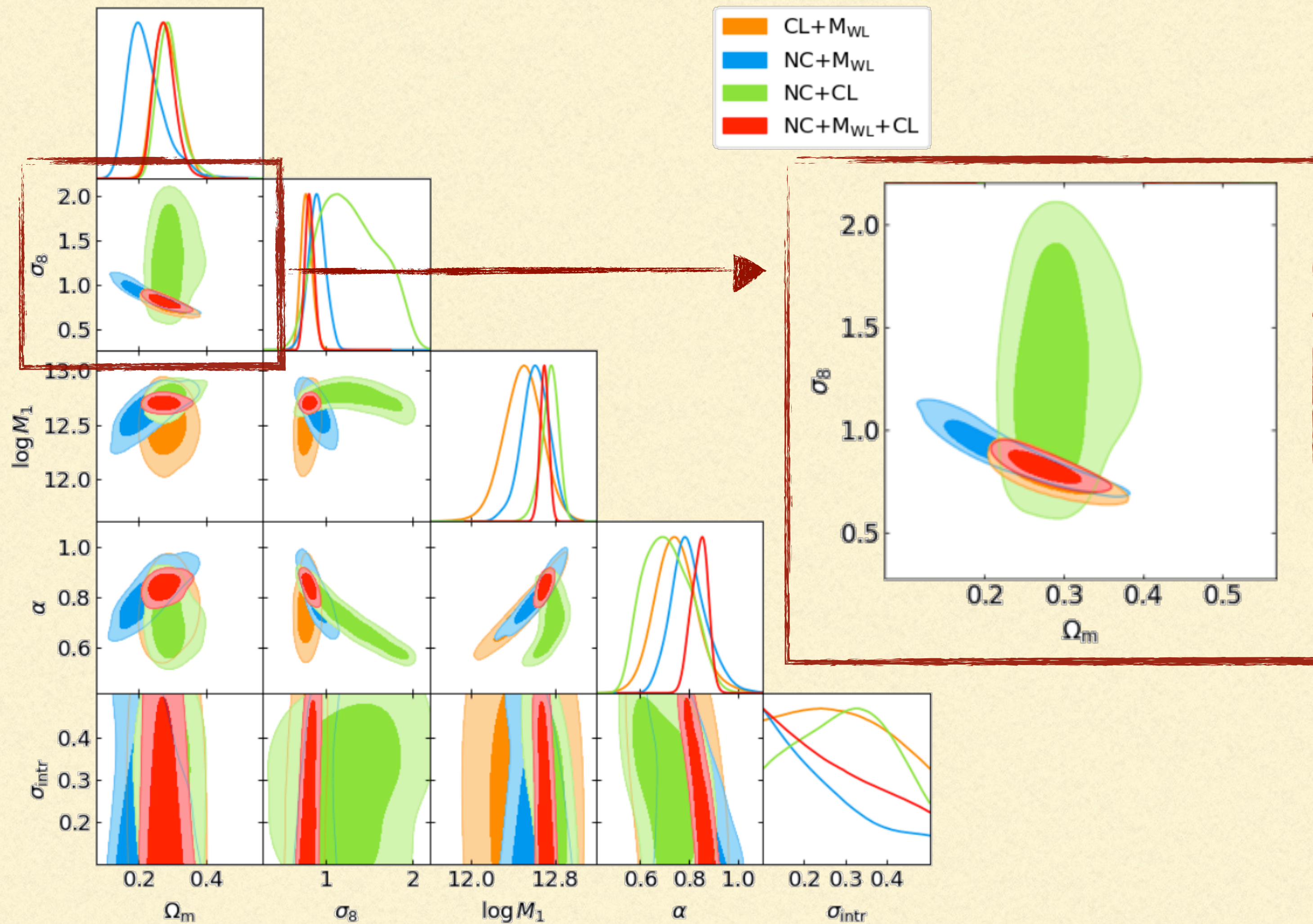
- Three **independent Gaussian likelihoods**
- Analytical **cosmo-dependent covariance** for counts (EC:Fumagalli+21)
- Analytical **cosmo-dependent covariance** for clustering (EC:Fumagalli+22)
- Numerical fixed covariance for WL masses (Costanzi+19)

Free parameters:

- $\Omega_m, A_s (\sigma_8), h, \Omega_v h^2, \Omega_b h^2, n_s, \alpha, M_l, M_{\min}, \sigma_{\text{intr}}, S_{\text{hmf}}, q_{\text{hmf}}$
- cosmology
scaling relation
model calibration

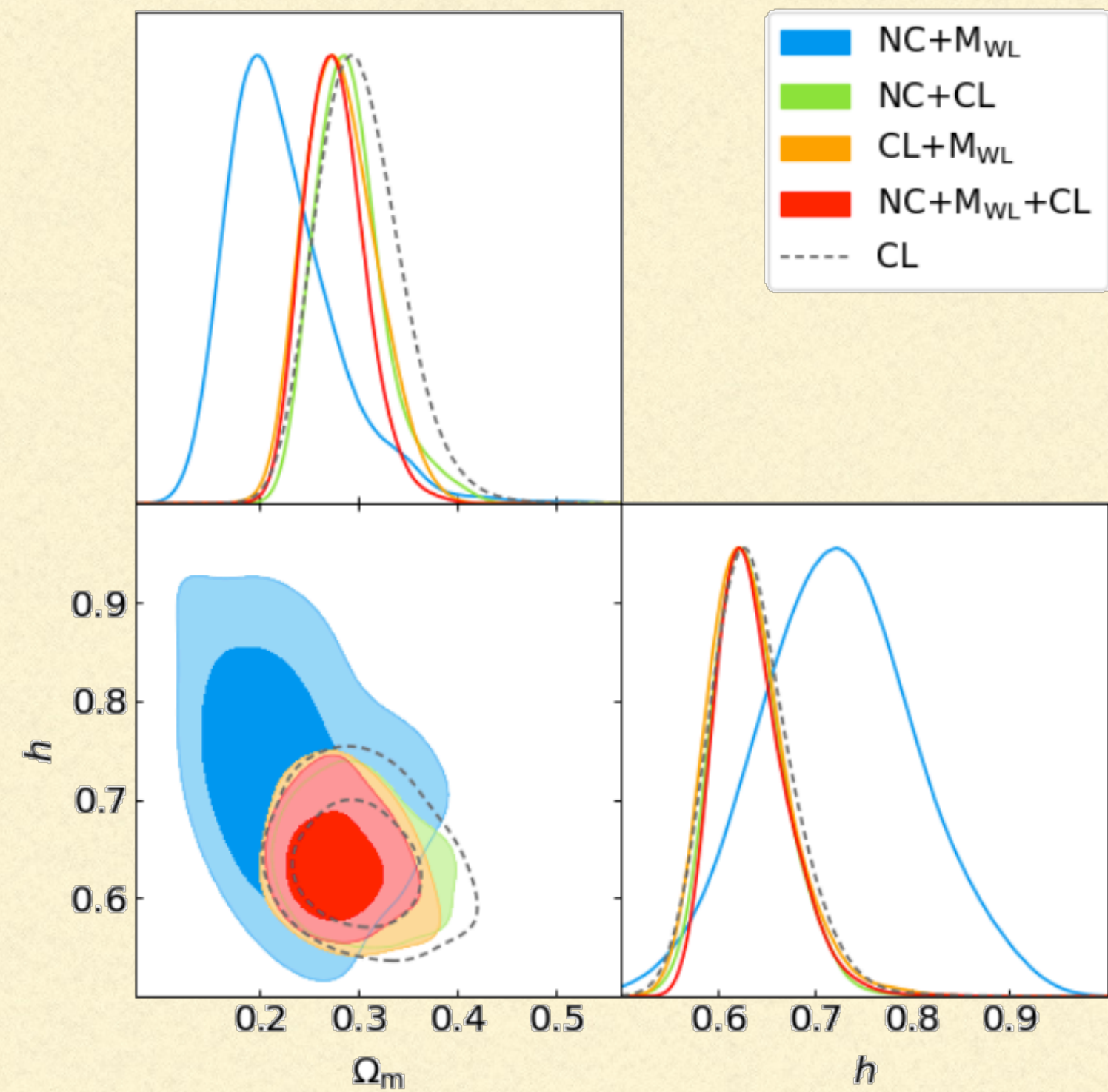


RESULTS: PARAMETER CONSTRAINTS



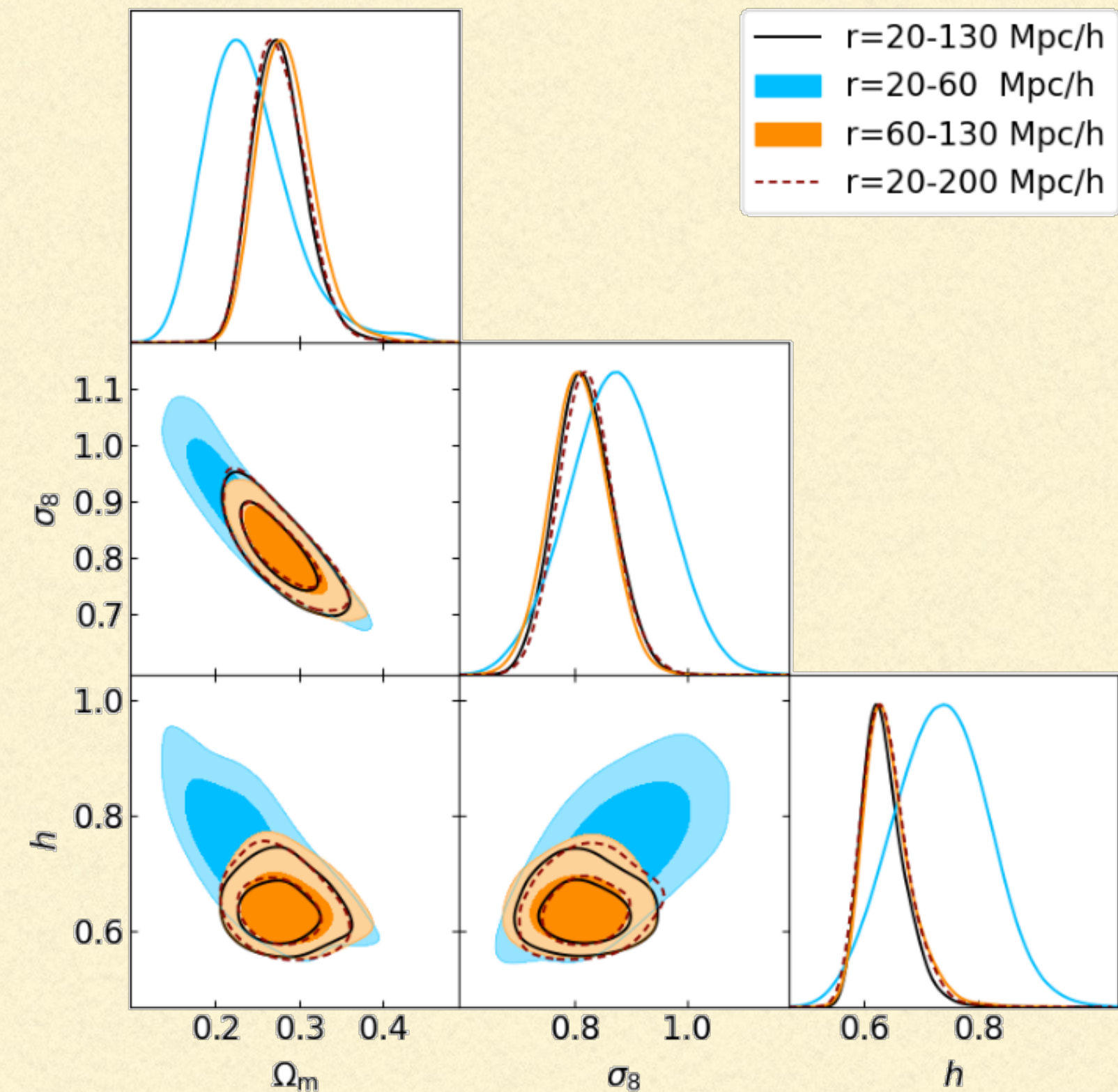
- CL helps to constrain Ω_m
- NC+CL don't constrain σ_8
- M_{WL}+CL better than NC+M_{WL}
- high improvement from NC+CL+M_{WL}

RESULTS: PARAMETER CONSTRAINTS

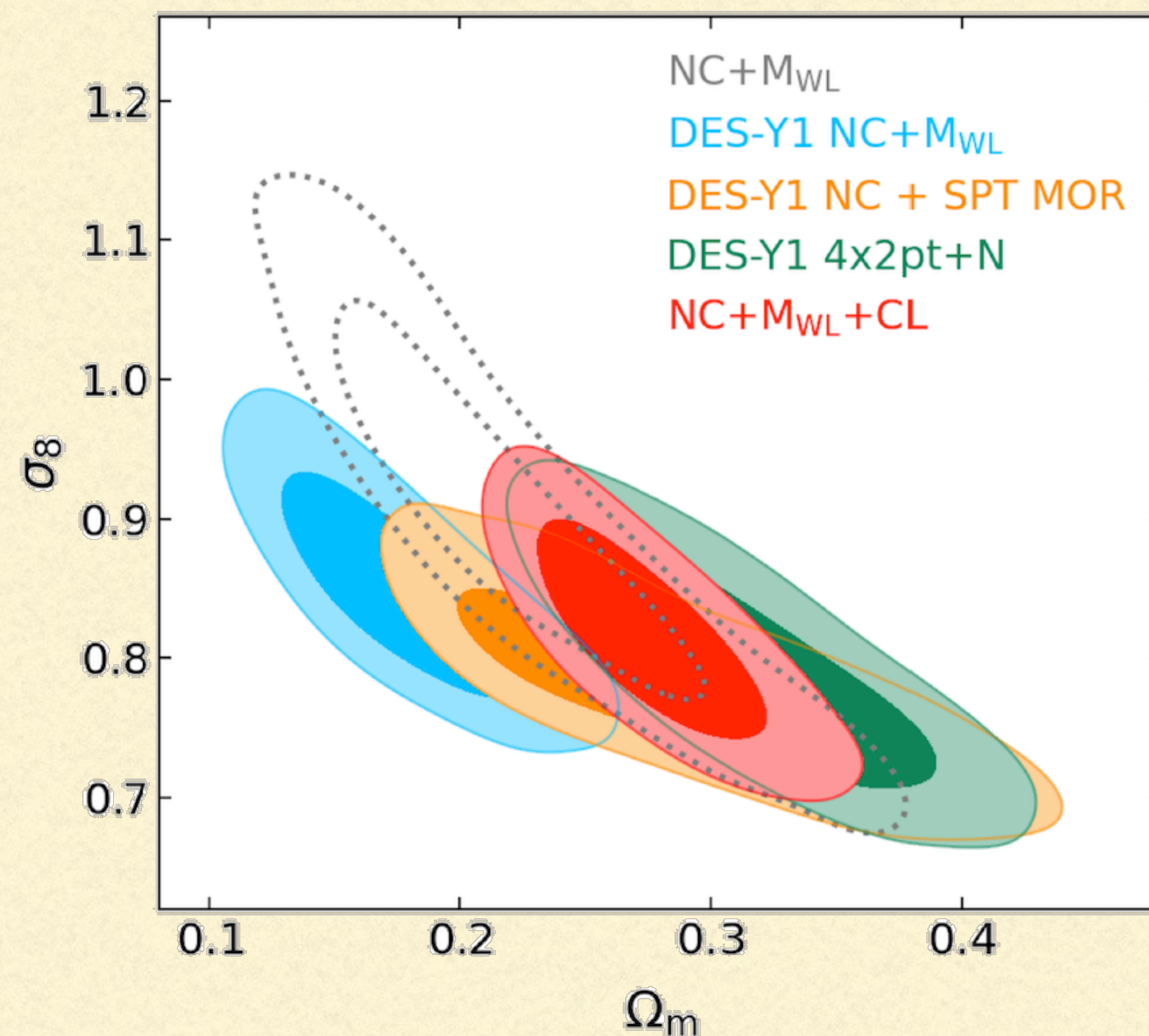
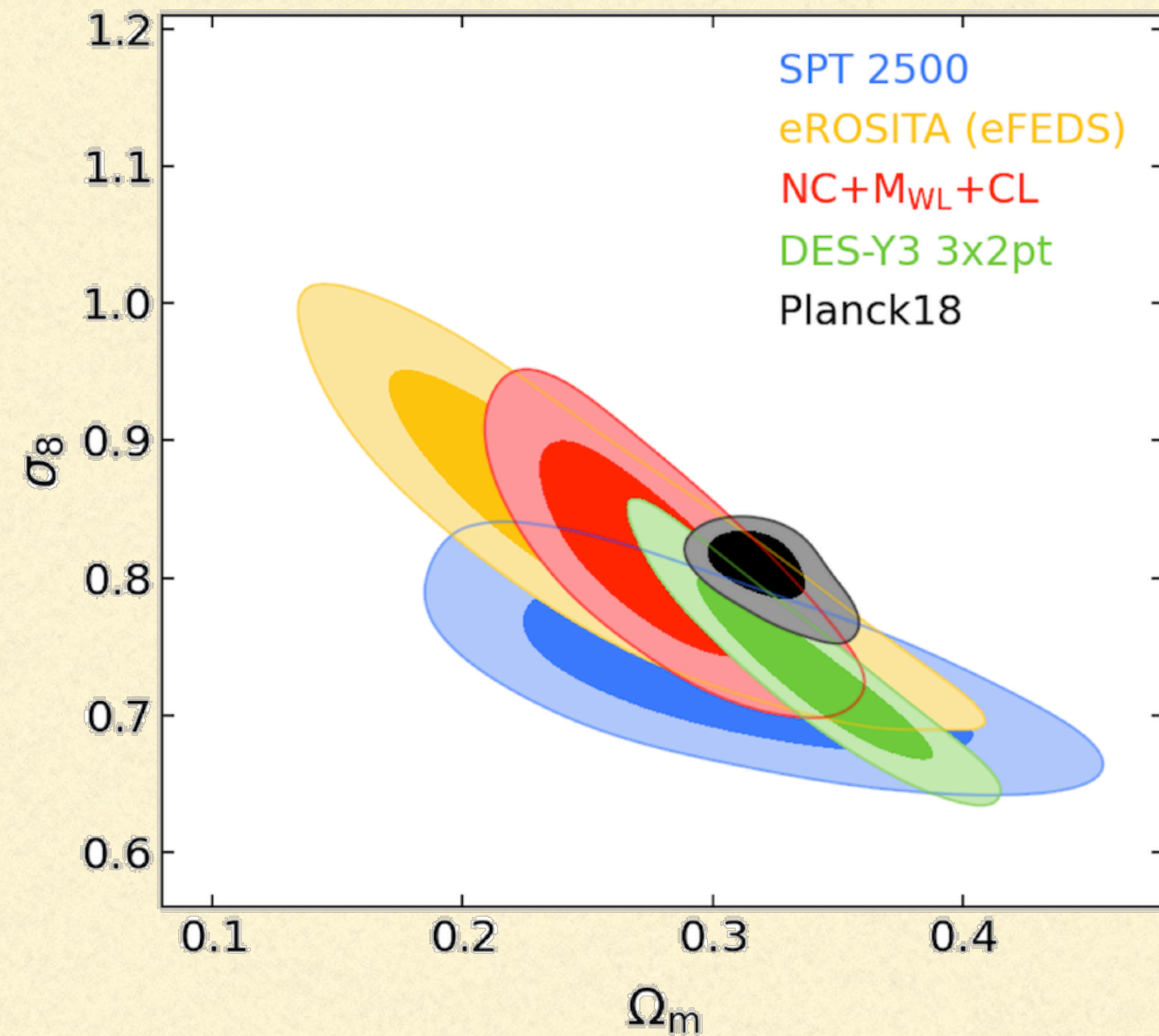


Cluster clustering helps to constraints the Hubble parameter ($h = 0.64 \pm 0.04$)

Information mainly from the BAO scales (~ 110 Mpc/h)



RESULTS: LITERATURE COMPARISON



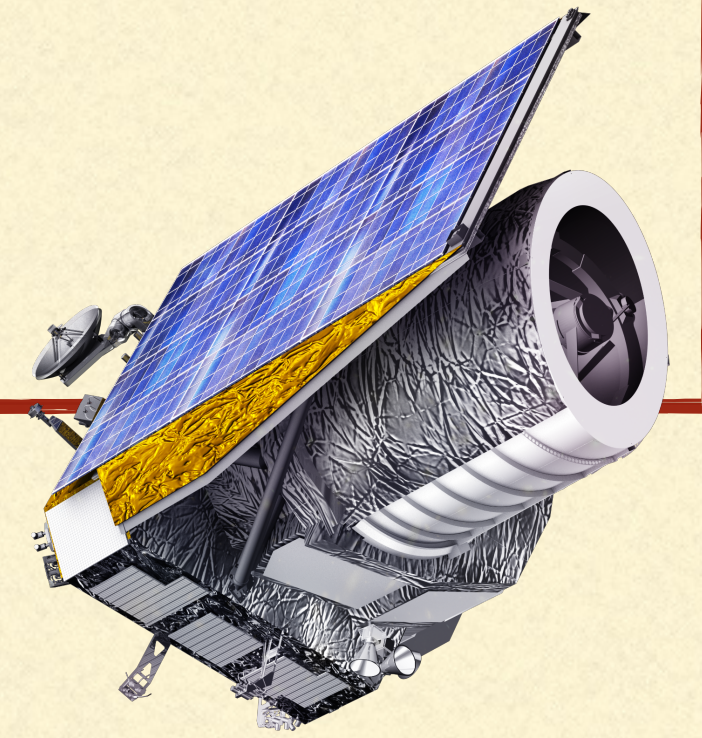
- Consistent with other cluster and non-cluster surveys
- Tension with DES-Y1 NC+M_{WL} (different WL modelling \Rightarrow different scaling relation)
- Competitive constraining power
- Tighter than DES-Y1 4x2pt (different CL and WL scales)

CONCLUSIONS

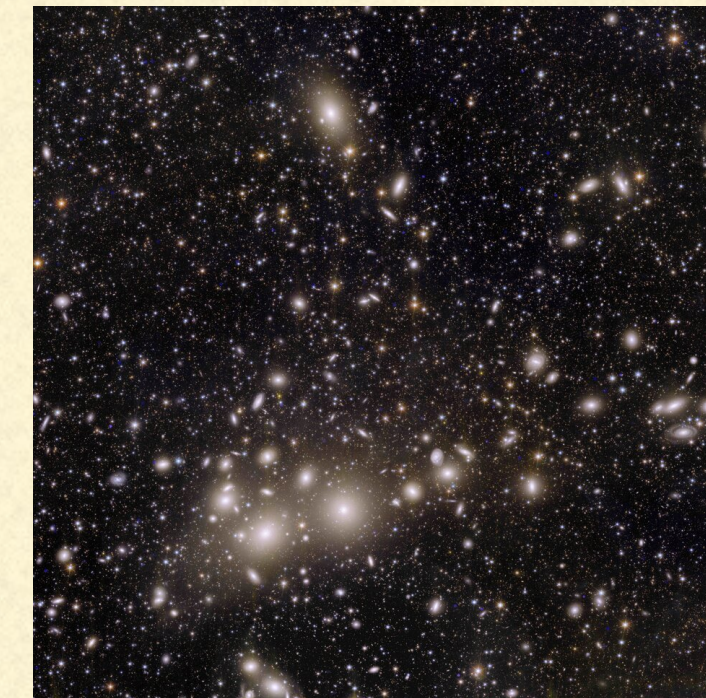
- Cluster clustering helps to constrain **cosmological parameters** (mainly Ω_m) and **scaling relations**, already from currently available catalogs (e.g. SDSS)
- Constraints from cluster cosmology including cluster clustering are **competitive** with other cosmological probes
- **Future surveys** (Euclid, LSST, ...) will allow us to better exploit cluster clustering information (redshift/richness binning, lower photo-z uncertainties,..)
- More information can be extracted by extending the analysis to **quadrupole and hexadecapole** and/or **higher-order statistics**

BACKUP SLIDES

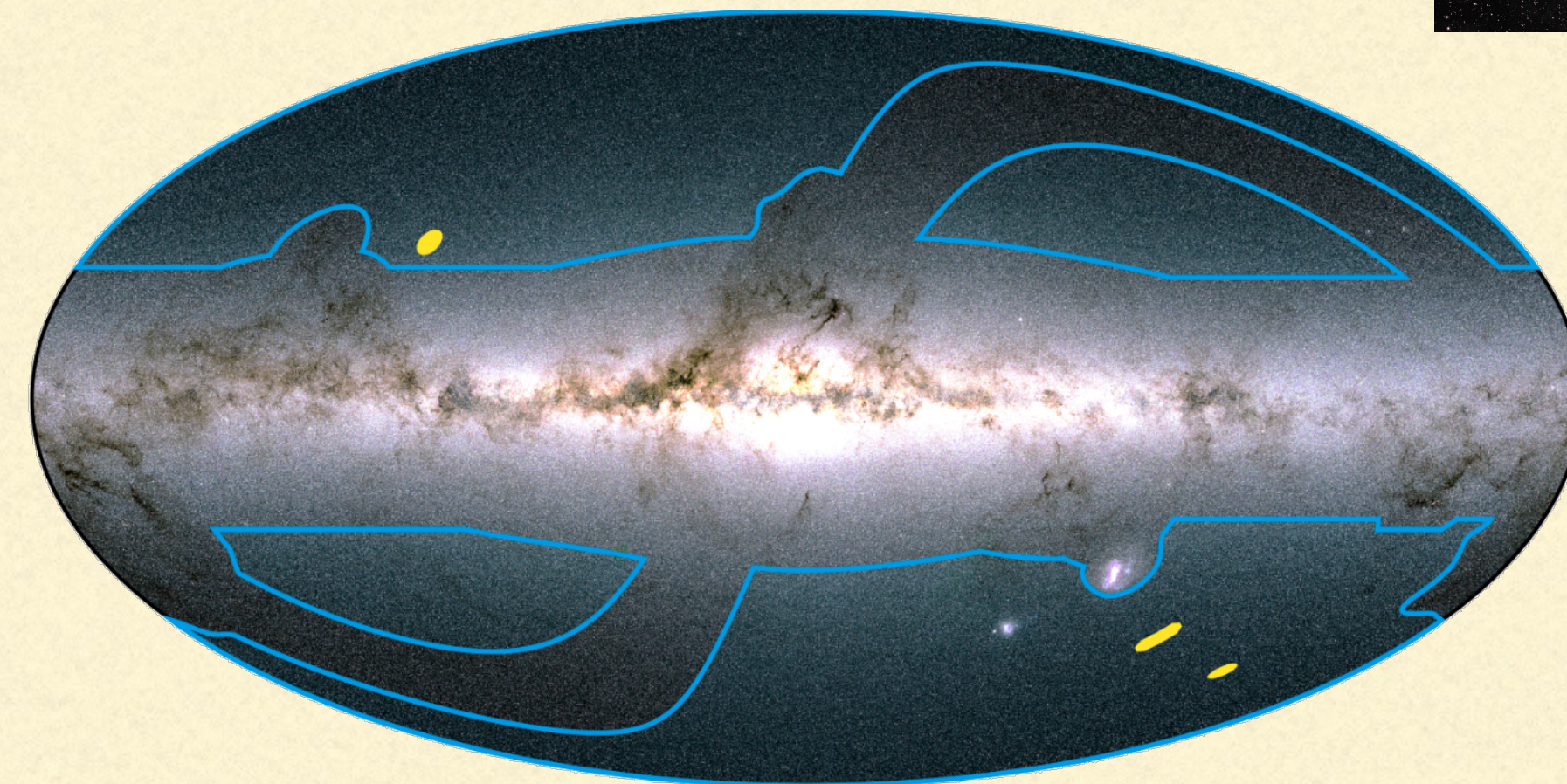
EUCLID



- **Visible/near-infrared** space telescope by the European Space Agency (**ESA**)
- Launched on July 1, 2023
- Two major surveys:
 - **Euclid Wide Survey**: 15 000 deg² of the extra-galactic sky
 - Euclid Deep Survey: ~53 deg² split over three fields
- Primary probes: weak lensing and galaxy clustering
Secondary probes: **galaxy clusters**, strong lensing, ...



- Redshift range $z = 0.2 - 2$
- Mass range $M > 1.0 \times 10^{14} M_{\odot}$
- Number of clusters $\geq 10^5$
- Cluster detected via photometric data, spectroscopic data, weak gravitational lensing



SIMULATIONS

Covariance matrices requires **large sets of simulations ($\sim 10^3$)**:

- not feasible with N-body simulations due to high computational costs
- approximate methods: less accurate but faster

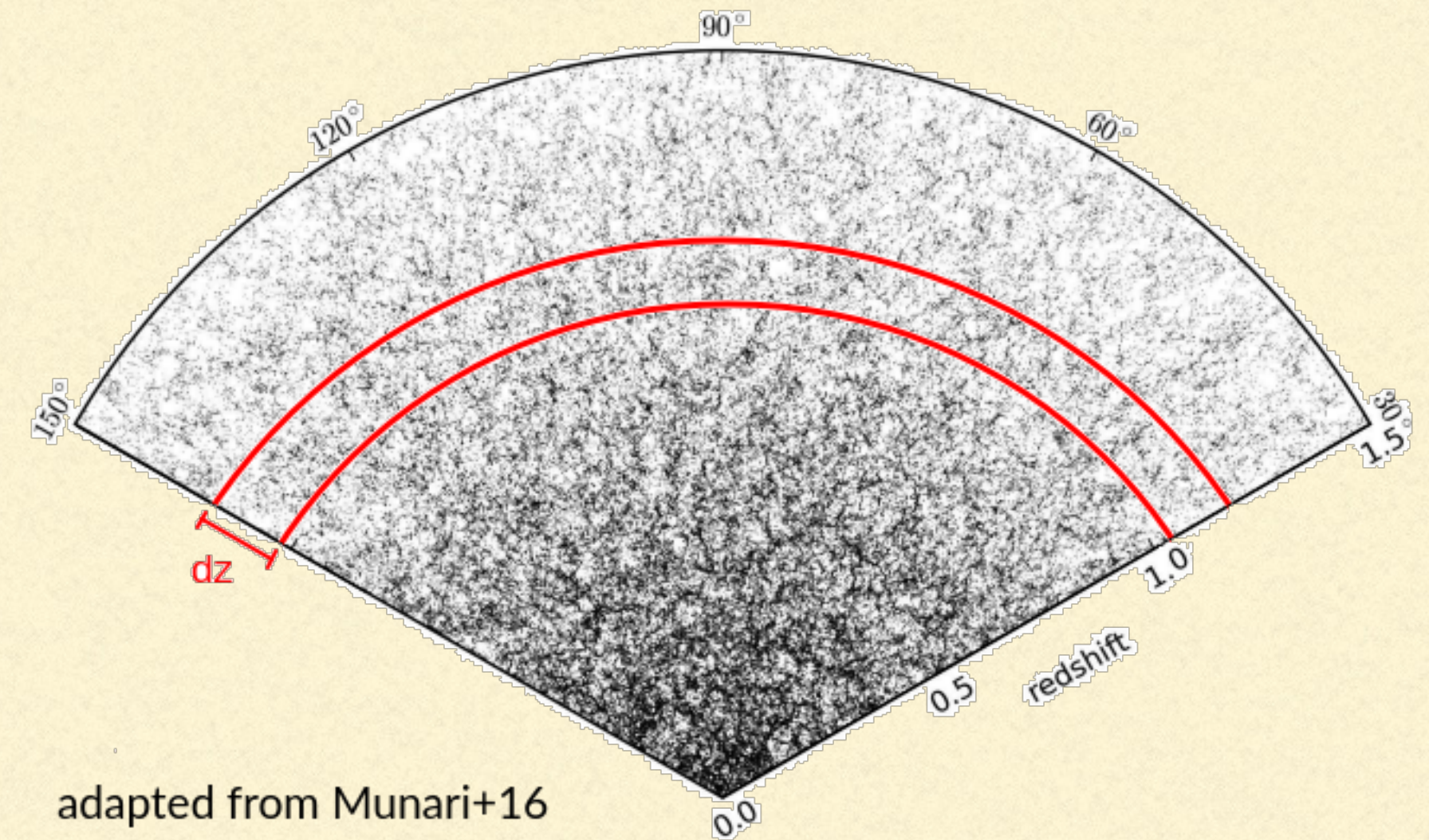
In this work (Euclid Collaboration: Fumagalli+22):

dark matter halo catalogs from cosmological simulations generated with LPT-based methods by the **PINOCCHIO** algorithm



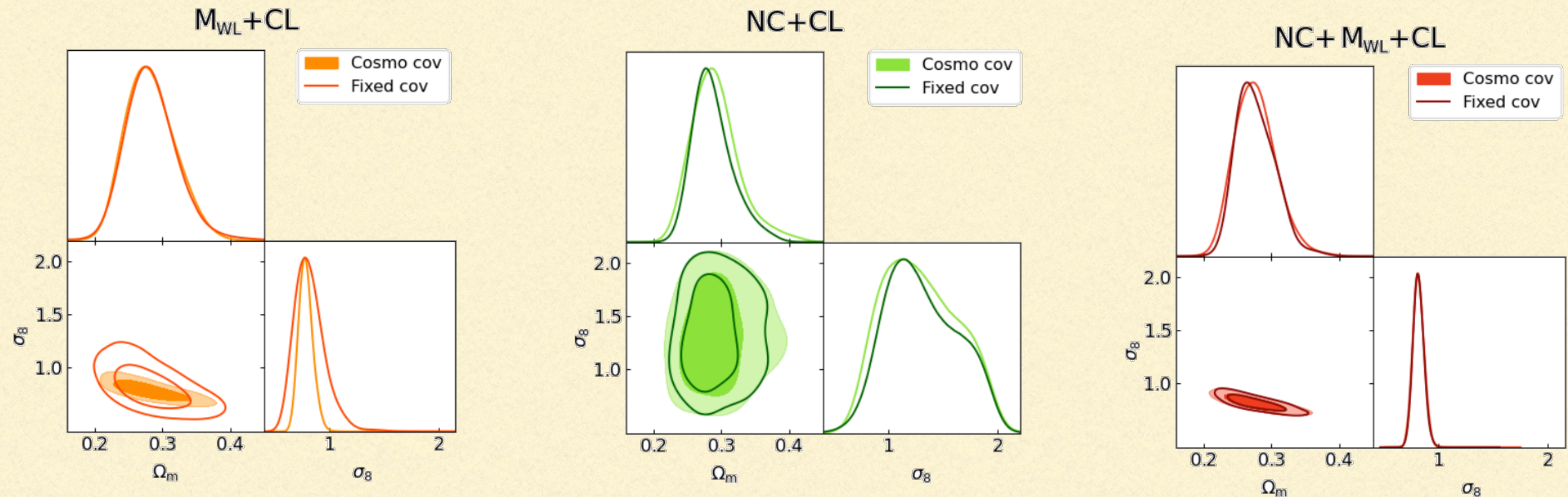
1000 Euclid-like lightcones with

- Redshift range $z = 0 - 2$
- Mass range $M_{\text{vir}} \geq 5 \times 10^{13} M_{\odot}/h$
- Number of objects $\sim 3 \times 10^5$



adapted from Munari+16

RESULTS: COSMO-DEPENDENT COVARIANCE



- ~ 100% improvement
- Information from shot-noise (integrated HMF)
- Only brings information on σ_8

- No difference between fixed and cosmo-dep covariance
- Information already extracted from number counts data
- No further tightening (= no double count of information)